APPLICATION OF DIFFERENTIAL TRANSFORMATION METHOD FOR THE STATIC ANALYSIS OF EULER-BERNOULLI BEAM

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ABSTRACT

This paper deals with the static analysis of uniform Euler-Bernoulli beam under various supporting condition using Differential Transformation Method (DTM). Deflection and bending moment are calculated for different cases of boundary conditions. MATLAB code has been developed to solve the differential equation of the beam. Comparison of results with the previous solutions has been made and found that the deflection and bending moment for all the boundary conditions are in excellent agreement with available results.

Keywords: DTM; Euler-Bernoulli Beam; Static Analysis; Taylor series method;

I. INTRODUCTION

Many problems in science and engineering fields can be described by the partial differential equations. A variety of numerical and analytical methods has been developed to obtain accurate approximate solutions for the problems in the literature. The classical Taylor series method is one of the earliest analytic techniques to many problems, especially ordinary differential equations. However, since it requires a lot of symbolic calculation for the derivatives of functions, it takes a lot of computational time for higher order derivatives. Here, an updated version of the Taylor series method is introduced which is called the Differential Transform Method (DTM).

This method is useful to obtain the solutions of linear and nonlinear differential equations. There is no need to linearization or discretization. Large computational work and round-off errors are avoided. It has been used to solve effectively, easily and accurately a large class of linear and nonlinear problems with approximations. For problems of complex nature, the exact solution cannot be obtained or is hard to obtain. In such cases, approximation method is resorted. But by applying differential transformation method, even for the complex problems, the exact solution can be obtained. The method is capable of modelling any beam whose cross-sectional area and moment of inertia vary along its length. Hence DTM can be effectively used in most of engineering applications.

DTM was first used by Zhou[(3)] to solve both linear and nonlinear initial value problems in electric circuit analysis. C.K chen[(1)] solved eigen value problems using DTM. Narhari Patil1 and Avinash Khambayat[(2)] used Differential Transform Method for system of Linear Differential Equations. Farshid Mirzaee[(4)] Solved Linear and Nonlinear Systems of Ordinary Differential Equations using DTM.Ibrahim Ozkol & Aytac Arikoglu[(5)] ,Solved the differential equations by using differential transform method.

In this paper, the static analysis of uniform Euler-Bernoulli beams has been done by using DTM for various boundary conditions.

II. DIFFERENTIAL TRANSFORMATION METHOD

The Differential transformation of function $y(x)$ is defined as follows:

$$Y(k) = \frac{1}{k!}\left[\frac{d^k y(x)}{dx^k}\right]_{x=0} \tag{2.1}$$

In (2.1), $y(x)$ is the original function and $Y(k)$ is the transformed function, which is called the T-function in brief.

Differential inverse transformation of $Y(k)$ is defined as follows:

$$y(x) = \sum_{k=0}^{\infty} x^k Y(k) \tag{2.2}$$

In fact, from (2.1) and (2.2), we obtain:

$$y(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}\left[\frac{d^k y(x)}{dx^k}\right]_{x=0} \tag{2.3}$$

Equation (2.3) implies, that the concept of differential transformation is derived from the Taylor series expansion. In this study, lower-case letters are used to represent the original functions and upper-case letters to stand for the transformed function (T- functions).

From the definitions of (2.1) and (2.2), it is easy to prove that the transformed functions comply with the following basic mathematics operations:

Table 1: Some Basic Mathematical Operations of DTM

Original Function	*Transformed Function*	
$w(x) = y(x) \mp z(x)$	$W(k) = Y(k) \mp Z(k)$	(2.4)
$z(x) = \lambda y(x)$	$Z(k) = \propto Y(k)$	(2.5)
$w(x) = y(x)\, z(x)$	$W(k) = \sum_{l=0}^{k} Y(l) Z(k-l)$	(2.6)
$z(x) = \frac{dy(x)}{dx}$	$Z(k) = (k+1)Y(k+1)$	(2.7)
$w(x) = \frac{d^m y(x)}{dx^m}$	$W(k) = (k+1)(k+2)..(k+m)Y(k+m)$	(2.8)
$w(x) = x^m$	$W(k) = \delta(k-m) = \begin{cases}1 & k=m \\ 0 & k \neq m\end{cases}$	(2.9)

In real applications, the function $y(x)$is expressed by a finite series and (2) can be written as:

$$y(x) = \sum_{k=0}^{m} x^k Y(k) \tag{2.10}$$

III. ANALYSIS OF EULER-BERNOULLI BEAM

The governing equation for Euler-Bernoulli beam:

$$EI\frac{d^4v}{dx^4} = q$$

Where $'v'$ is deflection of the beam. To make the equation to a non-dimensional value, x $\rightarrow \frac{x}{l}$.

The limits change from: $0 < x < l \text{ to } 0 < x < 1$

Thus the equation becomes:

$$\frac{d^4v}{dx^4} = \frac{ql^4}{EI}$$

Differential transformation is done according to the theorem given below:

$$V(k+4) = \frac{\delta(k)}{(k+1)(k+2)(k+3)(k+4)}$$

$putting\ k = 0,$

$$V[4] = \frac{1}{24}$$

$$v[5] = v[6] = \cdots v[n] = 0$$

3.1 Pinned-Pinned Support

The Boundary Conditions are:

$$v(0) = 0\ ;\ v''(0) = 0 \quad (3.1.1)$$

$$v(1) = 0\ ;\ v''(1) = 0 \quad (3.1.2)$$

The DT equivalents of (3.1.1) gives:

$V[0] = 0\ ; V[2] = 0$

Let $V[1] = A$; $V[3] = B$; Then,

$V[0] = 0;\ V[1] = A;\ V[2] = 0;\ V[3] = B$

The DT equivalents of (3.1.2) gives :

$$\sum_{k=0}^{\infty} V(k) = 0 \quad (3.1.3)$$

$$\sum_{k=0}^{\infty}(k)(k-1)V(k) = 0 \quad (3.1.4)$$

Solving (3.1.3) & (3.1.4), the value of 'A' & 'B' are obtained.

Substituting the value of $V(k)$ in the equation (3.1.5), the deflection of Euler-Bernoulli beam can be obtained.

$$v(x) = \sum_{k=0}^{\infty} V(k)x^k \quad (3.1.5)$$

3.2 Fixed-Fixed Support

The Boundary Conditions are :

$$v(0) = 0\ ;\ v'(0) = 0 \quad (3.2.1)$$

$$v(1) = 0\ ;\ v'(1) = 0 \quad (3.2.2)$$

The DT equivalents of (3.2.1) gives :

$V[0] = 0\ ; V[1] = 0$

Let $V[2] = A$; $V[3] = B$; Then,

$V[0] = 0;\ V[1] = 0;\ V[2] = A;\ V[3] = B$

The DT equivalents of (3.2.2) gives :

$$\sum_{k=0}^{\infty} V(k) = 0 \tag{3.2.3}$$

$$\sum_{k=0}^{\infty} (k)V(k) = 0 \tag{3.2.4}$$

Solving (3.2.3) & (3.2.4), the value of ‘A’ & ‘B’ are obtained.

Substituting the value of $V(k)$ in the equation (3.1.5), the deflection of Euler-Bernoulli beam can be obtained.

3.3 Fixed-Roller Support

The Boundary Conditions are:

$$v(0) = 0\ ;\ v'(0) = 0 \tag{3.3.1}$$

$$v(1) = 0\ ;\ v''(1) = 0 \tag{3.3.2}$$

The DT equivalents of (3.3.1) gives:

$V[0] = 0\ ;\ V[1] = 0$

Let $V[2] = A$; $V[3] = B$; Then,

$V[0] = 0;\ V[1] = 0;\ V[2] = A;\ V[3] = B$

The DT equivalents of (3.3.2) gives:

$$\sum_{k=0}^{\infty} V(k) = 0 \tag{3.3.3}$$

$$\sum_{k=0}^{\infty} (k)(k-1)V(k) = 0 \tag{3.3.4}$$

Solving (3.3.3) & (3.3.4), the value of ‘A’ & ‘B’ are obtained.

Substituting the value of $V(k)$ in the equation (3.1.5), the deflection of Euler-Bernoulli beam can be obtained.

3.4 Fixed-Free Supports

The Boundary Conditions are:

$$v(0) = 0\ ;\ v'(0) = 0 \tag{3.4.1}$$

$$v''(1) = 0\ ;\ v'''(1) = 0 \tag{3.4.2}$$

The DT equivalents of (3.4.1) gives:

$V(0) = 0\ ;\ V(1) = 0$

Let $V[2] = A$; $V[3] = B$; Then,

$V[0] = 0;\ V[1] = 0;\ V[2] = A;\ V[3] = B$

The DT equivalents of (3.4.2) gives:

$$\sum_{k=0}^{\infty} k(k-1)V(k) = 0 \tag{3.4.3}$$

$$\sum_{k=0}^{\infty} (k)(k-1)(k-2)V(k) = 0 \tag{3.4.4}$$

Solving (3.4.3) & (3.4.4), the value of ‘A’ & ‘B’ are obtained.

Substituting the value of $V(k)$ in the equation (3.1.5), the deflection of Euler-Bernoulli beam can be obtained.

IV. NUMERICAL RESULTS

The solution obtained from the Differential Transformation Method and the closed form solution are compared and are tabulated in Table 2.

Table 2: Comparison of DTM Solution with Closed Form Solution

Support conditions	DTM solution		Closed form solution	
	Max deflection	Max BM in span	Max deflection	Max BM in span
Pinned-pinned	$\frac{5wL^4}{384EI}$	$\frac{wL^2}{8}$	$\frac{5wL^4}{384EI}$	$\frac{wL^2}{8}$
Fixed-fixed	$\frac{wL^4}{384EI}$	$\frac{wL^2}{12}$	$\frac{wL^4}{384EI}$	$\frac{wL^2}{12}$
Roller-fixed	$\frac{wL^4}{185EI}$	$\frac{9wL^2}{128}$	$\frac{wL^4}{185EI}$	$\frac{9wL^2}{128}$

V. CONCLUSION

The static analysis of uniform Euler-Bernoulli beam is done using Differential Transformation Technique. Maximum value of deflection and bending moment for various boundary conditions are obtained. The method is successfully implemented in Mat lab. It is found that the deflections and bending moments obtained are in excellent agreement with the closed-form solutions. Based on the results presented, it can be demonstrated that the Differential Transformation Method is a convenient and efficient method for the static analysis of beams with good accuracy.

It is expected that DT will be more promising for further development into efficient and flexible numerical techniques for solving practical engineering problems in future.

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