FREQUENCY DOMAIN FLUTTER ANALYSIS OF AIRCRAFT WING IN SUBSONIC FLOW

Ms.K.Niranjana[1], Mr.A.Daniel Antony[2]

[1]*UG Student, Department of Aerospace Engineering, Karunya University, (India)*

[2]*Assistant professor, Department of Aerospace Engineering, Karunya University, (India)*

ABSTRACT

This paper presents simple binary flutter model developed with the simplified unsteady terms. Binary flutter is the most important of all aero elastic phenomena and it is difficult to predict. The Aero elastic equation of motion which is frequency dependent, along with its dynamic aero elastic behavior for the flapping and pitching of the wing is derived. Further the equation is scripted as a MATLAB code and the figure obtained while the command is executed. From which the frequencies and damping ratio at different airspeed is calculated and the velocity at the flutter point is found.

Keywords: Cessna 152,Crictical Velocity, Damping ratio,Flutter point, Frequency

I INTRODUCTION

Flutter is a dynamic aero elasticity phenomena and in most cases it is very difficult to predict it is an unstable self-excited vibration in which the structure extracts energy from the air stream and often results in catastrophic structural failure [1]. It is a dynamic instability of an elastic structure in a fluid flow, caused by positive feedback between the body deflection and the force exerted by the fluid flow. Classical binary flutter occurs when the aerodynamics forces associated with the motion in two modes of vibration because the modes to couple in unfavorable manner that causes flutter In a linear system 'flutter point' is the point at which the structure is undergoing simple harmonic motion-zero net damping and so any further decrease in net damping will result in self-oscillation and eventual failure.Net damping is the sum of the structure's natural positive damping and the negative damping of the aerodynamics force[2].

Flutter is classified into two types: hard flutter and soft flutter.

- Hard flutter is net damping decreases very suddenly and very close to flutter point.
- Soft flutter is net damping decrease gradually [1].

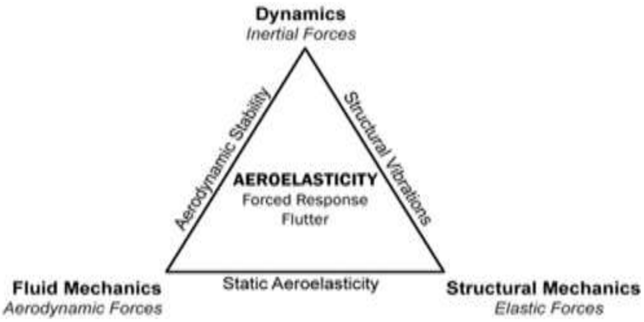

Fig.1.1Collar's triangle diagram

1.1 Literature Review

Based on aerodynamic identification technology in which unsteady CFD (computational fluid dynamics) is used, the unsteady aerodynamic reduced order models (ROM) are constructed. Coupled with structural equations, we get the analyzable models for transonic aeroelasticity in state-space. A Mach number flutter trend of a typical airfoil section with a control surface is analyzed and agrees well with that of CFD/CSD (computational structural dynamics) direct coupling method. Then we study the effect of the structural parameters (natural frequency and the flap static unbalance) of the control surface on the transonic flutter system [3].

The aerodynamic lift and moment deduced from the aerodynamic theories, one Gaussian white noise force was also added to the lift force. Then the spectral density of response was calculated with respect to the frequency response of the system as well as the spectral density of the excitation. The variance of the response was determined with respect to the airspeed. The flutter speed was obtained by investigating the variation of the response-variance against the airspeed [4].

II EQUATION OF MOTION

Consider 2D aerofoil with flexural axis positioned a distance ec after of the aerodynamics centre and ab after mid chord.

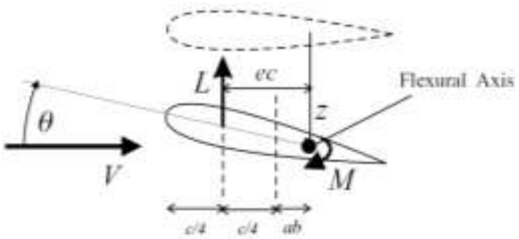

Fig 2.1 2-D aerofoil with flexural axis

Classical solution for lift and moment about flexural axis, per unit span for an aerofoil undergoing oscillatory harmonic motion (flutter)

$$L=\pi\rho b^2\left[\ddot{z} + V\dot{\theta} - ba\ddot{\theta}\right] + 2\pi\rho b c(k)\left[\dot{z} + V\theta + b\left(\tfrac{1}{2} - a\right)\dot{\theta}\right] \tag{1}$$

$$M=\pi\rho b^2\left[ba\ddot{z} - Vb\left(\tfrac{1}{2} - a\right)\theta - b^2\left(\tfrac{1}{8} + a\right)\ddot{\theta}\right] + 2\pi\rho V b^2\left(\tfrac{1}{2} + a\right)c(k)\left[\dot{z} + V\theta + b\left(\tfrac{1}{2} - a\right)\dot{\theta}\right] \tag{2}$$

Aerodynamic damping and stiffness

$k=\frac{\omega b}{V}$; $z=z_0 e^{i\omega t}$; $\dot{z} = i\omega z_0 e^{i\omega t}$; $\theta=\theta_0 e^{i\omega t}$; $\dot{\theta}= i\omega\theta_0 e^{i\omega t}$

By substituting aerodynamic damping and stiffness in equation 1 & 2

We get,

The lift and moment per unit span for an airfoil for a particular reduced frequency

$$L=\rho V^2\left(L_z Z + L_{\dot{z}}\frac{b\dot{z}}{V} + L_\theta b\theta + L_{\dot{\theta}}\frac{b^2\dot{\theta}}{V}\right) \tag{3}$$

$$M=\rho V^2\left(M_z bz + M_{\dot{z}}\frac{b^2\dot{z}}{V} + M_\theta b^2\theta + M_{\dot{\theta}}\frac{b^3\dot{\theta}}{V}\right) \tag{4}$$

Where V- True airspeed, ρ - Density

Pitching term (M_θ) is negative but initially it is considered to be constant and also it differs numerically by a factor of four, which occurs because the unsteady aerodynamic derivatives are derived in terms of the reduced frequency k rather than the frequency parameter v.

III AERO ELASTIC EQUATION OF MOTION

3.1 Derivation

Consider a rectangular wing of span s and chord c is rigid. It has a 2 rotational spring at root to provide flap (k) and pitch (θ)degrees of freedom. The spring is attached in the flexural axis shown in fig 3.1.1. The wing is assumed to have uniform mass distribution and the mass axis lies on the mid chord.

The displacement z on the wing due to aileron and flap movement

$$z(x, y, t) = yk(t) + (x - x_f)\theta(t) \quad (5)$$

Where

k – Generalized coordinate for pitch angle

θ – Generalized coordinate for flap angle

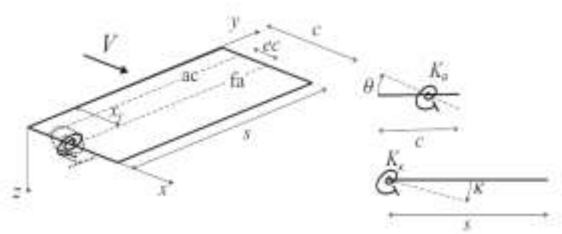

Fig 3.1.1 Aircraft wing with springs attached

The equation of motion can be found using LaGrange's equations. Total energy can be calculated by kinetic energy and potential energy.

The kinetic energy due to dynamic motion

$$T = \int_{wing} \frac{1}{2} dm\, \dot{z}^2 \quad (6)$$

Differentiating z w.r.t x&y

$$\dot{z} = y\dot{k} + (x - x_f)\dot{\theta}$$

$$T = \int_0^s \int_0^c [y\dot{k} + (x - x_f)\dot{\theta}]^2\, dx\, dy \quad (7)$$

The potential energy due to spring at the root of the wing

$$U = \frac{1}{2} K_k k^2 + \frac{1}{2} K_\theta \theta^2 \quad (8)$$

Applying Lagrange's equation (i.e. Kinetic energy & Potential energy) for both generalised coordinates

We get,

$$\frac{dT}{dt}\left(\frac{\partial T}{\partial k}\right) = m\left[\frac{s^3 c}{3}\ddot{k} + \frac{s^2}{2}\left(\frac{c^2}{2} - x_f c\right)\ddot{\theta}\right] \quad (9)$$

$$\frac{dT}{dt}\left(\frac{\partial T}{\partial \theta}\right) = m\left[\frac{s^2}{2}\left(\frac{c^2}{2} - x_f c\right)\ddot{k} + s\left(\frac{c^3}{3} - c^2 x_f + x_f^2 c\right)\ddot{\theta}\right] \quad (10)$$

$$\frac{\partial U}{\partial k} = K_k k \quad (11)$$

$$\frac{\partial U}{\partial \theta} = K_\theta \theta \quad (12)$$

Where

K_k- Flapping coefficient

K_θ-Pitching coefficient.

Writing the equations in a matrix form which leading to equation of motion for the wing, without any aerodynamics forces

$$\begin{bmatrix} \frac{ms^3c}{3} & \frac{ms^2}{2}\left(\frac{c^2}{2} - x_f c\right) \\ \frac{ms^2}{2}\left(\frac{c^2}{2} - x_f c\right) & ms\left(\frac{c^3}{3} - c^2 x_f + x_f{}^2 c\right) \end{bmatrix} \begin{Bmatrix} \ddot{k} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} K_k & 0 \\ 0 & K_\theta \end{bmatrix} \begin{Bmatrix} k \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Assuming inertia matrix

$$\begin{bmatrix} \frac{ms^3c}{3} & \frac{ms^2}{2}\left(\frac{c^2}{2} - x_f c\right) \\ \frac{ms^2}{2}\left(\frac{c^2}{2} - x_f c\right) & ms\left(\frac{c^3}{3} - c^2 x_f + x_f{}^2 c\right) \end{bmatrix} \quad \begin{bmatrix} I_k & I_{k\theta} \\ I_{k\theta} & I_\theta \end{bmatrix}$$

Where,

$$I_k = \int_0^s y^2 dm \quad (13)$$

$$I_\theta = \int_0^c \int_0^s (x - x_f) y dm \quad (14)$$

$$I_{k\theta} = \int_0^c (x - x_f)^2 dm \quad (15)$$

These terms are the moment of inertia in flap and pitch and the product moment of inertia

The pitch and flap natural frequencies are

$$\omega_k = \sqrt{\frac{K_k}{I_k}} \quad (16)$$

$$\omega_\theta = \sqrt{\frac{K_\theta}{I_\theta}} \quad (17)$$

Generalised forces $Q_k$and Q_θ (i.e. assumed unsteady aerodynamic forces) will act on the wing. It is written in terms of aerodynamics derivatives for a particular reduced frequency.

Applying strip theory,

Lift and pitching moment about the flexural axis for each elemental strip theory dy of

$$dL = \frac{1}{2}\rho V^2 c dy a_w \left(\frac{\dot{y}}{k} + \theta\right) \tag{18}$$

$$dM = \frac{1}{2}\rho V^2 c^2 \left[e a_w \left(\frac{y\dot{k}}{V} + \theta\right) + M_{\dot{\theta}} \frac{\dot{\theta} c}{4V}\right] \tag{19}$$

Where

$y\dot{k}$- Effective heave velocity

The incremental work done over the wing due to aerodynamic force and moment through incremental deflection δk & $\delta\theta$ of the wing is

$$\delta W = \int_{wing} [dL(-y\delta k) + dM\delta\theta$$

And therefore the generalised forces are

$$Q_k = \frac{\partial(\delta W)}{\partial(\delta k)} = -\int_0^s y dL$$

$$Q_k = -\frac{1}{2}\rho V^2 c s^2 a_w \left(\frac{\dot{k}s}{3V} + \frac{\theta}{2}\right) \tag{20}$$

$$Q_\theta = \frac{\partial(\delta W)}{\partial(\delta\theta)} = \int_0^s dM$$

$$Q_\theta = \frac{1}{2}\rho V^2 s c^2 \left[e a_w \left(\frac{\dot{k}s}{2V} + \theta\right) + M_{\dot{\theta}} \frac{\dot{\theta} c}{4V}\right] \tag{21}$$

Above equations can be written in the matrix to form,

The full aero elastic equation of motion become

$$\begin{bmatrix} I_k & I_{k\theta} \\ I_{k\theta} & I_\theta \end{bmatrix}\begin{Bmatrix} \ddot{k} \\ \ddot{\theta} \end{Bmatrix} + \rho V \begin{bmatrix} \frac{cs^3 a_w}{6} & 0 \\ -\frac{e a_w s^2 c^2}{4} & -\frac{sc^3}{8} M_{\dot{\theta}} \end{bmatrix}\begin{Bmatrix} \dot{k} \\ \dot{\theta} \end{Bmatrix} + \left\{\rho V^2 \begin{bmatrix} 0 & \frac{cs^2 a_w}{4} \\ 0 & -\frac{e a_w s c^2}{2} \end{bmatrix} + \begin{bmatrix} K_k & 0 \\ 0 & K_\theta \end{bmatrix}\right\}\begin{Bmatrix} k \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

The mass and the stiffness matrices are symmetric while the aerodynamic matrices are non-symmetric. Thus the 2 DoF are coupled and it is coupled that can give rise to flutter.

General Form of the Aero Elastic Equation

$$A\ddot{q} + (\rho V B + D)\dot{q}(\rho V^2 C + E)q = 0$$

Where,

A – Structural inertia

B – Aerodynamic damping

C – Aerodynamic stiffness

D – Structural damping

E – Structural stiffness matrices

3.2 MATLAB Baseline parameter

The dynamic aero elastic behavior for the flapping/pitching wing can be determined at different airspeed and altitude for each flight condition and then calculating the corresponding frequencies and damping ratio.

The baseline system parameters considered, (Cessna 152) [7]

- Mass axis is at the semi-chord x_m = 0.5c=0.725m
- Flexural axis is at x_f = 0.48c=0.696m from the leading edge.
- Semi-span s=5.1m
- Chord c=1.45m
- Mass per unit area=100kg/m^2
- Flap stiffness (K_k)= $I_k(5 * 2\pi)^2$

  Sub the values in equation 13, we get

  $I_k = 6411.465m$

  Hence, K_k= 6327862.318m

  Sub these 2 values in equation 16,$\omega_k = 10$ hertz
- Pitch stiffness (K_θ)= $I_k(10 * 2\pi)^2$

  Sub the values in equation 14, we get $I_\theta = 130.188$m

  Hence, K_θ= 513963.52m

  Sub these 2 values in equation 17,$\omega_\theta = 5$ hertz
- Lift slope curve a=2π
- Non dimensional pitch damping derivative M_θ= -1.2

Using the aero elastic equation of motion, the MATLAB program have been scripted and the baseline system parameter of Cessna 152 is given as the input, the frequency and damping ratio has been found for different airspeed.

IV RESULTS AND DISCUSSION

The figure given below obtained from MATLAB code, when the command is executed.

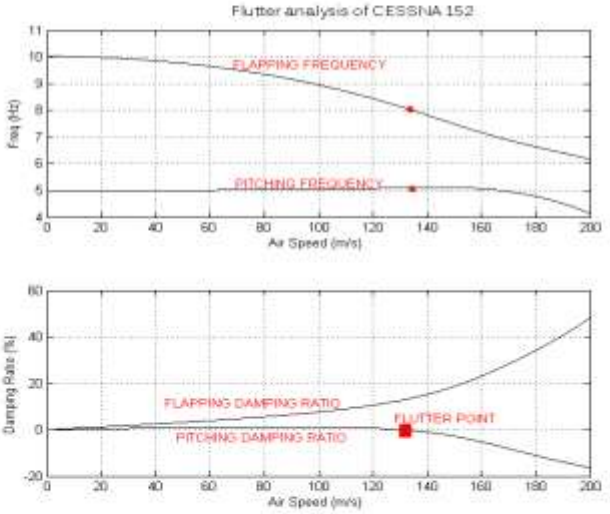

Fig 3.2.1 MATLAB Figure

The figure 3.2.1 shows how frequency and damping ratio values varies for this classical binary flutter behavior. As the airspeed increases, the frequencies begins to converge. Initially both of the damping ratio increases, but the flapping damping ratio continuous to increase, the pitching damping ratio starts to decrease and become zero at the flutter speed of 132m/s. Beyond this airspeed the pitching damping ratio becomes negative and flutter occurs. The two frequencies do not coalesce, but rather move close enough in frequency for the two modes to couple unfavorably.

Limitation:

Using this method only approximate flutter velocity can be obtain not the exact critical flutter velocity.

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