DGLAP EVOLUTION EQUATIONS IN LEADING ORDER AND NEXT-TO-LEADING ORDER AND HADRON STRUCTURE FUNCTIONS AT LOW-X

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ABSTRACT

We present unique solutions of singlet and non-singlet Dokshitzer-Gribov-Lipatov- Altarelli-Parisi (DGLAP) evolution equations in leading order (LO) and next-to-leading order (NLO) at low-x. We obtain t-evolutions of deuteron, proton, neutron and difference and ratio of proton and neutron structure functions and x- evolution of deuteron structure function at low-x from DGLAP evolution equations. The results of t-evolutions are compared with HERA and NMC low-x low-Q^2 data and x-evolution are compared with NMC low-x low-Q^2 data. And also we compare our results of t-evolution of proton structure functions with a recent global parameterization.

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***Keywords: Altarelli-Parisi Equation, Complete Solution, Low-X Physics, Particular Solution, Structure Function, Unique Solution*.**

I. INTRODUCTION

In some earlier papers [1-4], particular solutions of the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations [5-8] for t and x-evolutions of singlet and non-singlet structure functions in leading order (LO) and next-to-leading order (NLO) at low-x have been reported, which are non unique solutions. The present paper reports unique solutions of DGLAP evolution equations computed from complete solutions in LO and NLO at low-x and calculation of t and x- evolutions for singlet and non-singlet structure functions, and hence t-evolution of deuteron, proton, neutron, difference and ratio of proton and neutron structure functions, and x-evolution of deuteron structure functions. These results are compared with HERA [9], NMC [10] low-x low Q^2 data and also with recent global parameterization [11]. Here Section 1, Section 2, and Section 3 will give the introduction, the necessary theory and the results and discussion respectively.

II. THEORY

Though the necessary theory has been discussed elsewhere [2-4], here we have mentioned some essential steps for clarity. The DGLAP evolution equations with splitting functions [12-13] for singlet and non-singlet structure functions are in the standard forms [14 -15]

$$\frac{\partial F_2^S(x,t)}{\partial t} - \frac{\alpha_{s1}(t)}{2\pi}\Big[\frac{2}{3}\{3+4\ln(1-x)\}F_2^S(x,t) + \frac{4}{3}\int_x^1 \frac{dw}{1-w}\{(1+w^2)F_2^S\left(\frac{x}{w},t\right) - 2F_2^S(x,t)\}$$

$$+N_f\int_x^1\{w^2+(1-w)^2\}G(\frac{x}{w},t)dw\}]=0, \qquad (1)$$

$$\frac{\partial F_2^{NS}(x,t)}{\partial t}-\frac{\alpha_{s1}(t)}{2\pi}[\frac{2}{3}\{3+4\ln(1-x)\}F_2^{NS}(x,t)+\frac{4}{3}\int_x^1\frac{dw}{1-w}\{(1+w^2)F_2^{NS}\left(\frac{x}{w},t\right)-2F_2^{NS}(x,t)\}]=0, \qquad (2)$$

for LO, and

$$\frac{\partial F_2^{S}(x,t)}{\partial t}-\frac{\alpha_{s}(t)}{2\pi}[\frac{2}{3}\{3+4\ln(1-x)\}F_2^{S}(x,t)+\frac{4}{3}\int_x^1\frac{dw}{1-w}\{(1+w^2)F_2^{S}\left(\frac{x}{w},t\right)-2F_2^{S}(x,t)\}$$

$$+N_f\int_x^1\{w^2+(1-w)^2\}G(\frac{x}{w},t)dw\}]-\left(\frac{\alpha_s(t)}{2\pi}\right)^2[(x-1)F_2^S(x,t)\int_0^1 f(w)dw+\int_x^1 f(w)F_2^S(\frac{x}{w},t)dw$$

$$+\int_x^1 F_{qq}^S(w)F_2^S(\frac{x}{w},t)dw+\int_x^1 F_{qg}^S(w)G(\frac{x}{w},t)dw]=0, \qquad (3)$$

$$\frac{\partial F_2^{NS}(x,t)}{\partial t}-\frac{\alpha_{s}(t)}{2\pi}\left[\frac{2}{3}\{3+4\ln(1-x)\}F_2^{NS}(x,t)+\frac{4}{3}\int_x^1\frac{dw}{1-w}\{(1+w^2)F_2^{NS}\left(\frac{x}{w},t\right)-2F_2^{NS}(x,t)\}\right]$$

$$-\left(\frac{\alpha_s(t)}{2\pi}\right)^2\left[(x-1)F_2^{NS}(x,t)\int_0^1 f(w)dw+\int_x^1 f(w)F_2^{NS}\left(\frac{x}{w},t\right)dw\right]=0, \qquad (4)$$

for NLO, where,

$$t=\ln\frac{Q^2}{\Lambda^2},\quad \alpha_{s1}(t)=\frac{4\pi}{\beta_0 t},\quad \alpha_s(t)=\frac{4\pi}{\beta_0 t}\left[1-\frac{\beta_1\ln t}{\beta_0^2 t}\right],\quad \beta_0=\frac{33-2n_f}{3}\text{ and }\beta_1=\frac{306-38n_f}{3},$$

N_f being the number of flavours. Here,

$$f(w)=C_F^2[P_F(w)-P_A(w)]+\frac{1}{2}C_F C_A[P_G(w)+P_A(w)]+C_F T_R N_f P_{N_F}(w)$$

and

$$F_{qq}^S(w)=2C_F T_R N_f F_{qq}(w)\text{ and }F_{qg}^S(w)=C_F T_R N_f F_{qg}^1(w)+C_G T_R N_f F_{qg}^2(w).$$

The explicit forms of higher order kernels are taken form references [12-13]. Here $C_A=C_G=N_C=3$, $C_F=(N_C^2-1)/2N_C$ and $T_R=1/2$.

Using Taylor expansion method and neglecting higher order terms of x as discussed in our earlier works [1-4, 17-19], $F_2^S(x/w,t)$ can be approximated for low-x as

$$F_2^S(x/w,t)\cong F_2^S(x,t)+x\sum_{k=1}^{\infty}u^k\frac{\partial F_2^S(x,t)}{\partial x}.$$

where,

$$u=1-w\quad\text{and}\quad\frac{x}{1-u}=x\sum_{l=0}^{\infty}u^l.$$

Similarly, G(x / w, t) and F_2^{NS}(x / w, t) can be approximated for small-x. Then putting these values of F_2^S (x/w, t), G(x/w, t) and F_2^{NS} (x/w, t) in equation (1) and (3) and performing u-integrations we get,

$$\frac{\partial F_2^S(x,t)}{\partial t} - \frac{\alpha_{s_1}(t)}{2\pi}\left[A_1(x)F_2^S(x,t) + A_2(x)G(x,t) + A_3(x)\frac{\partial F_2^S(x,t)}{\partial x} + A_4(x)\frac{\partial G(x,t)}{\partial x}\right] = 0 \qquad (5)$$

in LO and

$$\frac{\partial F_2^S(x,t)}{\partial t} - \left[\frac{\alpha_s(t)}{2\pi}A_1(x) + \left(\frac{\alpha_s(t)}{2\pi}\right)^2 B_1(x)\right]F_2^S(x,t) - \left[\frac{\alpha_s(t)}{2\pi}A_2(x) + \left(\frac{\alpha_s(t)}{2\pi}\right)^2 B_2(x)\right]G(x,t)$$
$$-\left[\frac{\alpha_s(t)}{2\pi}A_3(x) + \left(\frac{\alpha_s(t)}{2\pi}\right)^2 B_3(x)\right]\frac{\partial F_2^S(x,t)}{\partial x} - \left[\frac{\alpha_s(t)}{2\pi}A_4(x) + \left(\frac{\alpha_s(t)}{2\pi}\right)^2 B_4(x)\right]\frac{\partial G(x,t)}{\partial x} = 0, \qquad (6)$$

in NLO, where,

$$A_1(x) = \frac{2}{3}\{3 + 4\ln(1-x) + (x-1)(x+3)\},$$

$$A_2(x) = N_f[\frac{1}{3}(1-x)(2-x+2x^2)],$$

$$A_3(x) = \frac{2}{3}\{x(1-x^2) + 2x\ln(\frac{1}{x})\},$$

$$A_4(x) = N_f x\{\ln\frac{1}{x} - \frac{1}{3}(1-x)(5-4x+2x^2)\}$$

$$B_1(x) = x\int_0^1 f(w)dw - \int_0^x f(w)dw + \frac{4}{3}N_f\int_x^1 F_{qq}(w)dw,$$

$$B_2(x) = \int_x^1 F_{qg}^S(w)dw,$$

$$B_3(x) = x\int_x^1\{f(w) + \frac{4}{3}N_f F_{qq}(w)\}\frac{1-w}{w}dw,$$

$$B_4(x) = x\int_x^1\frac{1-w}{w}F_{qg}^S(w)dw.$$

Let us assume for simplicity [1-4, 17-19]

$$G(x, t) = K(x)\, F_2^S(x, t) \qquad (7)$$

where K(x) is a function of x. In this connection, earlier we considered [1, 4] K(x) = k, ax^b, ce^{-dx}, where k, a, b, c, d are constants. Agreement of the results with experimental data is found to be excellent for k = 4.5, a = 4.5, b = 0.01, c = 5, d = 1 for low-x in LO and a =10, b = 0.016, c = 0.5, d =-3.8 for low-x in NLO. Therefore equation (5) and (6) becomes

$$\frac{\partial F_2^S(x,t)}{\partial t} - \frac{\alpha_{s_1}(t)}{2\pi}\left[L_1(x)F_2^S(x,t) + L_2(x)\frac{\partial F_2^S(x,t)}{\partial x}\right] = 0, \qquad (8)$$

in LO and

$$\frac{\partial F_2^S(x,t)}{\partial t}-\left[\frac{\alpha_s(t)}{2\pi}L_1(x)+\left(\frac{\alpha_s(t)}{2\pi}\right)^2 M_1(x)\right]F_2^S(x,t)-\left[\frac{\alpha_s(t)}{2\pi}L_2(x)+\left(\frac{\alpha_s(t)}{2\pi}\right)^2 M_2(x)\right]\frac{\partial F_2^S(x,t)}{\partial x}=0, \tag{9}$$

in NLO.

For simplicity, we can write equation (8) as

$$\frac{\partial F_2^S(x,t)}{\partial t}-\left[L_1^{/}(x,t)F_2^S(x,t)+L_2^{/}(x,t)\frac{\partial F_2^S(x,t)}{\partial x}\right]=0, \tag{10}$$

where,

$$L_1(x)=A_1(x)+K(x)A_2(x)+A_4(x)\frac{\partial K(x)}{\partial x},$$
$$L_2(x)=A_3(x)+K(x)A_4(x),$$
$$M_1(x)=B_1(x)+K(x)B_2(x)+B_4(x)\frac{\partial K(x)}{\partial x},$$
$$M_2(x)=B_3(x)+K(x)B_4(x),$$
$$L_1^{/}(x,t)=\frac{\alpha_{S_1}(t)}{2\pi}L_1(x), L_2^{/}(x,t)=\frac{\alpha_{S_1}(t)}{2\pi}L_2(x),$$

For a possible solution of equation (9), we assume [2-4, 15] that

$$\left(\frac{\alpha_s(t)}{2\pi}\right)^2=T_0\left(\frac{\alpha_s(t)}{2\pi}\right), \tag{11}$$

where, T_0 is a numerical parameter to be obtained from the particular Q^2-range under study. By a suitable choice of T_0 we can reduce the error to a minimum. Now equation (9) can be recast as

$$\frac{\partial F_2^S(x,t)}{\partial t}-\left[P(x,t)F_2^S(x,t)+Q(x,t)\frac{\partial F_2^S(x,t)}{\partial x}\right]=0, \tag{12}$$

in NLO, where,

$$P(x,t)=\frac{\alpha_s(t)}{2\pi}\left[L_1(x)+T_0M_1(x)\right] \text{ and } Q(x,t)=\frac{\alpha_s(t)}{2\pi}\left[L_2(x)+T_0M_2(x)\right].$$

The general solutions [20-21] of equation (10) is F (U, V) = 0, where F is an arbitrary function and $U(x, t, F_2^S) = C_1$ and $V(x, t, F_2^S) = C_2$ where, C_1 and C_2 are constants and they form a solutions of equations

$$\frac{dx}{L_2^{/}(x,t)}=\frac{dt}{-1}=\frac{dF_2^S(x,t)}{-L_1^{/}(x,t)F_2^S(x,t)} \tag{13}$$

Solving equation (13) we obtain,

$$U\left(x,t,F_2^S\right)=t\exp\left[\frac{1}{A_f}\int\frac{1}{L_2(x)}dx\right] \text{ and } V\left(x,t,F_2^S\right)=F_2^S(x,t)\exp\left[\int\frac{L_1(x)}{L_2(x)}dx\right].$$

where, $A_f = 4/(33-2N_f)$. Since U, and V are two independent solutions of equation (13) and if α and β are arbitrary constants, then V = αU + β may be taken as a complete solution [20-21] of equation (10). We take this form as this is the simplest form of a complete solution which contains both the arbitrary constants α and β.

Then the complete solution [20-21]

$$F_2^S(x,t)\exp\left[\int\frac{L_1(x)}{L_2(x)}dx\right]=\alpha t\exp\left[\frac{1}{A_f}\int\frac{L_1(x)}{L_2(x)}dx\right]+\beta, \tag{14}$$

is a two-parameter family of planes.

Due to conservation of the electromagnetic current, F_2 must vanish as Q^2 goes to zero [22-23]. Also $R\to 0$ in this limit. Here R indicates ratio of longitudinal and transverse cross-sections of virtual photon in DIS process. This implies that scaling should not be a valid concept in the region of very low Q^2. The exchanged photon is then almost real and the close similarity of real photonic and hadronic interactions justifies the use of the Vector Meson Dominance (VMD) concept [24-25] for the description of F_2. In the language of perturbation theory this concept is equivalent to a statement that a physical photon spends part of its time as a "bare", point-like photon and part as a virtual hadron (s) [23]. The power and beauty of explaining scaling violations with field theoretic methods (i.e., radiative corrections in QCD) remains, however, unchallenged in as much as they provide us with a framework for the whole x-region with essentially only one free parameter Λ [26]. For Q^2 values much larger than Λ^2, the effective coupling is small and a perturbative description in terms of quarks and gluons interacting weakly makes sense. For Q^2 of order Λ^2, the effective coupling is infinite and we cannot make such a picture, since quarks and gluons will arrange themselves into strongly bound clusters, namely, hadrons [22] and so the perturbation series breaks down at small-Q^2 [27]. Thus, it can be thought of Λ as marking the boundary between a world of quasi-free quarks and gluons, and the world of pions, protons, and so on. The value of Λ is not predicted by the theory; it is a free parameter to be determined from experiment. It should expect that it is of the order of a typical hadronic mass [22]. Since the value of Λ is so small we can assume at $Q = \Lambda$, $F_2^S(x, t) = 0$ due to conservation of the electromagnetic current [22-23]. This dynamical prediction agrees with most ad hoc parameterizations and with the data [26]. Using this boundary condition in equation (14) we get $\beta = 0$ and

$$F_2^S(x,t)=\alpha t\exp\left[\int\left(\frac{1}{A_f L_2(x)}-\frac{L_1(x)}{L_2(x)}\right)dx\right]. \tag{15}$$

Now, defining

$$F_2^S(x,t_0)=\alpha t_0\exp\left[\int\left(\frac{1}{A_f L_2(x)}-\frac{L_1(x)}{L_2(x)}\right)dx\right],$$

at $t = t_0$, where, $t_0 = \ln(Q_0^2/\Lambda^2)$ at any lower value $Q = Q_0$, we get from equations (15)

$$F_2^S(x,t)=F_2^S(x_0,t)\left(\frac{t}{t_0}\right), \tag{16}$$

which gives the t-evolutions of singlet structure function $F_2^S(x, t)$ in LO. Proceeding in the same way we get

$$F_2^S(x,t)=F_2^S(x,t_0)\left(\frac{t^{(b/t+1)}}{t_0^{(b/t_0+1)}}\right)\exp\left[b\left(\frac{1}{t}-\frac{1}{t_0}\right)\right], \tag{17}$$

which gives the t-evolutions of singlet structure function $F_2^S(x, t)$ in NLO, where $b = \beta_1/\beta_o^2$

We observed that the Lagrange's auxiliary system of ordinary differential equations (12) occurred in the formalism can not be solved without the additional assumption of linearization (equation 11) and introduction of an adhoc parameter T_0 [2-4, 15]. This parameter does not effect in the results of t- evolution of structure functions.

Proceeding exactly in the same way, we get for non-singlet structure functions

$$F_2^{NS}(x,t) = F_2^{NS}(x,t_0)\left(\frac{t}{t_0}\right), \tag{18}$$

$$F_2^{NS}(x,t) = F_2^{NS}(x,t_0)\left(\frac{t^{(b/t+1)}}{t_0^{(b/t_0+1)}}\right)\exp\left[b\left(\frac{1}{t}-\frac{1}{t_0}\right)\right] \tag{19}$$

which gives the t-evolutions of non-singlet structure functions $F_2^{NS}(x, t)$ in LO and NLO respectively.

We observe that if b tends to zero, then equation (17) and (19) tends to equation (16) and (18) respectively, i.e., solution of NLO equations goes to that of LO equations. Physically b tends to zero means number of flavours is high. Again defining,

$$F_2^{S}(x_0,t) = \alpha t \exp\left[\int\left(\frac{1}{A_f L_2(x)} - \frac{L_1(x)}{L_2(x)}\right)dx\right]_{x=x_0}$$

we obtain from equation (15)

$$F_2^{S}(x,t) = F_2^{S}(x_0,t)\exp\left[\int_{x_0}^{x}\left(\frac{1}{A_f L_2(x)} - \frac{L_1(x)}{L_2(x)}\right)dx\right], \tag{20}$$

which gives the x-evolutions of singlet structure function $F_2^S(x, t)$ in LO. Similarly we get

$$F_2^{S}(x,t) = F_2^{S}(x_0,t)\exp\int_{x_0}^{x}\left[\frac{1}{a}.\frac{1}{L_2(x)+T_0M_2(x)} - \frac{L_1(x)+T_0M_1(x)}{L_2(x)+T_0M_2(x)}\right]dx, \tag{21}$$

which gives the x-evolutions of singlet structure function $F_2^S(x, t)$ in NLO, where $a = 2/\beta_o$.

Proceeding in the same way, we get

$$F_2^{NS}(x,t) = F_2^{NS}(x_0,t)\exp\left[\int_{x_0}^{x}\left(\frac{1}{A_f A_3(x)} - \frac{A_1(x)}{A_3(x)}\right)\right]dx \tag{22}$$

and

$$F_2^{NS}(x,t) = F_2^{NS}(x_0,t)\exp\int_{x_0}^{x}\left[\frac{1}{a}.\frac{1}{A_5(x)+T_0B_5(x)} - \frac{A_6(x)+T_0B_6(x)}{A_5(x)+T_0B_5(x)}\right]dx \tag{23}$$

which gives the x-evolution of non-singlet structure functions $F_2^{NS}(x, t)$ in LO and NLO respectively. Here,

$$A_5(x) = \frac{2}{3}\{x(1-x^2) + 2x\ln(\frac{1}{x})\}, \qquad B_5(x) = x\int_x^1 \frac{1-w}{w} f(w)dw,$$

$$A_6(x) = \frac{2}{3}\{3 + 4\ln(1-x) + (x-1)(x+3)\}, \qquad B_6(x) = -\int_0^x f(w)dw + x\int_0^1 f(w)dw.$$

In our earlier communications [1-4] we observed that if in the relation $\beta = \alpha^y$, y varies between minimum to a maximum value, the powers of (t/t_o) in LO and powers of $t^{b/t+1} / t_0^{b/t_o+1}$, co-efficient of $b(1/t - 1/t_o)$ of exponential part in NLO in t-evolutions and the numerator of the first term in the integral sign in x-evolution in both LO and NLO varies between 2 to 1. Then it is understood that the particular solutions of DGLAP evolution equations in LO and NLO obtained by that methodology were not unique and so the t- evolutions of deuteron, proton and neutron structure functions, and x- evolution of deuteron structure function obtained by this methodology were not unique. Thus by this methodology, instead of having a single solution we arrive a band of solutions, of course the range for these solutions is reasonably narrow.

Now deuteron, proton and neutron structure functions measured in deep inelastic electro-production can be written in terms of singlet and non-singlet quark distribution functions [22] as

$F_2^d(x, t) = 5/9\, F_2^S(x, t),$ (24)

$F_2^p(x, t) = 5/18\, F_2^S(x, t) + 3/18\, F_2^{NS}(x, t),$ (25)

$F_2^n(x, t) = 5/18\, F_2^S(x, t) - 3/18\, F_2^{NS}(x, t),$ (26)

$F_2^p(x, t) - F_2^n(x, t) = 1/3\, F_2^{NS}(x, t).$ (27)

Now using equations (16), (17) and (20) and (21) in equation (24) we will get t and x-evolution of deuteron structure function $F_2^d(x, t)$ at low-x as

$$F_2^d(x,t) = F_2^d(x,t_0)\left(\frac{t}{t_0}\right), \tag{28}$$

in LO

$$F_2^d(x,t) = F_2^d(x,t_0)\left(\frac{t^{(b/t+1)}}{t_0^{(b/t_0+1)}}\right)\exp\left[b\left(\frac{1}{t} - \frac{1}{t_0}\right)\right] \tag{29}$$

in NLO and

$$F_2^d(x,t) = F_2^d(x_0,t)\exp\left[\int_{x_0}^{x}\left(\frac{1}{A_f L_2(x)} - \frac{L_1(x)}{L_2(x)}\right)dx\right], \tag{30}$$

in LO

$$F_2^d(x,t) = F_2^d(x_0,t)\exp\int_{x_0}^{x}\left[\frac{1}{a}.\frac{1}{L_2(x)+T_0M_2(x)} - \frac{L_1(x)+T_0M_1(x)}{L_2(x)+T_0M_2(x)}\right]dx, \tag{31}$$

in NLO.

Similarly using equations (16), (18) and (17), (19) in equations (25), (26) and (27) we get the t– evolutions of proton, neutron, and difference and ratio of proton and neutron structure functions at low-x in LO and NLO as

$$F_2^p(x,t) = F_2^p(x,t_0)\left(\frac{t}{t_0}\right), \tag{32}$$

$$F_2^p(x,t) = F_2^p(x,t_0)\left(\frac{t^{(b/t+1)}}{t_0^{(b/t_0+1)}}\right)\exp\left[b\left(\frac{1}{t}-\frac{1}{t_0}\right)\right], \tag{33}$$

$$F_2^n(x,t) = F_2^n(x,t_0)\left(\frac{t}{t_0}\right), \tag{34}$$

$$F_2^n(x,t) = F_2^n(x,t_0)\left(\frac{t^{(b/t+1)}}{t_0^{(b/t_0+1)}}\right)\exp\left[b\left(\frac{1}{t}-\frac{1}{t_0}\right)\right], \tag{35}$$

$$F_2^p(x,t) - F_2^n(x,t) = [F_2^p(x,t_0) - F_2^n(x,t_0)](\frac{t}{t_0}), \tag{36}$$

$$F_2^p(x,t) - F_2^n(x,t) = [F_2^p(x,t_0) - F_2^p(x,t_0)]\left(\frac{t^{(b/t+1)}}{t_0^{(b/t_0+1)}}\right)\exp\left[b\left(\frac{1}{t}-\frac{1}{t_0}\right)\right] \tag{37}$$

and

$$\frac{F_2^p(x,t)}{F_2^n(x,t)} = \frac{F_2^p(x,t_0)}{F_2^n(x,t_0)} = R(x). \tag{38}$$

where R(x) is a constant for fixed-x. It is observed that ratio of proton and neutron is same for both NLO and LO and it is independent of t for fixed-x. We also observed that unique solutions of GLDAP evolution equations in LO and NLO are same with particular solutions in LO and NLO for y maximum in $\beta = \alpha^y$ relation [1-4].

III. RESULTS AND DISCUSSION

In the present paper, we compare our results of t-evolution of deuteron, proton, neutron and difference and ratio of proton and neutron structure functions with the HERA [9] and NMC [10] low-x and low-Q^2 data and results of x- evolution of deuteron structure functions with NMC low-x and low-Q^2 data. In case of HERA data proton and neutron structure functions are measured in the range $2 \le Q^2 \le 50$ GeV^2. Moreover here $P_T \le 200$ MeV, where P_T is the transverse momentum of the final state baryon. In case of NMC data proton and neutron structure functions are measured in the range $0.75 \le Q^2 \le 27$GeV^2. We consider number of flavours $N_f = 4$. We also compare our results of t-evolution of proton structure functions with recent global parameterization [11]. This parameterization includes data from H1-96 \ 99, ZEUS- 96/97(X0.98), NMC, E665, data.

In Fig. 1(a), (b), (c), (d), we present our results of t-evolutions of deuteron, proton, neutron, and difference of proton and neutron structure functions (solid lines for NLO and dashed lines for LO) for the representative values of x given in the figure. Data points at lowest-Q^2 values in the figures are taken as inputs to test the evolution equations. Agreement with the data [9-10] is found to be good. We observe that t-evolutions are slightly steeper in LO calculations than those of NLO.

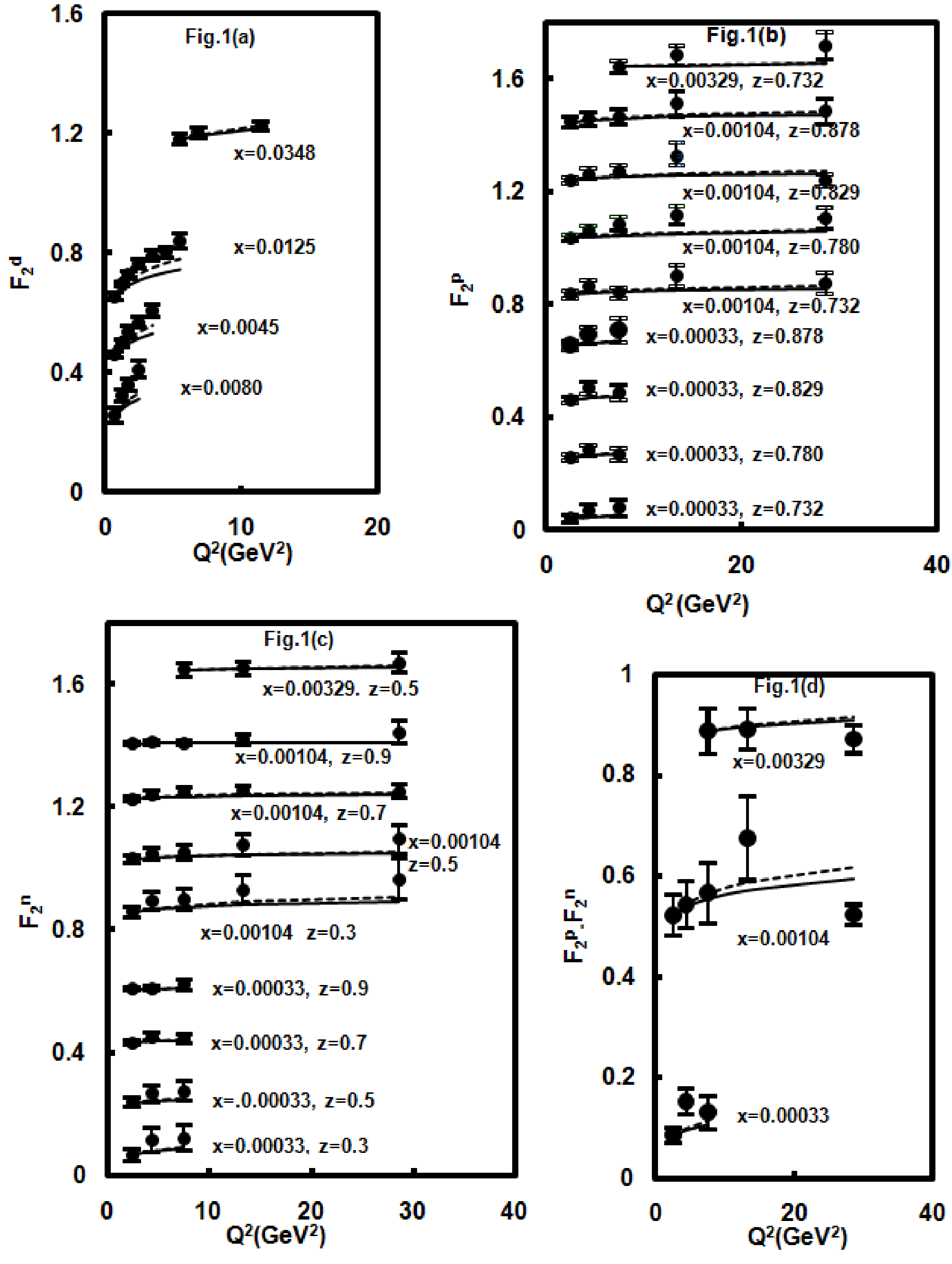


Fig.1: Results of t-evolutions of deuteron, proton, neutron and difference of proton and neutron structure functions (dashed lines for LO and solid lines for NLO) for the representative values of x in LO and NLO for NMC data. For convenience, value of each data point is increased by adding 0.2i, where i = 0, 1, 2, 3 ... are the numberings of curves counting from the bottom of the lowermost curve as the 0-th order. Data points at lowest-Q^2 values in the figures are taken as input.

In fig. 2, we compare our results of t-evolutions of proton structure functions F_2^p (solid lines for NLO and dashed lines for LO) with recent global parameterization [11] (long dashed lines) for the representative values of x given in the figure. Data points at lowest-Q^2 values in the figures are taken as input to test the evolution equation. We observe that t-evolutions are slightly steeper in LO calculations than those of NLO. Agreement with the LO results is found to be better than with the NLO results.

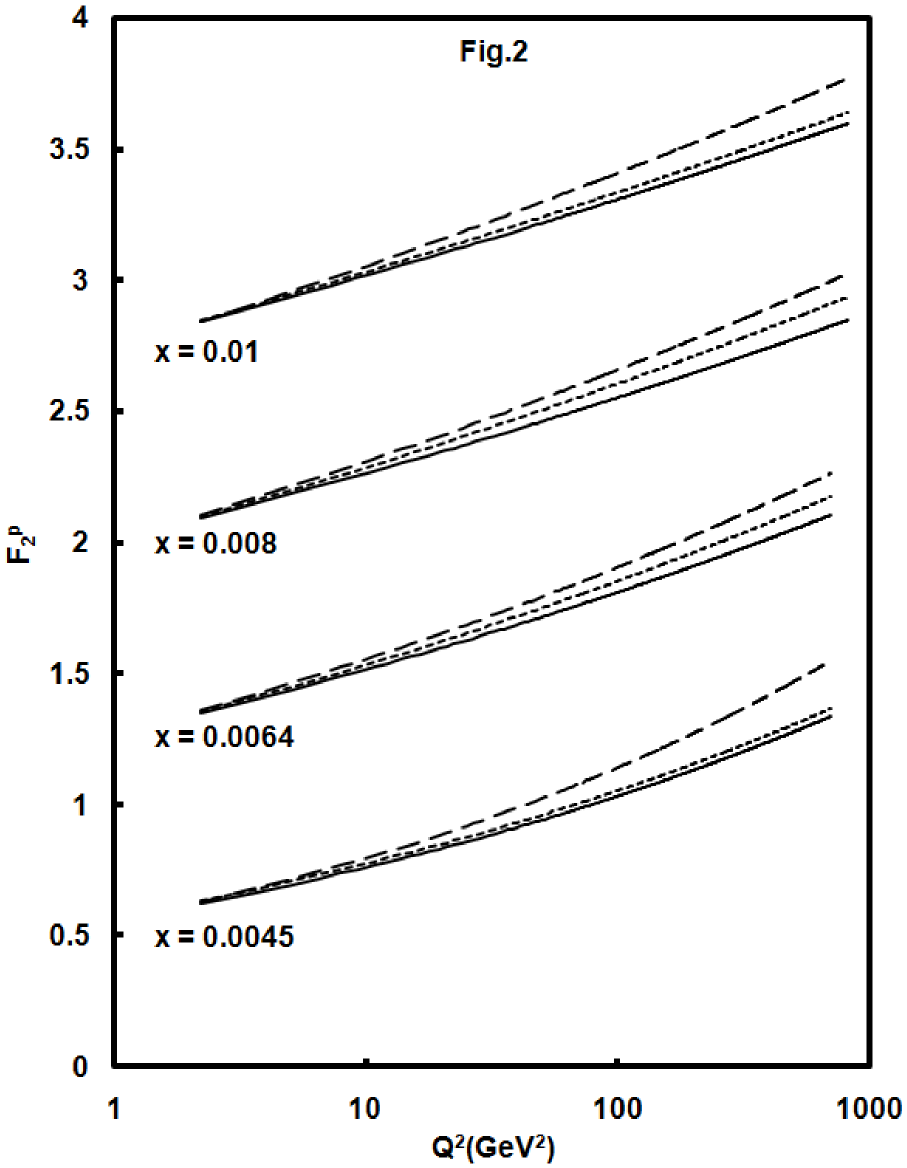
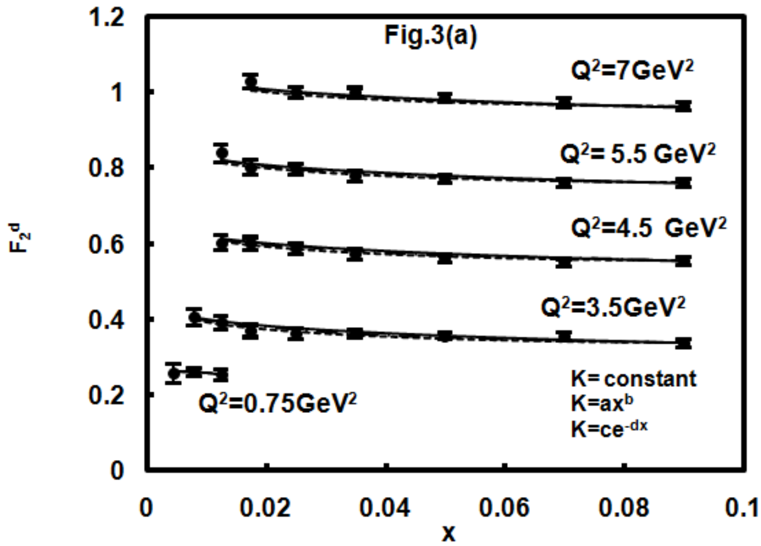
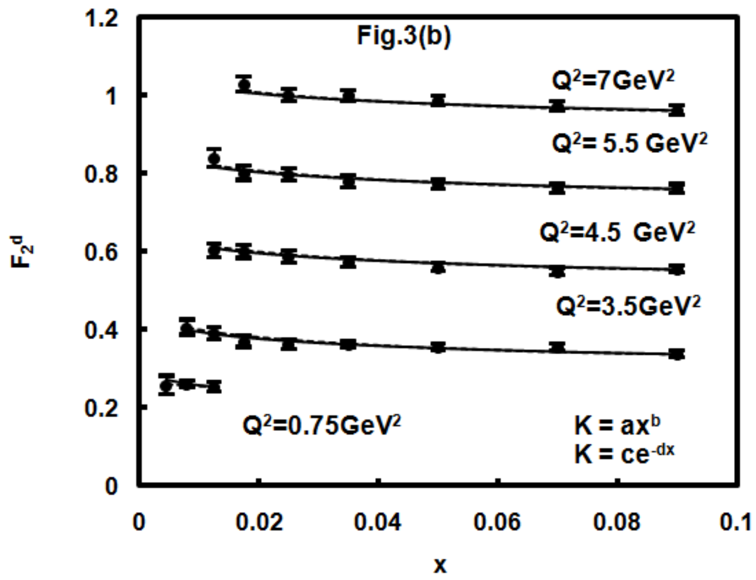


Fig.2: Results of t-evolutions of proton structure functions F_2^p (dashed lines for LO and solid lines for NLO) with recent global paramatrization (long dashed lines) for the representative values of x given in the figures. Data points at lowest-Q^2 values in the figures are taken as input. For convenience, value of each data point is increased by adding 0.5i, where i = 0, 1, 2, 3, ... are the numberings of curves counting from the bottom of the lowermost curve as the 0-0-th order. Data points at lowest-Q^2 values in the figures are taken as input.

In figs.3(a), 3(b) we present our results of x-distribution of deuteron structure functions F_2^d in LO for K(x) = constant (solid lines), K(x) = ax^b (dashed lines) and for K(x) = ce^{-dx} (dotted lines), and in NLO for K(x) = ax^b (solid lines) and for K(x) = ce^{-dx} (dashed lines) where a, b, c and d are constants and for representative values of Q^2 given in each figure , and compare them with NMC deuteron low-x low- Q^2 data [10]. In each data point for x- value just below 0·1 has been taken as input F_2^d (x_0, t). In case of LO, if we take K(x) = 4.5, then agreement





Figs.3(a) and 3(b): Results of x-distribution of deuteron structure functions F_2^d in LO for K(x) = constant (solid lines), K(x) = ax^b (dashed lines) and for K(x) = ce^{-dx} (dotted lines), where K(x) = 4.5, a = 4.5, b = 0.01, c = 5, b = 1 and in NLO for K(x) = ax^b (solid lines), and for K(x) = ce^{-dx} (dotted lines), where a = 5.5, b = 0.016, c = 0.28, and d = -3.8 and for representative values of Q^2 given in each figure, and compare them with NMC deuteron low-x low-Q^2 data. In each the data point for x- value just below 0.1 has been taken as input $F_2^d(x_0, t)$. For convenience, value of each data point is increased by adding 0.2i, where i = 0, 1, 2, 3, ... are the numberings of curves counting from the bottom of the lowermost curve as the 0-th order.

of the result with experimental data is found to be excellent. On the other hand, if we take K(x) = ax^b, then agreement of the results with experimental data is found to be good at a = 4.5, b = 0.01. Again if we take K(x) = ce^{-dx}, then agreement of the results with experimental data is found to be good at c=5, b=1. For x- evolutions of deuteron structure function, results of unique solutions and results of particular solutions have not any significance difference in LO [1].

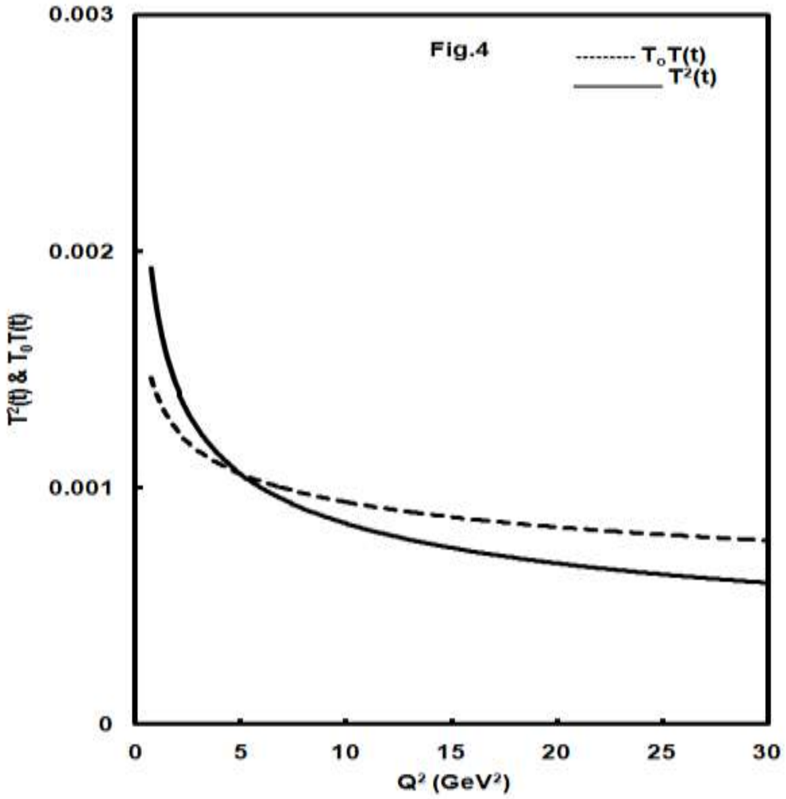


Fig.4: $T(t)^2$ and $T_0T(t)$, where $T(t) = \alpha_s(t)/2\pi$ against Q^2 in the Q^2 range $0 \leq Q^2 \leq 30$ GeV^2.

In case of NLO, if we take $K(x) = ax^b$, then agreement of the result with experimental data is found to be excellent at a = 5.5, b = 0.016. On the other hand if we take $K(x) = ce^{-dx}$, then agreement of the results with experimental data is found to be good at c= 0.28, d = -3.8. But agreement of the results with experimental data is found to be very poor for any constant value of K(x). Therefore we do not present our result of x-distribution at K(x) = constant in NLO.

In fig.4, we plot $T(t)^2$ and $T_0T(t)$, where $T(t) = \alpha_s(t)/2\pi$ against Q^2in the Q^2 range $0 \leq Q^2 \leq 30$ GeV^2 as required by our data used. Though the explicit value of T_0 is not necessary in calculating t-evolution of, yet we observe that for $T_0 = 0.027$, errors become minimum in the Q^2 range $0 \leq Q^2 \leq 30$ GeV^2.

IV. CONCLUSION

We derive *t* and *x*-evolutions of various structure functions and compare them with global data and parameterizations with satisfactory phenomenological success. It has been observed that though we have derived a unique *t*-evolution for deuteron, proton, neutron, difference and ratio of proton and neutron structure functions in LO and NLO, yet we can not establish a completely unique *x*-evolution for deuteron structure function in LO and NLO due to the relation $K(x)$ between singlet and gluon structure functions. $K(x)$ may be in the forms of a constant, an exponential function or a power function and they can equally produce required *x*-distribution of deuteron structure functions. But unlike many parameter arbitrary input *x*-distribution functions generally used in the literature, our method requires only one or two such parameters. On the other hand, our methods are mathematically simpler with less number of approximations. Explicit form of $K(x)$ can actually be obtained only by solving coupled GLDAP evolution equations for singlet and gluon structure functions, and works are going on in this regard.

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