APPLICATION OF DIFFERENTIAL TRANSFORMATION METHOD FOR STABILITY ANALYSIS OF EULER BERNOULLI COLUMN

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ABSTRACT

In this paper, a relatively new approach called the Differential Transformation Method (DTM) is applied for stability analysis of Euler- Bernoulli column with uniform cross section. Buckling loads are calculated for different cases of boundary conditions. MATLAB code has been developed to solve the differential equation of the column using the Differential Transformation Method. It is found that the buckling loads for all the boundary conditions are in excellent agreement with published results.

Keywords: Differential Transformation Method, Eigen Value Problem, Euler Bernoulli Column, Stability Analysis, Taylor Series

I. INTRODUCTION

Many problems in science and engineering fields can be described by the partial differential equations. A variety of numerical and analytical methods has been developed to obtain accurate/ approximate solutions for the problems in the literature. The classical Taylor series method is one of the earliest analytic techniques to solve many problems, especially ordinary differential equations. However, since it requires a lot of symbolic calculations for the derivatives of functions, it takes a lot of computational time for higher order derivatives. Here, an updated version of the Taylor series method which is called the differential transform method (DTM) is introduced.

This method is useful to obtain the solutions of linear and nonlinear differential equations. There is no need to linearization or discretization. Large computational work and round-off errors are avoided. It has been used to solve effectively, easily and accurately a large class of linear and nonlinear problems with approximations.

For problems of complex nature, the exact solution cannot be obtained or is hard to obtain. In such cases, approximation method is resorted. But by applying differential transformation method, even for the complex problems, the exact solution can be obtained. The method is capable of modeling any beam whose cross-sectional area and moment of inertia vary along its length. Hence DTM can be effectively used in most of engineering applications.

DTM was first used by Zhou(1) to solve both linear and nonlinear initial value problems in electric circuit analysis. C.K chen(2) solved eigen value problems using DTM. Narhari Patil1 and Avinash Khambayat(3) used Differential Transform Method for system of Linear Differential Equations. Ülker Erdönmez, Ibrahim Özkol(5) solved Optimal shape analysis of a column structure under various loading conditions by using differential transform method. Chai, Y.H.; Wang, C.M[9] done stability analysis of heavy columns using DTM.

In this paper the stability analysis of uniform Euler Bernoulli columns has been done by using DTM and the buckling loads for various boundary conditions have been found.

II. DIFFERENTIAL TRANSFORMATION METHOD

In order to solve Eigenvalue problem by differential transformation its basic theory is started in brief in the section the differential transformation of the k^{th} derivative of function Y(x) is defined as follows,

$$Y(K) = \frac{1}{k!}\left[\frac{d^k Y(x)}{dx^k}\right] x = x_0 \qquad (2.1)$$

In eq. (1) Y(x) is the original function and Y(K) is the transformed function, which is called the T- function in brief.

The differential inverse transformation of Y (k) is defined as follows:

$$y(x) = \sum_{k=0}^{\infty} Y(K)\,(x - x_0)^k \qquad (2.2)$$

Combining Eqs. (1) and (2), we have

$$Y(K) = \sum_{k=0}^{\infty} \frac{(x-x_0)^k}{k!}\left[\frac{d^k Y(x)}{dx^k}\right] x = x_0 \qquad (2.3)$$

Eqn (2.2) is the Taylor series of y(x) at x = x_0 Thus eq. (2.3) implies that the concept of differential transformation is derived from the Taylor series expansion. In this study lower-case letters are used to represent the original functions and upper-case letters to stand for the transformed functions (T functions).

From definitions of (2.1) and (2.2), it is very easy to prove that the transformed functions comply with the following basic mathematic operations. . It is easy to prove that the transformed functions comply with the basic

mathematical operations as shown in Table -1

Table-1 Some Basic Mathematical Operations of DTM

Original Function	*Transformed Function*	
$w(x) = y(x) \mp z(x)$	$W(k) = Y(k) \mp Z(k)$	(2.4)
$z(x) = \lambda y(x)$	$Z(k) = \propto Y(k)$	(2.5)
$w(x) = y(x)\, z(x)$	$W(k) = \sum_{l=0}^{k} Y(l) Z(k-l)$	(2.6)
$z(x) = \frac{dy(x)}{dx}$	$Z(k) = (k+1) Y(k+1)$	(2.7)
$w(x) = \frac{d^m y(x)}{dx^m}$	$W(k) = (k+1)(k+2)..(k+m) Y(k+m)$	(2.8)
$w(x) = x^m$	$W(k) = \delta(k-m) = \begin{cases} 1 & k = m \\ 0 & k \neq m \end{cases}$	(2.9)

In real applications, the function y(x) is expressed by a finite series and (2.2) can be written as

$$y(x) = \sum_{k=0}^{n} x^k Y(k) \quad (2.10)$$

Equation (2.10) implies that $\sum_{k=n+1}^{\infty} x^k Y(k)$ is negligibly small. In fact, n is decided by the convergence of the Eigen value in this study

III. BUCKLING OF EULER- BERNOULLI COLUMN

Consider a homogeneous column with flexural rigidity El, length L and end axial compressive load P. The governing Euler column buckling equation is given by,

$$EI\frac{d^4v}{dx^4} + \lambda\frac{d^2v}{dx^2} = 0\ ,\quad \lambda = \frac{Pl^2}{EI} \quad (3.1)$$

Where, $x = \frac{x}{l}$ a non-dimensional co-ordinate varying from 0to 1

ie, $$\frac{d^4v}{dx^4} = -\lambda^2\frac{d^2v}{dx^2} \quad (3.2)$$

DT of eq (3.2) is written as

$$\mathrm{V}\,(\mathrm{K}+4) = \frac{-\lambda^2\,(\mathrm{K}+1)\,(\mathrm{K}+2)\,\mathrm{V}\,(\mathrm{K}+2)}{(\mathrm{K}+1)\,(\mathrm{K}+2)\,(\mathrm{K}+3)\,(\mathrm{K}+4)} \quad (3.3)$$

3.1 Simply Supported At Both the Ends

The boundary conditions are v(0)=0 ; v”(0)=0 ; v(1)=0 ; v”(1)=0 (3.1.1)

The DT equivalents are

V[0]=0 ; V[1]=c ; V[2]=0 ; V[3]=d (3.1.2)

v(1)=0 leads to $\sum_{k=0}^{\infty} V(k) = 0$ (3.1.3)

v”(1)=0 leads to $\sum_{k=0}^{\infty} k(k-1)V(k) = 0$ (3.1.4)

By solving these equations, for a column with simply supported ends λ =9.86

That is buckling load for a column with simply supported ends is, $\mathrm{P} = \frac{9.86EI}{L^2}$

which agrees with closed form value.

3.2 Fixed and Roller Support

The boundary conditions are v(0)=0 ; v”(0)=0 ; v(1)=0 ; v’(1)=0 (3.2.1)

The DT equivalents are

V[0]=0 ; V[1]=c ; V[2]=0 ; V[3]=d (3.2.2)

v(1)=0 leads to $\sum_{k=0}^{\infty} V(k) = 0$ (3.2.3)

v’(1)=0 leads to $\sum_{k=0}^{\infty} kV(k) = 0$ (3.2.4)

By solving these equations, for a column with fixed and roller ends λ=20.19

That is buckling load for a column with fixed and roller ends is, $P = \frac{20.19EI}{L^2}$

which agrees with closed form value.

3.3 Both End Fixed

The boundary conditions are v(0)=0 ; v'(0)=0 ; v(1)=0 ; v'(1)=0 (3.3.1)

The DT equivalents are

V[0]=0 ; V[1]=0 ; V[2]=c ; V[3]=d (3.3.2)

v(1)=0 leads to $\sum_{k=0}^{\infty} V(k) = 0$ (3.3.3)

v'(1)=0 leads to $\sum_{k=0}^{\infty} kV(k) = 0$ (3.3.4)

By solving these equations, for a column with fixed ends λ=39.47

That is buckling load for a column with fixed ends is, $P = \frac{39.47EI}{L^2}$

which agrees with closed form value.

3.4 One End Fixed Other End Free

The boundary conditions are

v(0)=0 ; v'(0)=0 ; v''(1)=0 ; v'''(1)=0 (3.4.1)

The DT equivalents are

V[0]=0 ; V[1]=0 ; V[2]=c ; V[3]=d (3.4.2)

V''(1)=0 leads to $\sum_{k=0}^{\infty} V(k) = 0$ (3.4.3)

v'''(1)=0 leads to $\sum_{k=0}^{\infty} k(k-1)(k-2)V(k) = 0$ (3.4.4)

By solving these equations, for a column with one end fixed other end free λ=2.45

That is buckling load for a column with one end fixed other end free, $P = \frac{2.45EI}{L^2}$

which agrees with closed form value.

IV. CONCLUSION

Buckling load of uniform Euler Bernoulli column for various boundary conditions are obtained using differential transformation method. The results obtained from mathematical analysis was found to be exactly same as closed form solutions. Comparison of buckling load obtained from DTM with closed form solution is shown in table- 2

Table- 2 Comparison of DTM with Closed Form Solution

Support condition	**DTM solution**	**Closed form solution**
Both end pinned	$P = \frac{9.86EI}{L^2}$	$P = \frac{\pi^2 EI}{L^2}$
Roller and fixed	$P = \frac{20.19EI}{L^2}$	$P = \frac{2.045\pi^2 EI}{L^2}$
Both end fixed	$P = \frac{39.47EI}{L^2}$	$P = \frac{4\pi^2 EI}{L^2}$
One end fixed other end free	$P = \frac{2.45EI}{L^2}$	$P = \frac{\pi^2 EI}{4L^2}$

In this paper, buckling load of a uniform Euler–Bernoulli column is analyzed using differential transformation technique. Buckling load for various boundary conditions are obtained. The method is successfully implemented in Matlab. Number of terms required for convergence is generally taken as 30. It is found that the Buckling load obtained, are in excellent agreement with published results. This method is better than numerical methods, since it is free from rounding off error. It is expected that DT will be more promising for further development into efficient and flexible numerical techniques for solving practical engineering problems in future.

V.ACKNOWLEDGEMENT

The authors are thankful to the Management of Karunya University for providing necessary facilities to carry out the work reported in this paper.

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