AN INVENTORY MODEL FOR DETERIORATING ITEMS WITH TWO LEVELS OF STORAGE AND DISPLAYED STOCK-DEPENDENT DEMAND RATE

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ABSTRACT

Most of the consumer goods are subject to deterioration thus, we cannot ignore the fact that an item in stock will remain fully usable for satisfying the future demand in a perfect condition. Also with the boom of retail sector it can be generalized that the presence of a larger quantity of goods displayed may attract more customers. Above situation has been extended to two warehousing problem, where there is limited display space and the units are kept in backroom or warehouse. Furthermore, if the units demanded are consumed from backroom initially, the stock in display area will remain intact and hence the demand remains stable. Considering this situation, the problem is to determine the optimal lot size, when the demand is dependent on the displayed stock level. The model is formulated by assuming that the demand rate is a linear form of current inventory level. The results have been validated with the help of a numerical example. Sensitivity analysis has also been performed to study the impact of various parameters on the optimal solution.

***Keywords:* Displayed Stock-Dependent Demand, Deterioration, Inventory, Two-Warehouse System.**

I INTRODUCTION

The deterioration of inventory in stock during the storage period constitutes an important factor, which has attracted the attention of researchers. Deterioration, in general, maybe considered as the result of various effects on the stock, some of which are damage, spoilage, obsolescence, decay, decreasing usefulness, and many more. The first attempt to obtain optimal replenishment policies for deteriorating items was made by Ghare and Schrader [1], who derived a revised form of the economic order quantity (EOQ) model assuming exponential decay. Thereafter, a great deal of research efforts has been devoted to inventory models of deteriorating items.

Classical inventory models usually consider the consumption rate to be constant or time varying. However, it has been noted that classical models are unsuitable for representations of the reality of inventory control situation in the retail sector. As observed by Silver and Peterson [2], sales at the retail level tend to be proportional to inventory displayed and a large piles of goods displayed in a retail store will lead the customers to buy more. This implies that holding higher inventory level will probably make the retailer sell more items. Gupta and Vrat [3] first developed a model for consumption environment to minimize the cost with the assumption that consumption rate is a function of the initial stock level. Mandal and Phaujdar [4] then developed an inventory model for deteriorating items with uniform rate of production and linearly stock dependent demand. Some of the researchers who extensively examined these models are Giri and Chaudhuri [5], Chung [6], and others.

Moreover, the general assumption in inventory models is that the organization owns a single warehouse without capacity limitation. But, in practice, when a large stock is to be held, due to the limited capacity of the owned warehouse/display area (denoted by DA), one additional rented warehouse/backroom (RW/BR) is required. This additional warehouse may be a rented warehouse which is assumed to be available with abundant capacity. Zhou and Yang [7] developed a two warehouse inventory model for items with stock level dependent demand rate, where the stock in RW is transported to OW in an intermittent pattern. Research continues with Zhou [8], Chung and Huang [9], Das et al. [10], Dye et al. [11], Dey et al. [12], Hsieh et al. [13], Niu and Xie [14] and many more.

Taking into consideration the above mentioned aspects, i.e., deteriorating items, displayed stock dependent demand, and considering that the amount on shelf/ display area (DA) is assumed to be limited and the firm stores goods in backroom/ warehouse (BR), the objective of this paper is to determine the ordering schedule which maximizes the average profit per unit time yielded by the retailer. The inventory holding cost inside the shop may be higher as compared to in the backroom. As soon as the retailer receives the delivery of the items, some of the items are displayed in the shop while the rest of the items are kept in BR. According to this approach, the items stored in BR are consumed prior to the items stored in DA. Also Numerical examples are provided to illustrate the proposed model and the sensitivity analysis of the optimal solution with respect to parameters of the system has also been performed.

II ASSUMPTIONS AND NOTATIONS

The following assumptions are used in developing the model:

- Demand rate is a function of the stocks on display $Q_d(t)$. For simplicity we also assume that $R(Q_d(t)) = a + bQ_d(t)$, where a and b are non-negative constants. b is the stock dependent consumption rate parameter, $0 \leq b \leq 1$
- Replenishment is instantaneous
- The time horizon of the inventory system is infinite
- Lead-time is negligible
- The maximum allowable number of displayed stocks in the DA is W units; the BR has unlimited capacity
- The retailer orders quantity Q per order from a supplier and W items are displayed in the DA while rest of the items is kept in the BR. The units in BR are stored only when the capacity of DA has been utilized completely and the goods of DA are consumed only after consuming the goods kept in BR
- Shortages are not allowed

Notations adopted in this paper are as below:

$R(Q_d(t))$ the demand rate at time t.

W	the maximum allowable number of items in the DA
Q	the replenishment quantity per replenishment
α	the constant deterioration rate of inventory items in DA, $\alpha > 0$
β	the constant deterioration rate of inventory items in BR, $\beta > 0$
t_o	the time at which inventory level reaches zero in BR
T	the time at which inventory level reaches zero in DA
C	the purchasing cost per unit item
P	selling price per unit item
A	cost of placing per order
H	the holding cost per unit per unit time in DA
F	the holding cost per unit per unit time in BR
$Q_d(t)$	the inventory level at time t in DA
$Q_r(t)$	the inventory level at time t in BR

III MODEL DESCRIPTION AND ANALYSIS

3.1 Model formulation with capacity constraint

In the development of this model, two warehouse dispatch policy has been followed. In such a system, inventory items stored in BR will first be released for consumption until the inventory items are completely consumed, thereafter withdrawing from DA. The inventory situation can be represented as shown in Figure 1. At $t = 0$ of the period the lot size Q enters the system., out of these Q units W units are kept in DA and Q-W units in BR. As soon as the demand is received, the goods from BR are consumed. By time t_o inventory level in BR reaches to zero due to the combined effect of demand and deterioration and the inventory level in DA also reduces due to effect of deterioration. By time T inventory level in DA reaches to zero due to the combined effect of demand and deterioration.

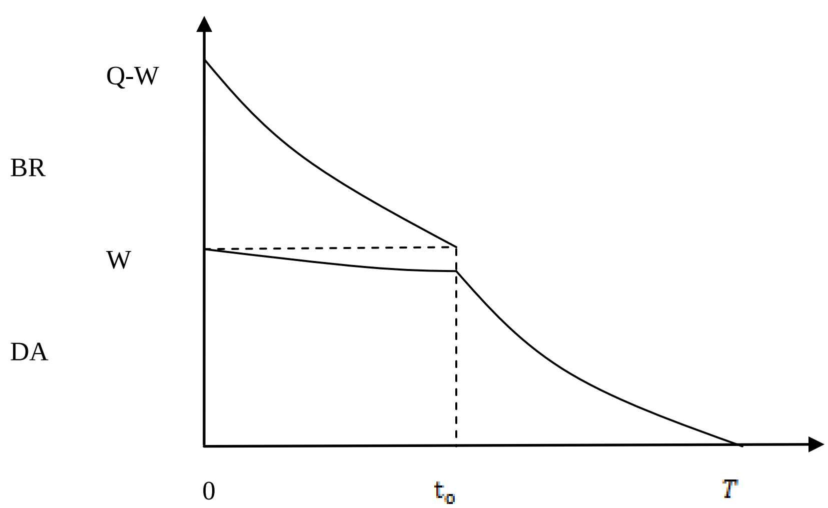

Fig. 1: Graphical representation of two warehouse inventory system

As described above, during the interval $(0, t_o)$ all the W units in DA are kept unused, but they are subject to deterioration at the rate α, the state of inventory in this interval is given by

$$\frac{dQ_d(t)}{dt} + \alpha Q_d(t) = 0 \qquad \text{for } 0 \le t \le t_o \qquad (1)$$

Noting that at $t = 0$, $Q_d(t) = W$ we get

$$Q_d(t) = W e^{-\alpha t} \qquad \text{for } 0 \le t \le t_0 \qquad (2)$$

Now, the inventory level in BR during the time interval $(0, t_o)$ decreases due to combined effect of demand and deterioration both, where the demand is dependent on the stock in DA i.e. $Q_d(t)$ units. The differential equation describing the inventory level in the BR during this interval is

$$\frac{d\left(Q_r(t)\right)}{dt} + \beta Q_r(t) = -\left(a + b\, Q_d(t)\right) \qquad for\ 0 \le t \le t_o \qquad (3)$$

using the initial condition $Q_r(t_o) = 0$

$$Q_r(t) = \frac{a}{\beta}\left\{e^{\beta(t_o - t)} - 1\right\} + \frac{bW}{\beta - \alpha}\left\{e^{-\beta t} e^{(\beta - \alpha)t_o} - e^{-\alpha t}\right\} \qquad \text{for } 0 \le t \le t_o \qquad (4)$$

Also at $t = 0$, $Q_r(0) = Q - W$ we get

$$Q = W + \frac{a}{\beta}\left(e^{\beta t_o} - 1\right) + \frac{bW}{\beta - \alpha}\left(e^{(\beta - \alpha)t_o} - 1\right) \qquad (5)$$

Again, during (t_o, T), the stock in DA decreases owing to demand and deterioration, where demand is further dependent on the items on display. The differential equation describing the state of inventory in this interval is given by

$$\frac{d\left(Q_d(t)\right)}{dt} + \alpha Q_d(t) = -\left(a + bQ_d(t)\right) \qquad \text{for } t_o \le t \le T \qquad (6)$$

After using the boundary condition $Q_d(T) = 0$, the solution is

$$Q_d(t) = \frac{a}{\alpha + b}\left[e^{(\alpha + b)(T - t)} - 1\right] \qquad \text{for } t_o \le t \le T \qquad (7)$$

Using continuity of $Q_d(t)$ at $t = t_o$, we get

$$T = t_0 + \frac{1}{\alpha + b}\ln\left[1 + \frac{(\alpha + b)}{a} W e^{-\alpha t_o}\right] \qquad (8)$$

The holding cost of items in BR is

$$HC_{BR} = F\int_0^{t_0} Q_r(t)dt$$

$$HC_{BR} = F\left[\frac{bW}{\alpha(\beta-\alpha)}\{e^{-\alpha t_0}-1\} + \frac{bWe^{-\alpha t_0}}{\beta(\beta-\alpha)}\{e^{\beta t_0}-1\} + \frac{a}{\beta^2}\{e^{\beta t_0}-1\} - \frac{at_0}{\beta}\right] \qquad (9)$$

The holding cost of items in DA is

$$HC_{DA} = H\left[\int_0^{t_0} Q_d(t)dt + \int_{t_0}^{T} Q_d(t)\,dt\right]$$

$$HC_{DA} = H\left[\frac{W}{\alpha} - \frac{bWe^{-\alpha t_0}}{\alpha(\alpha+b)} - \frac{a}{(\alpha+b)^2}\ln\left\{1+\frac{(\alpha+b)}{a}We^{-\alpha t_0}\right\}\right] \qquad (10)$$

The quantity deteriorated during the period is given by

$$D = Q - \int_0^T R(Q_d(t))dt$$

$$D = Q - \left[\alpha t_0 + \frac{\alpha a}{(\alpha+b)^2}\ln\left\{1+\frac{(\alpha+b)}{a}We^{-\alpha t_0}\right\} - \frac{bW}{\alpha}(e^{-\alpha t_0}-1) + \frac{bW}{(\alpha+b)}e^{-\alpha t_0}\right] \qquad (11)$$

The average profit for the system per unit time is thus, given by the following expression:

$$AP(t_0) = \frac{1}{T}[(P-C)Q - A - CD - HC_{BR} - HC_{DA}] \qquad (12)$$

After substituting $(5), (8), (9), (10), (11)$ into (12) we get the average profit for the system $AP(t_0)$ which is a function of one continuous variable t_0, as given below

$$AP(t_0) = \frac{1}{T}\Bigg[(P-C)Q - A - C\left[Q - \left\{\alpha t_0 + \frac{\alpha a}{(\alpha+b)^2}\ln\left\{1+\frac{(\alpha+b)}{a}We^{-\alpha t_0}\right\} - \frac{bW}{\alpha}(e^{-\alpha t_0}-1) + \frac{bW}{(\alpha+b)}e^{-\alpha t_0}\right\}\right] - H\left[\frac{W}{\alpha} - \frac{bWe^{-\alpha t_0}}{\alpha(\alpha+b)} - \frac{a}{(\alpha+b)^2}\ln\left\{1+\frac{(\alpha+b)}{a}We^{-\alpha t_0}\right\}\right] - F\left[\frac{bW}{\alpha(\beta-\alpha)}\{e^{-\alpha t_0}-1\} + \frac{bWe^{-\alpha t_0}}{\beta(\beta-\alpha)}\{e^{\beta t_0}-1\} + \frac{a}{\beta^2}\{e^{\beta t_0}-1\} - \frac{at_0}{\beta}\right]\Bigg] \qquad (13)$$

The optimal values of t_0 for maximum value of the Average Profit is any solution of the system

$$\frac{\partial AP(t_0)}{\partial t_0} = 0$$

which also satisfies the condition

$$\frac{\partial^2 AP(t_0)}{\partial {t_0}^2} \leq 0$$

Using the optimal value of t_0, the optimal values of Q, and the Average Profit can be obtained.

We have also proved concavity graphically as shown below:

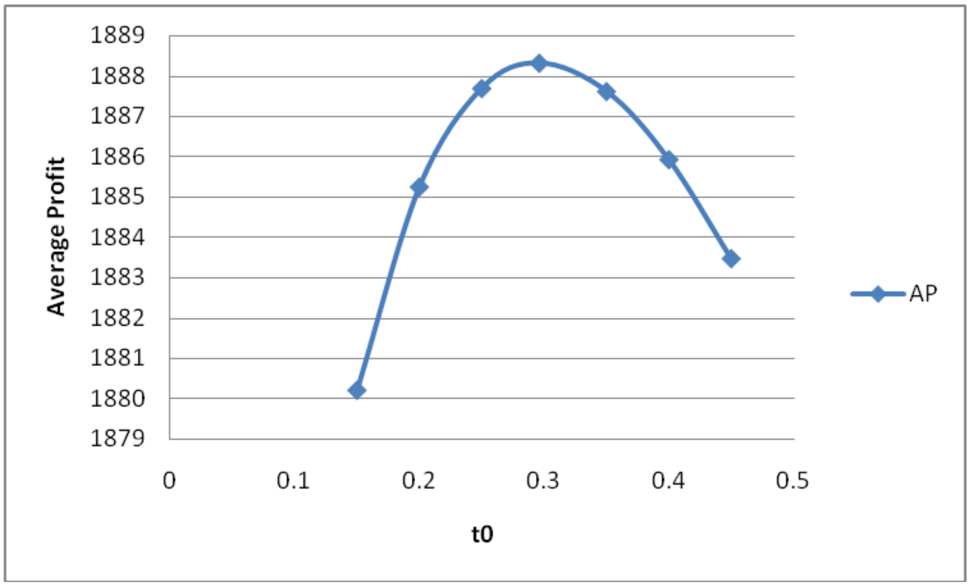

Fig. 2: Concavity of Profit Function

3.2 Inventory model without capacity constraint

In most of the conventional inventory models the capacity of the warehouse is considered to be limited. Therefore, the RW/backroom is not required. In such a scenario, the model gets reduced to the single warehouse inventory model. The inventory system with a single warehouse can be stated as follows: the inventory level of the system Q (i.e., order quantity) at the beginning of each replenishment cycle is kept in DA (unlimited capacity). Then inventory level gradually decreases because of meeting the stock dependent demand and deterioration until the inventory level reaches zero at the end of the replenishment cycle. Proceeding on the similar lines as in section 3 we can derive the following:

$$Q = \left[\frac{a}{\alpha + b} e^{(\alpha+b)T} - 1\right] \tag{15}$$

$$HC = \frac{Ha}{\alpha+b}\left[\frac{1}{(\alpha+b)}\left\{e^{(\alpha+b)T} - 1\right\} - T\right] \tag{16}$$

$$D = \frac{\alpha}{(\alpha + b)}(Q - aT) \tag{17}$$

$$AP = \frac{1}{T}\left[(P - C)Q - A - HC - DC\right]$$

$$AP(\mathrm{T}) = \frac{1}{\mathrm{T}}\left[\left\{(P-C) - \frac{H}{(\alpha+b)} - \frac{\alpha C}{(\alpha+b)}\right\}\left\{\frac{a}{(\alpha+b)}\left(e^{(\alpha+b)\mathrm{T}} - 1\right)\right\} + \left\{\frac{Ha}{(\alpha+b)} + \frac{a\alpha C}{(\alpha+b)}\right\}\mathrm{T} - \mathrm{A}\right] \quad (18)$$

Indicating that the profit function is a function of single variable T. Moreover, using solver it is proved that the second-order derivative of AP (T) with respect to T is less than zero. This implies that $AP(\mathrm{T})$ is a concave function of T. Therefore, known from the necessary condition for $AP(\mathrm{T})$ to be maximum, the optimal value of T satisfies $\frac{\partial AP(\mathrm{T})}{\partial \mathrm{T}} = 0$, which gives

$$\left\{(P-C) - \frac{(H+\alpha C)}{(\alpha+b)}\right\}\frac{a}{(\alpha+b)}\left[1 + e^{(\alpha+b)\mathrm{T}}\left\{\frac{\mathrm{T}}{(\alpha+b)} - 1\right\}\right] + A = 0$$

The single warehouse model does not impose the restriction that $Q \leq W$. Therefore the optimal replenishment quantity Q^* may be either greater than or less than W. If $Q^* < W$, it implies that the decision maker does not obtain more benefits from ordering more than Q^* units. In other words, the decision-maker will order only Q^* units but not consider renting warehouse to enlarge order quantity if $Q^* < W$. Otherwise, the decision-maker will need to consider the ordering policy of the two warehouse system because holding items more than W units by using RW/ BR in this situation may bring more profits to the decision-maker.

IV SOLUTION PROCEDURE

Now, depending on the total costs computed from single warehouse and two warehouse systems, decision is made whether to rent a warehouse or not. In order to find the optimal profit and the order quantity we propose the following algorithm:

Step 1 Solve single warehouse model and find the profit per unit time and the order quantity Q using (18) and (15), respectively.

Step 2 If Q (quantity ordered) calculated from the single warehouse model is less than W (capacity of DA), then use only single warehouse which implies that the profit and the quantity found in step 1 are optimal.

Step 3 If Q calculated from the single warehouse model is equal to W, then solve two-warehouse model explained in step 5. If the profit per unit time calculated in step 1 is more than the profit per time calculated in step 5, then use single warehouse only (i.e. the profit and quantity found in step 1 are optimal),otherwise use RW/BR also.

Step 4 If Q calculated from the single warehouse model is greater than W, then solve two-warehouse model explained in step 5.

Step 5 Calculate the profit per unit time and the order quantity using (13) and (5) respectively which are the optimal values if two-warehouse system is used.

V SOME SPECIAL CASES

The important special cases that influence the optimal value of average profit are described as follows:

5.1 Case 1

The deterioration of inventory items is not considered i.e. $\alpha, \beta = 0$.

When there is no deterioration then the total profit per unit time is given by

$$AP(T)=\frac{1}{T}\left[\left\{(P-C)W-A-\frac{HW}{b}+\frac{Ha}{b^2}\ln\left(1+\frac{Wb}{a}\right)\right\}+\{(P-C)(a+bW)-HW\}\left\{T-\frac{1}{b}\ln\left(1+\frac{Wb}{a}\right)\right\}-\frac{F(a+bW)}{2}\left\{T-\frac{1}{b}\ln\left(1+\frac{Wb}{a}\right)\right\}^2\right]$$

By using the same solution procedure as to the proposed model, the optimal replenishment policy can be found.

5.2 Case 2

The deterioration rate of inventory items is equal in BR and DA i.e. $\alpha=\beta=\theta(say)$.

When the rate of deterioration is equal in both the warehouses, then the profit per unit time is given by

$$AP(t_0)=\Bigg[(P-C)\left\{W+bWt_0+\frac{a}{\theta}\left(e^{\theta t_0}-1\right)\right\}-A-C\left[\left\{W+bWt_0+\frac{a}{\theta}\left(e^{\theta t_0}-1\right)\right\}-\left\{at_0+\frac{\theta a}{(\theta+b)^2}\ln\left\{1+\frac{(\theta+b)}{a}We^{-\theta t_0}\right\}-\frac{bW}{\theta}\left(e^{-\theta t_0}-1\right)+\frac{bW}{(\theta+b)}e^{-\theta t_0}\right\}\right]-H\left[\frac{W}{\theta}-\frac{bWe^{-\theta t_0}}{\theta(\theta+b)}-\frac{a}{(\theta+b)^2}\ln\left\{1+\frac{(\theta+b)}{a}We^{-\theta t_0}\right\}\right]-\frac{F}{\theta^2}\left[a\left(e^{\theta t_0}-1\right)-\theta t_0(a-bW)+bW\left(e^{-\theta t_0}-1\right)\right]\Bigg]\Big/\left\{t_0+\frac{1}{\theta+b}\ln\left(1+\frac{(\theta+b)}{a}We^{-\theta t_0}\right)\right\}$$

5.3 Case 3

The stock-dependent consumption rate is neglected i.e. $b=0$.

When stock-dependent demand parameter is zero i.e. demand is constant, the profit per unit time is given by

$$AP(t_0)=\left[(P-C)\left\{W+\frac{a}{\beta}\left(e^{\beta t_0}-1\right)\right\}-A-C\left[\left\{W+\frac{a}{\beta}\left(e^{\beta t_0}-1\right)\right\}-\left\{at_0+\frac{\alpha a}{\alpha^2}\ln\left(1+\frac{\alpha}{a}We^{-\alpha t_0}\right)\right\}\right]-H\left\{\frac{W}{\alpha}-\frac{a}{\alpha^2}\ln\left(1+\frac{\alpha}{a}We^{-\alpha t_0}\right)\right\}-F\left\{\frac{a}{\beta^2}\left(e^{\beta t_0}-1\right)-\frac{at_0}{\beta}\right\}\right]\Big/\left\{t_0+\frac{1}{\alpha}\ln\left(1+\frac{\alpha}{a}We^{-\alpha t_0}\right)\right\}$$

VI NUMERICAL EXAMPLES

6.1 Example 1

To illustrate the proposed model, let us consider an inventory system with the following data:

a = 1000 units/unit time, b = 0.2, H = Rs 0.6/unit/unit time, F = Rs 0.3/unit/unit time, W=200 units, A = Rs 30, $\alpha=0.03$ units/unit time, $\beta=0.05$ units/unit time, C = Rs 1.0 per unit and P = Rs 3.0 per unit. Optimal solution obtained is shown in TABLE 1. The optimal lot size per unit time and the total average profit of the system are 510 units and Rs1888.321 respectively.

6.2 Example 2

Also *when* $\alpha,\beta=0$ i.e. there is no deterioration, then keeping all the parameters same, results are given in TABLE 2.

Table.1 Optimal solution when there is no deterioration

t_0	T	Q	HC(BR)	HC(DA)	Profit
0.2572	0.4533	468	10.3174	42.5499	1879.762

When deterioration of items is not considered ($\alpha, \beta = 0$), the order quantity declines significantly but the average profit decreases marginally.

6.3 Example 3

When deterioration rate of inventory items is equal in BR and DA i.e. $\alpha = \beta = \theta(say)$, we obtain the following results.

Table.2 Optimal solution when deterioration rate is equal

	t_0	T	Q	HC(BR)	HC(DA)	Profit
when α=β=0.02	0.2728	0.4675	485	11.6276	44.1793	1884.256

It can be observed from TABLE 2 that order quantity decreases but the average profit decreases marginally as compared to the case when deterioration is not equal.

6.4 Example 4

When the stock-dependent consumption rate is neglected i.e. $b = 0$, and demand is constant we get the following results.

Table.3 Optimal solution demand is constant (b=0)

	t_0	T	Q	HC(BR)	HC(DA)	Profit
when b=0	0.2356	0.4336	437	8.3584	39.9562	1827.203

When demand is constant it is observed that there is substantial decline in order quantity and average profit as compared to the case when demand is stock dependent.

VII SENSITIVITY ANALYSIS

By using the numerical values provided in Example 1 the optimal values of T, Q and AP have been computed for different values of demand parameters. The computed results are given in Table 4. It can be observed that higher the values of demand parameters (a) and (b), the more profitable the inventory system using the two storage levels.

Table.4 Optimal solution of the model for different values of demand parameters

a	b	t_0	T	Q	HC(BR)	HC(DA)	Profit
500	0.2	0.3175	0.6967	373	8.2052	60.1277	922.6716
	0.3	0.3848	0.7565	418	12.5097	67.5068	951.0243
	0.4	0.4412	0.8058	459	17.0381	73.6175	980.174
750	0.2	0.3102	0.5667	447	11.4582	52.1478	1404.137

	0.3	0.3486	0.6016	485	14.8415	56.426	1434.265
	0.4	0.3823	0.6321	520	18.3018	60.1648	1464.895
1000	0.2	0.2961	0.49	510	13.7432	46.8184	1888.321
	0.3	0.3216	0.5135	544	16.5289	49.6899	1919.59
	0.4	0.3447	0.5346	575	19.3471	52.2753	1951.213

It can be concluded from TABLE 4 that as the base demand (*a*) increases there is a steep rise in the order quantity, and hence the average profit. Also, when stock dependent parameter of the demand function (*b*) increases for fixed base demand (*a*), there is increase in order quantity which ultimately increases the average profit per unit time. Now, in order to take advantage of this increased sale, it is economical for the decision maker to order more quantity and go for a RW/ BR.

Using the numerical example given in the preceding section, the sensitivity analysis of various parameters has been done. In this section, we have discussed the effects of major parameters W, α, β and A on the optimal profit derived from the proposed model, and the results are shown in TABLE 5.

Table.5 Sensitivity Analysis with respect to various parameters

W	A	t_0	T	Q	HC(BR)	HC(DA)	Profit
150	10	0.1432	0.2901	298	3.1757	19.4036	1937.446
	30	0.3406	0.4866	504	18.0169	36.9604	1885.96
	50	0.4776	0.623	648	35.5151	49.0924	1849.914
	70	0.589	0.7339	766	54.1128	58.9173	1820.439
	90	0.6852	0.8297	868	73.34	67.3748	1794.859
200	10	0.1032	0.2981	308	1.6634	23.9353	1939.059
	30	0.2961	0.485	510	13.7432	46.8184	1888.321
	50	0.4315	0.6246	654	29.2472	62.798	1852.437
	70	0.5419	0.7344	771	46.2088	75.7802	1823.004
	90	0.6374	0.8293	873	64.024	86.9732	1797.424
250	10	0.0659	0.3085	319	0.6839	27.8606	1940.145
	30	0.2536	0.4949	518	10.1715	55.6919	1890.372
	50	0.387	0.6274	660	23.7379	75.3742	1854.735
	70	0.4963	0.7358	777	39.0926	91.4281	1825.395
	90	0.5909	0.8298	879	55.5082	105.2974	1799.848
300	10	0.031	0.3209	333	0.1531	31.3536	1940.75
	30	0.213	0.5013	527	7.2388	63.7189	1892.115

	50	0.3442	0.6314	668	18.9365	86.9366	1856.807
	70	0.4521	0.7384	784	32.7213	105.9628	1827.607
	90	0.5457	0.8313	886	47.7606	122.4392	1802.125

Table.6 Sensitivity Analysis with respect to various parameters (α and β)

α	β	t_0	T	Q	HC(BR)	HC(DA)	Profit
0.03	0.05	0.2961	0.4900	510	13.7432	46.8184	1888.321
	0.08	0.3223	0.5160	540	16.3403	49.9133	1891.228
	0.10	0.3430	0.5366	563	18.5589	52.3583	1893.383
	0.20	0.5276	0.7202	778	44.9771	74.1018	1908.369
0.05	0.08	0.3259	0.5180	543	16.7032	50.0348	1894.279
	0.10	0.3468	0.5387	567	18.9718	52.4812	1896.472
	0.20	0.5337	0.7238	785	46.0269	74.2170	1911.726
0.08	0.10	0.3521	0.5415	572	19.5598	52.6145	1901.081
	0.20	0.5419	0.7285	795	47.4626	74.2661	1916.711
0.10	0.20	0.5468	0.7310	800	48.3231	74.2202	1920.000

The following inferences can be made from the results obtained:

- The increase in deterioration rates α and β results in significant increase in order quantity but average profit increases marginally, due to rise in deterioration cost.
- The increase in ordering cost results in substantial rise in order quantity but there is major decline in average profit.
- The order quantity and average profit is not much sensitive to maximum allowable number of units in display area W.

VIII CONCLUSION

Considering the current scenario of retail industry, major inventory problem is to how, when and where to stock the goods. Usually it is assumed that the display area has unlimited capacity. But accounting various factors like stock dependent demand, the retail outlet orders more than the capacity of display area, and these excess units are stored in Backroom or Rented Warehouse. In this paper an inventory model has been presented by incorporating some of the realistic features viz. deterioration, displayed stock dependent demand and two storage facilities. Moreover, holding cost in display area is considered to be higher than that of Backroom along with different rates of deterioration due to different preservation environment. A solution procedure has also been presented, which helps the decision maker to decide under what circumstances to rent a warehouse. Findings have also been validated with the help of some numerical examples. Sensitivity analysis has also been performed on the demand parameters and the different deterioration rates (α, β) of both the storage facilities.

The proposed model can be extended by incorporating shortages, discount and inflation rates. In addition demand can be considered as a function of price, quality as well as time varying.

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