OPERATIONS ON INTERVAL-VALUED FUZZY GRAPHS

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ABSTRACT

To discuss the Cartesian Product Composition, union and join on Interval-valued fuzzy graphs. We also introduce the notion of Interval-valued fuzzy complete graphs. Some properties of self complementary graph.

Keywords: Complement Of A Fuzzy Graph, Interval-Valued Fuzzy Graph, Self Complementary Interval Valued Fuzzy Complete Graphs, Strong Interval Valued Fuzzy Graphs

I INTRODUCTION

In 1975, Zadeh [27] introduced the notion of interval-valued fuzzy sets as an extension of fuzzy sets [26] in which the values of the membership degrees are intervals of numbers instead of the numbers. Interval-valued fuzzy sets provide a more adequate description of uncertainty than traditional fuzzy sets. It is therefore important to use interval-valued fuzzy sets in applications, such as fuzzy control. One of the computationally most intensive part of fuzzy control is defuzzification [15]. Since interval-valued fuzzy sets are widely studied and used, we describe briefly the work of Gorzalczany on approximate reasoning [10, 11], Roy and Biswas on medical diagnosis [22], Turksen on multivalued logic [25] and Mendel on intelligent control [15].

The fuzzy graph theory as a generalization of Euler's graph theory was first introduced by Rosenfeld [23] in 1975. The fuzzy relations between fuzzy sets were first considered by Rosenfeld and he developed the structure of fuzzy graphs obtaining analogs of several graph theoretical concepts. Later, Bhattacharya [5] gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Mordeson and Peng [19]. The complement of a fuzzy graph was defined by Mordeson [18] and further studied by Sunitha and Vijayakumar [24]. Bhutani and Rosenfeld introduced the concept of M-strong fuzzy graphs in [7] and studied some properties. The concept of strong arcs in fuzzy graphs was discussed in [8]. Hongmei and Lianhua gave the definition of interval-valued graph in [12].

In this paper, we define the operations of Cartesian product, composition, union and join on interval-valued fuzzy graphs and investigate some properties. We study isomorphism (resp. weak isomorphism) between interval-valued fuzzy graphs is an equivalence relation (resp. partial order). We introduce the notion of interval-valued fuzzy complete graphs and present some properties of self complementary and self weak complementary interval-valued fuzzy complete graphs.

The definitions and terminologies that we used in this paper are standard. For other notations, terminologies and applications, the readers are referred to [1, 2, 3, 4, 9, 13, 14, 17, 20, 21, 28].

II PRELIMINARIES

A graph is an ordered pair G* = (V, E), where V is the set of vertices of G* and E is the set of edges of G*. Two vertices x and y in a graph G* are said to be adjacent in G* if {x,y} is in an edge of G*. (For simplicity an edge {x,y} will be denoted by xy. A **simple graph** is a graph without loops and multiple edges. A **complete graph** is a simple graph in which every pair of distinct vertices is connected by an edge. The complete graph on n vertices has n vertices and n(n – 1)/2 edges. We will consider only graphs with the finite number of vertices and edges.

By a **complementary graph** $\overline{G^*}$ of a simple graph G* we mean a graph having the same vertices as G* and such that two vertices are adjacent in $\overline{G^*}$ if and only if they are not adjacent in G*.

An **isomorphism** of graphs G_1^* and G_2^* is a bijection between the vertex sets of G_1^* and G_2^* such that any two vertices v_1 and v_2 of G_1 are adjacent in G_1 if and only if $f(v_1)$ and $f(v_2)$ are adjacent in G_2. Isomorphic graphs are denoted by $G_1^* \approx G_2^*$.

Let $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ be two simple graphs, we can construct several new graphs. The first construction called the **Cartesian product** of G_1^* and G_2^* gives a graph $G_1^* \times G_2^* = (V, E)$ with $V = V_1 \times V_2$ and

$$E = \{(x, x_2)(x, y_2) \mid x \in V_1, x_2y_2 \in E_2\} \cup \{(x_1, z)(y_1, z) \mid x_1y_1 \in E_1, z \in V_2\}$$

The **composition of graphs** G_1^* and G_2^* is the graph $G_1^*[G_2^*] = (V_1 \times V_2, E^0)$, where

$$E^0 = E \cup \{(x_1, x_2)(y_1, y_2) \mid x_1y_1 \in E_1, x_2 \neq y_2\}$$

and E is defined as in $G_1^* \times G_2^*$. Note that $G_1^*[G_2^*] \neq G_2^*[G_1^*]$.

The union of graphs G_1^* and G_2^* is defined as $G_1^* + G_2^* = (V_1 \cup V_2, E_1 \cup E_2)$.

The join of G_1^* and G_2^* is the simple graph $G_1^* + G_2^* = (V_1 \cup V_2, E_1 \cup E_2 \cup E')$, where E' is the set of all edges joining the nodes of V_1 and V_2. In this construction it is assumed that $V_1 \cap V_2 \neq \emptyset$.

By a fuzzy subset μ on a set X is mean a map $\mu : X \to [0, 1]$. A map $v : X \times X \to [0,1]$ is called a fuzzy relation on X if $v(x, y) \leq \min(\mu(x), \mu(y))$ for all $x,y \in X$. A fuzzy relation v is symmetric if $v(x, y) = v(y, x)$ for all $x, y \in X$.

An **interval number D** is an interval $[a^-, a^+]$ with $0 \leq a^- \leq a^+ \leq 1$. The interval [a, a] is identified with the number $a \in [0,1]$. D[0, 1] denotes the set of all interval numbers.

For interval numbers $D_1 = [a_1^-, b_1^+]$ and $D_2 = [a_2^-, b_2^+]$, we define

- $r\min(D_1, D_2) = r\min([a_1^-, b_1^+], [a_2^-, b_2^+]) = [\min\{a_1^-, a_2^-\}, \min\{b_1^+, b_2^+\}]$,
- $r\max(D_1, D_2) = r\max([a_1^-, b_1^+], [a_2^-, b_2^+]) = [\max\{a_1^-, a_2^-\}, \max\{b_1^+, b_2^+\}]$,
- $D_1 + D_2 = [a_1^- + a_2^- - a_1^-.a_2^-, b_1^+ + b_2^+ - b_1^+.b_2^+]$,
- $D_1 \leq D_2 \Leftrightarrow a_1^- \leq a_2^- \text{ and } b_1^+ \leq b_2^+$,
- $D_1 = D_2 \Leftrightarrow a_1^- = a_2^- \text{ and } b_1^+ = b_2^+$,

- $D_1 < D_2 \Leftrightarrow D_1 \le D_2 \; and \; D_1 \ne D_2 \Rightarrow a_1^- < a_2^-, b_1^+ < b_2^+$
- $kD_1 = k[a_1^-, b_1^+] = [ka_1^-, b_1^+], where \quad 0 \le k \le 1$.

Then, (D[0,1], ≤, V, Λ) is a complete lattice with [0, 0] as the least element and [1,1] as the greatest.

The interval-valued fuzzy set A in V is defined by

$A = \{(x, [\mu_A^-(x), \mu_A^+(x)]) : x \in V\}$,

where $\mu_A^-(x)$ and $\mu_A^+(x)$ are fuzzy subsets of V such that $\mu_A^-(x) \le \mu_A^+(x)$ for all $x \in V$. For any two interval-valued sets $A = [\mu_A^-(x), \mu_A^+(x)]$ and $B = [\mu_B^-(x), \mu_B^+(x)]$ in V we define:

- $A \cup B = \{(x, \max\,(\mu_A^-(x), \mu_B^-(x)), \max\,(\mu_A^+(x)\mu_B^+(x))) : x \in V\}$,
- $A \cap B = \{(x, \min\,(\mu_A^-(x), \mu_B^-(x)), \min\,(\mu_A^+(x)\mu_B^+(x))) : x \in V\}$.

If G* = (V, E) is a graph, then by an **interval-valued fuzzy relation** B on a set E we mean an interval-valued fuzzy set such that

$\mu_B^-(xy) \le \min\,(\mu_A^-(x), \mu_A^-(y))$

$\mu_B^+(xy) \le \min\,(\mu_A^+(x), \mu_A^+(y))$

for all $xy \in E$.

If G* =(V,E) is a graph, then by **A strong interval valued fuzzy graph,** we mean

$\mu_B^-(xy) = \min\,(\mu_A^-(x), \mu_A^-(y)), \; \mu_B^+(xy) = \min\,(\mu_A^+(x), \mu_A^+(y))$

III OPERATIONS ON INTERVAL-VALUED FUZZY GRAPHS

Throughout in this paper, G* is a crisp graph, and G is an interval-valued fuzzy graph.

Definition 3.1. By an interval-valued fuzzy graph of a graph G* = (V, E) we mean a pair G = (A, B), where $A = [\mu_A^-, \mu_A^+]$ is an interval-valued fuzzy set on V and $B = [\mu_B^-, \mu_B^+]$ is an interval-valued fuzzy relation on E.

Definition 3.2. The Cartesian product $G_1 \times G_2$ of two lattice graphs $G_1 = (A_1,B_1)$ and $G_2 = (A_2,B_2)$ of the graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ is defined as a pair $(A_1 \times A_2, B_1 \times B_2)$ such that

i) $$\begin{cases} (\mu_{A_1}^- \times \mu_{A_2}^-)(x_1, x_2) = \min(\mu_{A_1}^-(x_1), \mu_{A_2}^-(x_2)) \\ (\mu_{A_1}^+ \times \mu_{A_2}^+)(x_1, x_2) = \min(\mu_{A_1}^+(x_1), \mu_{A_2}^+(x_2)) \end{cases}$$

for all $(x_1, x_2) \in V$,

ii) $$\begin{cases} (\mu_{B_1}^- \times \mu_{B_2}^-)(x, x_2)(x, y_2)) = \min(\mu_{A_1}^-(x), \mu_{B_2}^-(x_2 y_2)) \\ (\mu_{B_1}^+ \times \mu_{B_2}^+)(x, x_2), (x, y_2) = \min(\mu_{A_1}^+(x), \mu_{B_2}^+(x_2 y_2)) \end{cases}$$

for all $x \in V_1$, and $x_2y_2 \in E_2$,

iii) $$\begin{cases} (\mu^-_{B_1} \times \mu^-_{B_2})(x_1,z)(y_1,z)) = \min(\mu^-_{B_1}(x_1y_1), \mu^-_{A_2}(z)) \\ (\mu^+_{B_1} \times \mu^+_{B_2})(x_1,z),(y_1,z) = \min(\mu^+_{B_1}(x_1y_1), \mu^+_{A_2}(z)) \end{cases}$$

for all $z \in V_2$, and $x_1y_1 \in E_1$.

Definition 3.3 The complement of an interval-valued fuzzy graph G=(A,B) of G^*=(V,E) is an interval-valued fuzzy graph

$\overline{G} = (\overline{A}, \overline{B})$ on G^*, where $\overline{A} = A = [\mu^-_A, \mu^+_A]$ and $\overline{B} = [\overline{\mu^-_B}, \overline{\mu^+_B}]$ is defined by

$$\overline{\mu^-_B}(xy) = \begin{cases} 0, if \mu^-_B(xy) > 0. \\ \min(\mu^-_A(x), \mu^-_A(y), if \mu^-_B(xy) = 0 \end{cases}$$

$$\mu^+_B(xy) = \begin{cases} 0, if \mu^+_B(xy) > 0 \\ \min(\mu^+_A(x), \mu^+_A(y), if \mu^+_B(xy) = 0 \end{cases}$$

Definition 3.4 An interval valued fuzzy graph is self complementary,

if $\overline{\overline{G}} = G$

Example 3.5: Consider a graph G^* =(V,E) such that V=$\{a,b,c\}$,

E=$\{ab, bc\}$, then an interval valued fuzzy graph G=(A,B), where

$$A = \left\langle \left(\frac{a}{0.1}, \frac{b}{0.2}, \frac{c}{0.3}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5}\right) \right\rangle, B = \left\langle \left(\frac{ab}{0.1}, \frac{bc}{0.2}, \right), \left(\frac{ab}{0.3}, \frac{bc}{0.4}\right) \right\rangle$$

is self complementary.

Solution: $\overline{\mu^-_B}(ab) = 0, \overline{\mu^+_B}(ab) = 0, \overline{\mu^-_B}(bc) = 0, \overline{\mu^+_B}(bc) = 0$ (by definition)

$$\overline{\overline{\mu^-_B}}(ab) = 0.1 = \mu^-_B(ab), \overline{\overline{\mu^+_B}}(ab) = 0.3 = \mu^+_B(ab),$$
$$\overline{\overline{\mu^-_B}}(bc) = 0.2 = \mu^-_B(bc), \overline{\overline{\mu^+_B}}(bc) = 0.4 = \mu^+_B(bc)$$

Definition 3.6

Let G_1 and G_2 are Interval Valued Fuzzy Graphs

$G = G_1 + G_2 = \langle V_1 \cup V_2, E_1 \cup E_2 \cup E' \rangle$ defined by

$(\mu_1 + \mu_1')(v) = (\mu_1 \cup \mu_1')(v)$ if $v \in V_1 \cup V_2$

$(\gamma_1 + \gamma_1')(v) = (\gamma_1 \cup \gamma_1')(v)$ if $v \in V_1 \cup V_2$

$(\mu_2 + \mu_2')(v_iv_j) = (\mu_2 \cup \mu_2')(v_iv_j)$ if $v_iv_j \in E_1 \cup E_2$

$= (\mu_1(v_i).\mu_1'(v_j))$ if $v_iv_j \in E'$

Theorem 3.7

Let $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$ be two Interval Valued Fuzzy Graphs. Then

(i) $\overline{G_1 + G_2} \cong \overline{G_1} \cup \overline{G_2}$

(ii) $\overline{G_1 \cup G_2} \cong \overline{G_1} + \overline{G_2}$

Proof

Consider the identity map I : $V_1 \cup V_2 \to V_1 \cup V_2$,

To prove (i) it is enough to prove

(a) (i) $\overline{\mu_1 \cup \mu_1^{'}}(v_i) = \overline{\mu_1} \cup \overline{\mu_1^{'}}(v_i)$

(ii) $\overline{\gamma_1 + \gamma_1^{'}}(v_i) = \overline{\gamma_1} \cup \overline{\gamma_1^{'}}(v_i)$

(b) (i) $\overline{\mu_2 \cup \mu_2^{'}}(v_i, v_j) = \overline{\mu_2} \cup \overline{\mu_2^{'}}(v_i, v_j)$

(ii) $\overline{\gamma_2 + \gamma_2^{'}}(v_i, v_j) = \overline{\gamma_2} \cup \overline{\gamma_2^{'}}(v_i, v_j)$

(a) (i) $\left(\overline{\mu_1 \cup \mu_1^{'}}\right)(v_i) = \left(\mu_1 + \mu_1^{'}\right)(v_i)$, by Definition 4.1

$$= \begin{cases} \mu_1(v_i) & if \ v_i \in V_1 \\ \mu_1^{'}(v_i) & if \ v_i \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\mu_1}(v_i) & if \ v_i \in V_1 \\ \overline{\mu_1^{'}}(v_i) & if \ v_i \in V_2 \end{cases}$$

$$= \left(\overline{\mu_1} \cup \overline{\mu_1^{'}}\right)(v_i)$$

(ii) $\left(\overline{\gamma_1 + \gamma_1^{'}}\right)(v_i) = \left(\gamma_1 + \gamma_1^{'}\right)(v_i)$, by Definition 4.1

$$= \begin{cases} \gamma_1(v_i) & if \ v_i \in V_1 \\ \gamma_1^{'}(v_i) & if \ v_i \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\gamma_1}(v_i) & if \ v_i \in V_1 \\ \overline{\gamma_1^{'}}(v_i) & if \ v_i \in V_2 \end{cases}$$

$$= \left(\overline{\gamma_1} \cup \overline{\gamma_1^{'}}\right)(v_i)$$

(b) (i) $\left(\overline{\mu_2 + \mu_2^{'}}\right)(v_i, v_j) = \left(\mu_1 + \mu_1^{'}\right)(v_i).\left(\mu_1 + \mu_1^{'}\right)(v_j) - \left(\mu_2 + \mu_2^{'}\right)(v_i, v_j)$

$$= \begin{cases} \left(\mu_1 \cup \mu_1^{'}\right)(v_i).\left(\mu_1 \cup \mu_1^{'}\right)(v_j) - \left(\mu_2 \cup \mu_2^{'}\right)(v_i, v_j) & if \ (v_i, v_j) \in E_1 \cup E_2 \\ \left(\mu_1 \cup \mu_1^{'}\right)(v_i).\left(\mu_1 \cup \mu_1^{'}\right)(v_j) - \mu_1(v_i).\mu_1^{'}(v_j) & if \ (v_i, v_j) \in E' \end{cases}$$

$$= \begin{cases} (\mu_1)(v_i).\mu_1(v_j) - \mu_2(v_i, v_j) & \text{if } (v_i, v_j) \in E_1 \\ \mu_1^{'}(v_i).\mu_1^{'}(v_j) - \mu_2^{'}(v_i, v_j) & \text{if } (v_i, v_j) \in E_2 \\ (\mu_1)(v_i).\mu_1^{'}(v_j) - \mu_1(v_i).\mu_1^{'}(v_j) & \text{if } (v_i, v_j) \in E' \end{cases}$$

$$= \begin{cases} \overline{\mu_2}(v_i, v_j) & \text{if } (v_i, v_j) \in E_1 \\ \overline{\mu_2^{'}}(v_i, v_j) & \text{if } (v_i, v_j) \in E_2 \\ 0 & \text{if } (v_i, v_j) \in E' \end{cases}$$

$$= \left(\overline{\mu_2} \cup \overline{\mu_2^{'}}\right)(v_i, v_j)$$

(b) (ii) $\overline{\left(\gamma_2 + \gamma_2^{'}\right)}(v_i, v_j) = (\gamma_1 + \gamma_1^{'})(v_i).(\gamma_1 + \gamma_1^{'})(v_j) - (\gamma_2 + \gamma_2^{'})(v_i, v_j)$

$$= \begin{cases} (\gamma_1 \cup \gamma_1^{'})(v_i).(\gamma_1 \cup \gamma_1^{'})(v_j) - (\gamma_2 \cup \gamma_2^{'})(v_i, v_j) & \text{if } (v_i, v_j) \in E_1 \cup E_2 \\ (\gamma_1 \cup \gamma_1^{'})(v_i).(\gamma_1 \cup \gamma_1^{'})(v_j) - \gamma_1(v_i).\gamma_1^{'}(v_j) & \text{if } (v_i, v_j) \in E' \end{cases}$$

$$= \begin{cases} (\gamma_1)(v_i).\gamma_1(v_j) - \gamma_2(v_i, v_j) & \text{if } (v_i, v_j) \in E_1 \\ \gamma_1^{'}(v_i).\gamma_1^{'}(v_j) - \gamma_2^{'}(v_i, v_j) & \text{if } (v_i, v_j) \in E_2 \\ (\gamma_1)(v_i).\gamma_1^{'}(v_j) - \gamma_1(v_i).\gamma_1^{'}(v_j) & \text{if } (v_i, v_j) \in E' \end{cases}$$

$$= \begin{cases} \overline{\gamma_2}(v_i, v_j) & \text{if } (v_i, v_j) \in E_1 \\ \overline{\gamma_2^{'}}(v_i, v_j) & \text{if } (v_i, v_j) \in E_2 \\ 0 & \text{if } (v_i, v_j) \in E' \end{cases}$$

$$= \left(\overline{\gamma_2} \cup \overline{\gamma_2^{'}}\right)(v_i, v_j)$$

To prove (ii) it is enough to prove

(a) (i) $\overline{\left(\mu_1 \cup \mu_1^{'}\right)}(v_i) = \left(\overline{\mu_1} \cup \overline{\mu_1^{'}}\right)(v_i)$

(ii) $\overline{\left(\gamma_1 \cup \gamma_1^{'}\right)}(v_i) = \left(\overline{\gamma_1} + \overline{\gamma_1^{'}}\right)(v_i)$

(b) (i) $\overline{\left(\mu_2 \cup \mu_2^{'}\right)}(v_i, v_j) = \left(\overline{\mu_2} + \overline{\mu_2^{'}}\right)(v_i, v_j)$

(ii) $\overline{\left(\gamma_2 \cup \gamma_2^{'}\right)}(v_i, v_j) = \left(\overline{\gamma_2} \cup \overline{\gamma_2^{'}}\right)(v_i, v_j)$

Consider the identity map $I : V_1 \cup V_2 \to V_1 \cup V_2$

(a) (i) $\overline{\left(\mu_1 \cup \mu_1^{'}\right)}(v_i) = \left(\mu_1 \cup \mu_1^{'}\right)(v_i)$

$$= \begin{cases} \mu_1(v_i) & \text{if } v_i \in V_1 \\ \mu_1^{'}(v_i) & \text{if } v_i \in V_2 \end{cases} = \begin{cases} \overline{\mu_1}(v_i) & \text{if } v_i \in V_1 \\ \overline{\mu_1^{'}}(v_i) & \text{if } v_i \in V_2 \end{cases}$$

$$= \left(\overline{\mu_1} \cup \overline{\mu_1^{'}}\right)(v_i)$$

(ii) $$\overline{\left(\gamma_1 \cup \gamma_1^{'}\right)}(v_i) = \left(\gamma_1 \cup \gamma_1^{'}\right)(v_i)$$

$$= \begin{cases} \gamma_1(v_i) & if\ v_i \in V_1 \\ \gamma_1^{'}(v_i) & if\ v_i \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\gamma_1}(v_i) & if\ v_i \in V_1 \\ \overline{\gamma_1^{'}}(v_i) & if\ v_i \in V_2 \end{cases}$$

$$= \left(\overline{\gamma_1} \cup \overline{\gamma_1^{'}}\right)(v_i)$$

(b) (i) $$\overline{\left(\mu_2 \cup \mu_2^{'}\right)}(v_i, v_j) = \left(\mu_1 \cup \mu_1^{'}\right)(v_i).\left(\mu_1 \cup \mu_1^{'}\right)(v_j) - \left(\mu_2 \cup \mu_2^{'}\right)(v_i, v_j)$$

$$= \begin{cases} (\mu_1)(v_i).\mu_1(v_j) - \mu_2(v_i, v_j) & if\ (v_i, v_j) \in E_1 \\ \mu_1^{'}(v_i).\mu_1^{'}(v_j) - \mu_2^{'}(v_i, v_j) & if\ (v_i, v_j) \in E_2 \\ \mu_1(v_i).\mu_1^{'}(v_j) - 0 & if v_i \in v_1, v_j \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\mu_2}(v_i, v_j) & if\ (v_i, v_j) \in E_1 \\ \overline{\mu_2^{'}}(v_i, v_j) & if\ (v_i, v_j) \in E_2 \\ \mu_1(v_i).\mu_1^{'}(v_j) & if\ v_i \in V_1, v_j \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\mu_2} \cup \overline{\mu_2^{'}}(v_i, v_j) & if\ (v_i, v_j) \in E_1\ or\ E_2 \\ \mu_1(v_1).\mu_1^{'}(v_1) & if\ (v_i, v_j) \in E' \end{cases}$$

$$= \left(\ \overline{\mu_2} + \overline{\mu_2^{'}}\ \right)(v_i, v_j)$$

(b) (ii) $$\overline{\left(\gamma_2 \cup \gamma_2^{'}\right)}(v_i, v_j) = \left(\gamma_1 \cup \gamma_1^{'}\right)(v_i).\left(\gamma_1 \cup \gamma_1^{'}\right)(v_j) - \left(\gamma_2 \cup \gamma_2^{'}\right)(v_i, v_j)$$

$$= \begin{cases} (\gamma_1)(v_i).\gamma_1(v_j) - \gamma_2(v_i, v_j) & if\ (v_i, v_j) \in E_1 \\ \gamma_1^{'}(v_i).\gamma_1^{'}(v_j) - \gamma_2^{'}(v_i, v_j) & if\ (v_i, v_j) \in E_2 \\ \gamma_1(v_i).\gamma_1^{'}(v_j) - 0 & if v_i \in v_1, v_j \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\gamma_2}(v_i, v_j) & if\ (v_i, v_j) \in E_1 \\ \overline{\gamma_2^{'}}(v_i, v_j) & if\ (v_i, v_j) \in E_2 \\ \gamma_1(v_i).\gamma_1^{'}(v_j) & if\ v_i \in V_1, v_j \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\gamma_2} \cup \overline{\gamma_2^{'}}(v_i, v_j) & if\ (v_i, v_j) \in E_1\ or\ E_2 \\ \gamma_1(v_1).\gamma_1^{'}(v_1) & if\ (v_i, v_j) \in E' \end{cases}$$

$$= (\overline{\gamma_2} + \overline{\gamma_2^{'}})(v_i, v_j)$$

Theorem 3.8

Let $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$ be two Strong Interval Valued Fuzzy Graphs. Then $G_1 \circ G_2$ is a strong Interval Valued Fuzzy Graph.

Proof

Let $G_1 \circ G_2 = G = \langle V, E \rangle$ where V= V_1 x V_2 and

$$E = \{(u, u_2)(u, v_2) : u \in V_1, u_2 v_2 \in E_2\} \cup \{(u_1, w) : (v_1, w) : w \in V_2, u_1 v_1 \in E_1\}$$

$\cup \{(u_1, u_2)(v_1, v_2) : u_1 v_1 \in E_1, u_2 \neq v_2\}$.

(i) $\mu_2\{(u, u_2)(u, v_2)\} = \mu_1(u).\mu_2(u_2 v_2)$

$= \mu_1(u).\mu_1^{'}(u_2).\mu_1^{'}(v_2)$, since G_2 is strong

$= \mu_1(u).\mu_1^{'}(u_2).\mu_1(u).\mu_1^{'}(v_2)$

$= (\mu_1 \circ \mu_1^{'})(u, u_2).(\mu_1 \circ \mu_1^{'})(u, v_2)$

$\gamma_2\{(u, u_2)(u, v_2)\} = \gamma_1(u).\gamma_2(u_2 v_2)$

$= \gamma_1(u).\gamma_1^{'}(u_2).\gamma_1^{'}(v_2)$, since G_2 is strong

$= \gamma_1(u).\gamma_1^{'}(u_2).\gamma_1(u).\gamma_1^{'}(v_2)$

$= (\gamma_1 \circ \gamma_1^{'})(u, u_2).(\gamma_1 \circ \gamma_1^{'})(u, v_2)$

(ii) $\mu_2((u_1, w)(v_1, w)) = \mu_1^{'}(w).\mu_2(u_1, v_1)$

$= \mu_1^{'}(w).\mu_1(u_1).\mu_1(v_1)$, since G_1 is strong

$= \mu_1^{'}(w).\mu_1(u_1).\mu_1^{'}(w).\mu_1(v_1)$

$= (\mu_1 \circ \mu_1^{'})(u_1, w).(\mu_1 \circ \mu_1^{'})(v_1, w)$

$\gamma_2((u_1, w)(v_1, w)) = \gamma_1^{'}(w).\gamma_2(u_1, v_1)$

$= \gamma_1^{'}(w).\gamma_1(u_1).\gamma_1(v_1)$, since G_1 is strong

$= \gamma_1^{'}(w).\gamma_1(v_1).\gamma_1^{'}(w).\gamma_1(v_1)$

$= (\gamma_1 \circ \gamma_1^{'})(u_1, w).(\gamma_1 \circ \gamma_1^{'})(v_1, w)$

(iii) $\mu_2(u_1, u_2)(v_1, v_2) = \mu_2(u_1, v_1).\mu_1^{'}(u_2).\mu_1^{'}(v_2)$

$= \mu_1(u_1).\mu_1(v_1).\mu_1^{'}(u_2).\mu_1^{'}(v_2)$, since G_1 is strong

$= \mu_1(u_1) | .\mu_1^{'}(u_2).\mu_1(v_1).\mu_1^{'}(v_2)$

$= (\mu_1 \circ \mu_1^{'})(u_1, u_2).(\mu_1 \circ \mu_1^{'})(v_1, v_2)$

$$\gamma_2(u_1, u_2)(v_1, v_2) = \gamma_2(u_1, v_1).\gamma_2(u_1, v_1).\gamma_1^{'}(v_2)$$

$= \gamma_1(u_1).\gamma_1(v_1).\gamma_1^{'}(u_2).\gamma_1^{'}(v_2)$, since G_1 is strong

$= \gamma_1(u_1).\gamma_1^{'}(u_2).\gamma_1(v_1).\gamma_1^{'}(v_2)$

$= (\gamma_1 \circ \gamma_1^{'})(u_1, u_2).(\gamma_1 \circ \gamma_1^{'})(v_1, v_2)$

From (i), (ii), (iii), G_1 0 G_2 is a strong Interval valued Fuzzy Graph.

IV CONCLUSION

It is well known that interval-valued fuzzy sets constitute a generalization of the notion of fuzzy sets. The interval-valued fuzzy models give more flexibility and compatibility to the system as compared to the classical and fuzzy models. So, we have introduced interval-valued fuzzy graphs and have presented several properties in this paper. The further study of interval-valued fuzzy graphs may also be extended with the following projects.

- Data base theory
- Expert systems
- Neural Networks
- Shortest paths in networks

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