



# PERFORMANCE COMPARISON OF MRAC AND IMC CONTROLLERS FOR SPHERICAL TANK LEVEL PROCESS

Merin Mathews<sup>1</sup>, Avinash K K<sup>2</sup>

<sup>1</sup>PG Scholar, <sup>2</sup>Asst. Professor, Dept. of Electronics and Instrumentation,  
Vimal Jyothi Engineering College, Chemperi, Kannur, Kerala (India)

## ABSTRACT

Control of process parameters is one of the important problems in process industry. The process considered for modeling is spherical tank liquid level system. Control of liquid level in a spherical tank is highly non-linear due to variation in the area of cross section of level system with change in shape. The aim of this paper is to implement an optimum controller for a spherical tank. The objective of the controller is to maintain the level inside the process tank in a desired value. System identification of spherical tank system is done using black box model which is identified to be non-linear and approximated to be a First order plus dead time model. In this paper performance of model reference adaptive controller (MRAC) is compared with Internal Model Controller (IMC) based PID controller. The comparative study of set point tracking and performance estimation of a liquid level process is carried out with the help of performance indices like ITAE, IAE and ISE.

**Keywords:** Matlab Simulink, MRAC, IMC, Nonlinear System.

## I. INTRODUCTION

The industrial application of liquid level control is tremendous especially in refineries petroleum and chemical process industries. Usually, level control exists in some of the control loops of a process control system. An evaporator system is one example in which a liquid level control system is a part of control loop. Evaporators are used in many chemical process industries for the purpose of separation of chemical products. Level control is also very important for mixing reactant process. The quality of the product of the mixture depends on the level of the reactants in the mixing tank. Mixing reactant process is a very common process in chemical process industries and food processing industries. Many other industrial applications are concerned with level control, may it be a single loop level control or sometimes multi-loop level control. In some cases, level controls that are available in the industries are for interacting tanks. Hence, level control is one of the control system variables which are very important in process industries. Nowadays, chemical engineering systems are also at the heart of our economics. The process industries such as refineries petrol, petro-chemical industries, paper making and water treatment industries require liquids to be pumped, stored in tanks, and then pumped to another tank. In the design of control system, one often has a complicated mathematical model of a system that has been obtained from fundamental physics and chemistry. The above mentioned industries are the vital industries where liquid



level and flow control is essential. Many times the liquids will be processed by chemical or mixing treatment in the tanks, but always the level fluid in the tanks must be controlled, and the flow between tanks must be regulated. Level and flow control in tanks are the heart of all chemical engineering systems.

Chemical process present many challenging control problems due to nonlinear dynamic behavior, uncertain and time varying parameters, constraints on manipulated variable, interaction between manipulated and controlled variables, unmeasured and frequent disturbances, dead time on input and measurements. Because of the inherent nonlinearity, most of the chemical process industries are in need of conventional control techniques.

Spherical tanks find wide spread usage in gas plants. They are non-linear system because their area of cross-section keeps varying with the height of the tank. A sphere is a very strong structure. The even distribution of stresses on the sphere's surfaces means that there are no weak points. Moreover, they have a smaller surface area per unit volume than any other shape of vessel. This means, that the quantity of heat transferred from warmer surroundings to the liquid in the sphere, will be less than that for cylindrical or rectangular storage vessels. Thus causing less pressurization due to external heat. Control of a spherical tank is important, because the change in shape gives rise to the nonlinearity.

PI controller shows smooth response in An evaluation of Model based Controller Design for a Spherical Tank by S.Nithya, N.Sivakumaran, T.Balasubramanian and N.Anantharaman<sup>[1]</sup> gives the controller design is compared based on conventional Proportional Integral (PI) based on Ziegler- Nicholas settings with Internal Model Control (IMC) based on Skogestad's settings. A Digital PI controller design method has been proposed by G. Sakthivel, T. S. Anandhi and S. P.Natarajan<sup>[2]</sup> shows that for both set point and load changes the method is effective and can be used in the low cost data acquisition system. D.Pradeepkannan, Dr.S.Sathiyamoorthy have discussed State Feedback with integral controller to implement in a Non-linear Spherical Tank SISO process<sup>[3]</sup>. It has been shown that State feedback with integral controller can cope with the tank non-linear characteristics at all operating points. Rinu Raj R R and L.D Vijay Anand proposed CDM based PI controller for a Non-linear Spherical tank<sup>[4]</sup>. S.Rajendran, Dr.S.Palani proposed Fuzzy logic controller (FLC) for a spherical tank to control liquid level<sup>[5]</sup>. The evaluation of Fuzzy Model Reference Learning Control by S.Ramesh and S.AbrahamLincon<sup>[6]</sup> shows that the incorporation of FMRLC in the control loop in spherical tank system provides a superior tracking performance than the NNIMC and conventional PI mode.

D.Pradeepkannan, Dr.S.Sathiyamoorthy proposed Control of a Nonlinear Spherical Tank Process Using GA Tuned PID Controller<sup>[7]</sup>. Evolutionary Algorithms based Controller Optimization for a Real Time Spherical Tank System by GanapathySivagurunathan and KaliannanSaravanan<sup>[8]</sup> and from this the PSO tuned servo and regulatory operations compared to ZN and GA based PI controller on all the operating regions.

The paper is organized as follows: Section I discusses about non linear level process, in Section II and III discusses about experimental setup and modeling of the system and controller design like MRAC and Internal Model Controller respectively. The simulation results are presented in Section IV. The conclusions are given in Section V.

## II. PROPOSED WORK

### 2.1 Experimental Setup

The control parameter which we have chosen is the level. Capacity sensors and level transmitter arrangement senses the level from the process and converts into electrical signal. Then the electrical signal is fed to the I-V

converter which in turn provides corresponding voltage to the controller. The closed loop control system is one in which control action is dependent on the output. This maintain water level in storage tank. The system performs this task by continuously sensing the level in the tank and adjusting a supply valve to add more or less water to the tank. The actual storage tank level sensed by the level transmitter is fed back to the level controller. This feedback is compared with the desired level. Now the controller describes the control action and it is given to the I-V and then to I-P converter. The final control element is now controlled by the resulting air pressure. This in turn control the inflow to the spherical tank and level is maintained.

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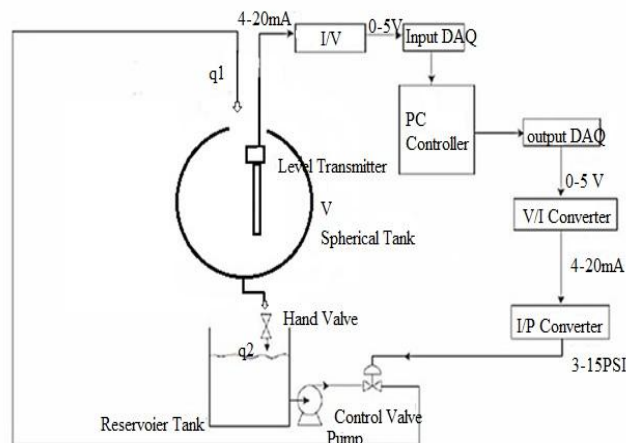


Figure 1: Block Diagram of Process

Figure 1 shows the block diagram of the system. The flow rate to the spherical tank is regulated by changing the stem position of the pneumatic valve by passing control signal from computer to the current to pressure converter through DAQ CARD and voltage to current converter. The operation current for regulating the valve position is 4-20mA, which is converted to 3-15psi of compressed air pressure. The water level inside the tank is measured with the differential pressure transmitter which is calibrated and is converted to an output current of 4-20mA. This output current is converted into 0-5V using I/V converter, which is given to the controller through DAQ CARD.

## 2.2 Mathematical Modelling

It is quite often the case that we have to design the control system for a process before the process has been constructed. In such a case we need a representation of the process in order to study its dynamic behavior. This representation is usually given in terms of a set of mathematical equations whose solution gives the dynamic or static behavior of the process. The process considered is the spherical tank in which the level of the liquid is desired to be maintained at a constant value. This can be achieved by controlling the input flow into the tank. The valve connection diagram for spherical tank is shown in figure 2.

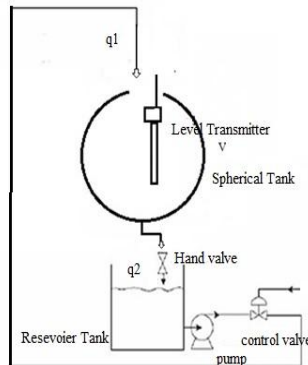


Figure:2 Valve Connection Diagram

Using the law of conservation of mass,

The rate of accumulation in tank system = rate of mass flow in- rate of mass flow out. 4  
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$$\frac{d\rho V}{dt} = \rho q_1 - \rho q_2 (A)$$

$$\rho \frac{dV}{dt} = \rho q_1 - \rho q_2 (B)$$

$$\frac{dV}{dt} = q_1 - q_2 (1)$$

Substituting parameters of \$q\_2\$ we get the equation as,

$$\frac{dV}{dt} = q_1 - c \sqrt{h} (2)$$

where \$V\$ volume of spherical tank at height \$h\$, Volume of sphere, \$V = \frac{4}{3} \pi r^3\$. Substituting the value of \$V\$ in equation (2) we get,

$$\frac{1}{6} \pi \frac{dh^3}{dt} = q_1 - c \sqrt{h} (3)$$

After all substitution and linearization get the transfer function model as,

$$\frac{H(s)}{Q_1(s)} = \frac{R_t}{(R_t S^{\frac{1}{2}} \frac{1}{2} \pi h_s^2 + 1)} (4)$$

$$\frac{H(s)}{Q_1(s)} = \frac{R_t}{\tau_s + 1} (5)$$

Where \$\tau\$-time constant.

The actual transfer function contains some time delay so it becomes FOPTD process.

$$\frac{H(s)}{Q_1(s)} = \frac{R_t}{\tau_s + 1} e^{-\theta s} (6)$$

Assume dead time = 2sec

For region 1: 0-2cm

$$G(s) = \frac{0.036}{2.55 + 1} e^{-2s} (7)$$

Region 2: 2-5cm

$$G(S) = \frac{0.057}{33S + 1} e^{-2s} \quad (8)$$

Region 3: 5-10cm

$$G(S) = \frac{0.094}{103S + 1} e^{-2s} \quad (9)$$

Region 4: 10-18cm

$$G(S) = \frac{0.1395}{228S + 1} e^{-2s} \quad (10)$$

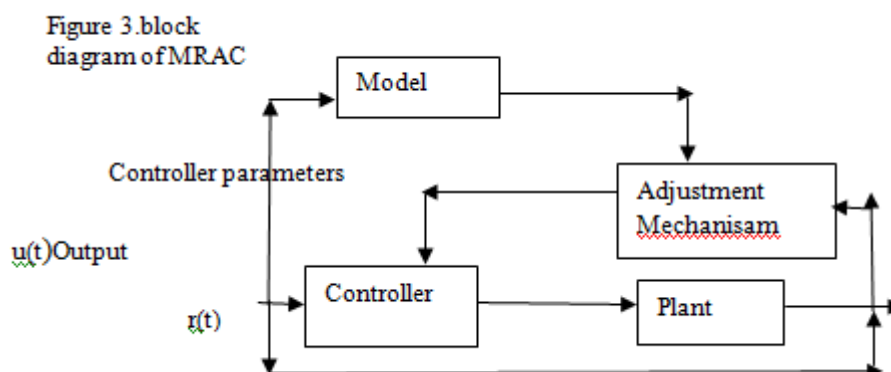
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### III. CONTROLLER DESIGN

#### 3.1 Model Reference Adaptive Controller

To adapt means to change behaviour to conform to a new circumstances. Adaptive controller is thus controller that can modify its behaviour in response to change in dynamics of process and the character of disturbances. Adaptive control is a specific type of control where the process is controlled in closed-loop, where knowledge about the system characteristics are obtained on-line while the system is operating. Based upon refreshed information obtained during normal operation, specific interventions in the control loop are made in order to fulfil the control goal. Adaptive Control covers a set of techniques which provide a systematic approach for automatic adjustment of controllers in real time, in order to achieve or to maintain a desired level of control system performance when the parameters of the plant dynamic model are unknown and/or change in time.

Adaptive control is the control method used by a controller which must adapt to a controlled system with parameters which vary, or are initially uncertain. For example, as an aircraft flies, its mass will slowly decrease as a result of fuel consumption; a control law is needed that adapts itself to such changing conditions. Adaptive control is different from robust controlling that it does not need apriority information about the bounds on these uncertain or time-varying parameters; robust control guarantees that if the changes are within given bounds the control law need not be changed, while adaptive control is concerned with control law changing themselves.



Reference model is used to give an idyllic response of the adaptive control system to the reference input. Controller is usually described by a set of adjustable parameters. In this paper only one parameter  $\theta$  is used to describe the control law. The value of  $\theta$  is primarily dependent on adaptation gain. Adjustment mechanism is used to alter the parameters of the controller so that actual plant could track the reference model. Mathematical approaches like MIT rule, Lyapunov theory and theory of augmented error can be used to develop the adjusting



mechanism. Outer loop adjusts the controller parameters in such a way that the error (y-ym) is small. The design of the controller is done now in order that the error between the output of the plant and the output of the reference model is identically.

It is used to adjust the parameters in the control law. Adaptation law searches for the parameters such that the response of the plant which should be same as the reference model. It is designed to guarantee the stability of the control system as well as conversance of tracking error to zero. Mathematical techniques like MIT rule, Lyapunov theory and augmented error theory can be used to develop the adaptation mechanism. In this paper the MIT rule is used for this purpose.

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To present the MIT rule, we will consider the closed loop system in which the controller has one adjustable parameter  $\theta$ . The desired closed-loop response is specified by a model whose output is  $y_m$ . Let e be the error between the output y of the closed loop system and the output  $y_m$  of the model. One possibility is to adjust parameters in such a way that the loss function is minimized.

$$J(\theta) = -\frac{1}{2}e^2 \tag{11}$$

$$\frac{d\theta}{dt} = -\gamma \frac{dJ}{d\theta} = -\gamma e \frac{de}{d\theta} \tag{12}$$

The gradient method gives,

$$\frac{d\theta}{dt} = -\gamma \frac{\partial e}{\partial \theta} \text{sign } e \tag{13}$$

Consider a system described by the model

$$\frac{dy}{dt} = -ay + bu \tag{14}$$

$$\frac{dy_m}{dt} = -a_m y_m + b_m u_c \tag{15}$$

$$u(t) = \theta_1 u_c(t) - \theta_2 y(t) \tag{16}$$

he parameters of the controller are,

$$\theta_1 = \theta_1^s = \frac{b_m}{b} \tag{17}$$

$$\theta_2 = \theta_2^s = \frac{a_m - a}{b} \tag{18}$$

So,  $\frac{dy}{dt} = \frac{dy_m}{dt}$

To apply the MIT rule, introduce the error

$e = y - y_m$  finally,

$$e = \frac{b \theta_1}{p + a + b \theta_2} u_c - \frac{b_m}{p + a_m} u_c \tag{19}$$

Then the following equation for updating the controller parameters:

$$\frac{d\theta_1}{dt} = -\gamma \left( \frac{a_m}{p + a_m} u_c \right) e \tag{20}$$

$$\frac{d\theta_2}{dt} = \gamma \left( \frac{a_m}{p + a_m} y \right) \varepsilon \quad 21$$

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In these equations we have combined parameters  $b$  and  $a_m$  with the adaptation gain  $\gamma'$ , since they appear as the product  $\gamma' b/a_m$ .

### 3.2 Internal Model Controller (Imc)

Internal Model Control(IMC) is a commonly used technique that provides a transparent mode for the design and tuning of various types of control. The ability of proportional-integral (PI) and proportional-integral-derivative (PID) controllers to meet most of the control objectives has led to their widespread acceptance in the control industry. The Internal Model Control (IMC)-based approach for controller design is one of them using IMC and its equivalent IMC based PID to be used in control applications in industries. It is because, for practical applications or an actual process in industries PID controller algorithm is simple and robust to handle the model inaccuracies and hence using IMC-PID tuning method a clear trade-off between closed-loop performance and robustness to model inaccuracies is achieved with a single tuning parameter.

Also the IMC-PID controller allows good set-point tracking but sulky disturbance response especially for the process with a small time-delay/time-constant ratio. But, for many process control applications, disturbance rejection for the unstable processes is much more important than set point tracking. Hence, controller design that emphasizes disturbance rejection rather than set point tracking is an important design problem that has to be taken into consideration.

In process control applications, model based control systems are often used to track set points and reject low disturbances. The internal model control (IMC) philosophy relies on the internal model principle which states that if any control system contains within it, implicitly or explicitly, some representation of the process to be controlled then a perfect control is easily achieved. In particular, if the control scheme has been developed based on the exact model of the process then perfect control is theoretically possible.

The IMC design procedure is exactly the same as the open loop control design procedure. Unlike open loop control, the IMC structure compensates for disturbances and model uncertainties. The IMC filter tuning parameter “ $\lambda$ ” is used to avoid the effect of model uncertainty. The normal IMC design procedure focuses on set point

responses but with good set point responses good disturbance rejection is not assured, especially those occurring at the process inputs. A modification in the design procedure is proposed to enhance input disturbance rejection and to make the controller internally stable.

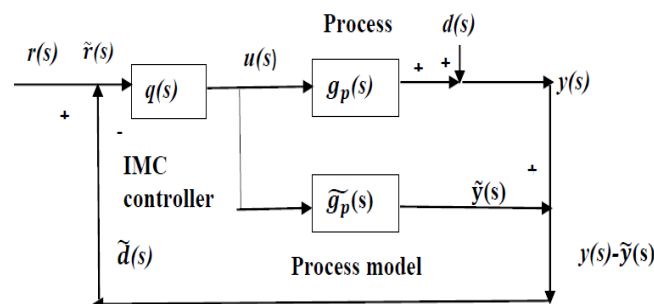


Figure 4: The IMC structure



Distinguishing characteristic of this structure I the process model, this is parallel to the actual process (plant). ‘~’ generally represent the signals associated with the model. Subscript (such as m) also represents the model. Consider a process model  $\tilde{g}_p(s)$  for an actual process or plant  $g_p(s)$ . The controller  $q(s)$  is used to control the process in which the disturbances  $d(s)$  enter into the system. The various steps in the Internal Model Control (IMC) system design procedure are:

Step1: Factorization

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It includes factorizing the transfer function into invertible and non invertible parts. The factor containing right hand poles, zeros or time delays become the poles when the process model is inverted leading to internal stability. So this is non invertible part which has to be removed from the transfer function. Mathematically, it is given as

$$\tilde{g}_p(s) = \tilde{g}_{p+}(s)\tilde{g}_{p-}(s) \quad 22$$

Where  $\tilde{g}_{p+}(s)$  – non invertible part

$\tilde{g}_{p-}(s)$ -invertible part

Step2: Form the idealized IMC controller.

The ideal internal model controller is the inverse of invertible portion of the process model.

$$\tilde{q}(s) = \tilde{g}_{p+}^{-1}(s) \quad 23$$

Step3: Adding filter

Now a filter is added to make the controller at least semi-proper because a transfer function is not stable if it is improper. A transfer function is known as proper if the order of the denominator is greater than the order of the numerator and for exactly of the same order the transfer function is known as semi-proper.

$$q(s) = \tilde{q}(s)f(s) = \tilde{g}_{p+}^{-1}(s)f(s) \quad 24$$

Where,

$$f(s) = \frac{1}{\lambda s + 1} \quad 25$$

Step4: Adjust the filter tuning parameter.

Adjust the filter tuning parameter  $\lambda$  to vary the speed of response of closed loop system.

In IMC formulation the controller  $q(s)$  based directly on the good part of the process transfer function. There only one tuning parameter the closed loop time constant,  $\lambda$ . The IMC based PID tuning parameters are then a function of this closed loop time constant. The selection of the closed loop time constant is directly related to the robustness (sensitivity to model error) of the closed loop system.

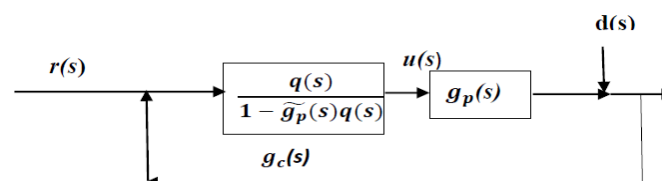


Figure 5: The Equivalent Feedback form to IMC





The equivalent feedback form to IMC is shown in figure 5 .This reformation is advantageous because we find that a PID controller often results when the IMC design procedure is used. Also the standard IMC block diagram cannot be used for unstable systems. So this feedback form be used for those cases. The standard feedback controller is a function of the internal model  $\bar{g}_p(s)$  and internal model controller  $q(s)$  shown in equation below.

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$$g_c(s) = \frac{q(s)}{1 - \bar{g}_p(s)q(s)} \quad 26$$

Now find the PID equivalent

$$g_c(s) = \frac{q(s)}{1 - \bar{g}_p(s)q(s)} = \frac{\tilde{q}(s)f(s)}{1 - \bar{g}_p(s)\tilde{q}(s)f(s)} \quad 27$$

$$= \frac{\tilde{q}(s)f(s)}{1 - \bar{g}_p - (s)\bar{g}_p + (s)\bar{g}_p - (s)^{-1}f(s)} \quad 28$$

$$= \frac{\tilde{q}(s)f(s)}{1 - \bar{g}_p + (s)f(s)} = \left[ \frac{1}{K_p} \right] \frac{(\tau_p s + 1)(0.5\theta s + 1)}{(\lambda + 0.5\theta)s} \quad 29$$

It is compared with conventional PID as

$$PID(s) = K_c \left( 1 + \frac{1}{T_i s} + sT_d \right) \quad 30$$

$$PID(s) = K_c \left( 1 + \frac{1}{T_i s} + sT_d \right) \quad 31$$

$$K_c = \frac{\tau + \frac{\theta}{2}}{K(\lambda + \frac{\theta}{2})} \quad 32$$

$$T_i = \frac{\theta}{2} + \tau \quad 33$$

$$T_d = \frac{\frac{\theta}{2} + \tau}{2(\frac{\theta}{2} + \tau)} \quad 34$$

For fast response and good robustness the tuning parameter is given by equation [1]  $\lambda = \theta$ .

#### IV. SIMULATION RESULT

At first, the mathematical model of conical tank level process is derived in terms of differential equation and an open loop response is obtained by performing step test in Matlab.

The process is identified and closed loop control performances of various MRAC and Adaptive PID controllers were studied and results are presented in figures for two regions. Response and comparison of conventional controllers and fuzzy controllers are below.

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For region 1:

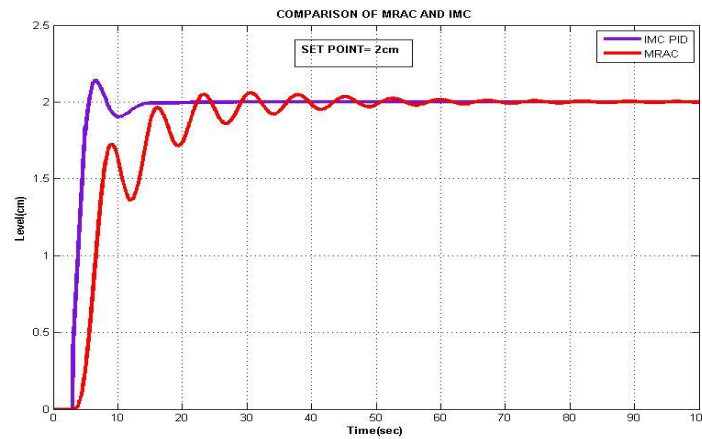


Figure6: Comparison of IMC PID with MRAC region1 with set point 2cm

For region 2:

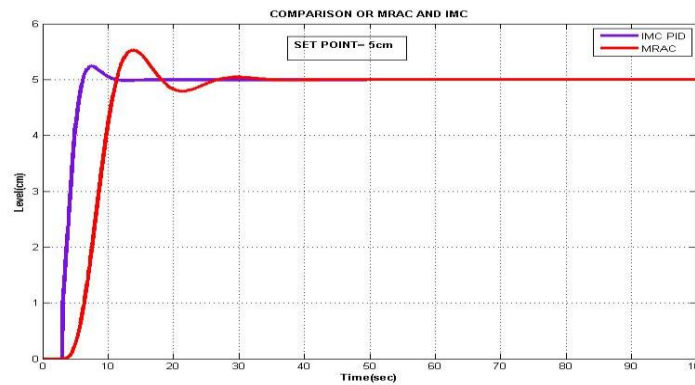


Figure7: Comparison of IMC PID with MRAC region1 with set point 5cm

For region 3:

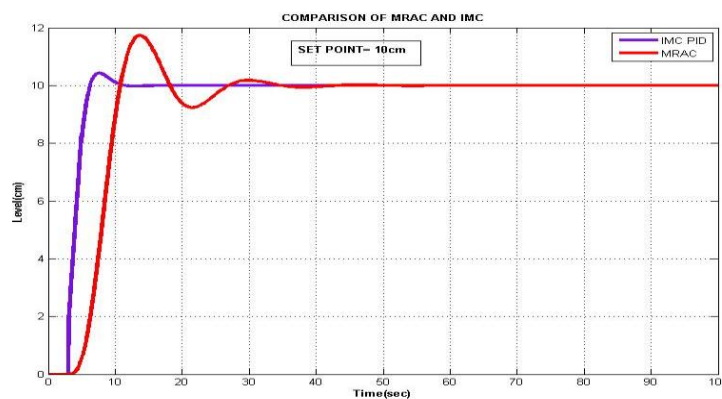


Figure8: Comparison of IMC PID with MRAC Region1 with Set Point 10cm

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For region 4:

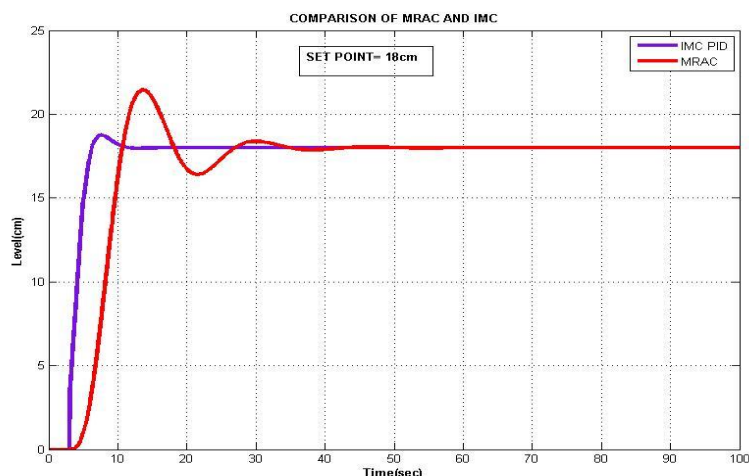


Figure9: Comparison of IMC PID with MRAC Region1 with Set Point 10cm

Table1. Performance Indices Comparison of Different Controllers

REGION	CONTROLLER	ITAE	IAE	ISE
REGION1	MRAC	140.5	16.63	21.11
	IMC PID	13.85	3.237	2.514
REGION2	MRAC	230.8	38.68	150.9
	IMC PID	57.97	16.27	64.61
REGION3	MRAC	564.9	82.04	603.7
	IMC PID	98.96	32.3	259
REGION4	MRAC	1082	150.6	1962
	IMC PID	173.2	58.08	839.9

## V. CONCLUSION

A modified IMC structure has been proposed for FOPDT process. This method ensures smooth and noise free process. A PID controllers are designed in terms of process model parameters and low pass filter time constant. The controllers perform well for set point tracking. The simulation results are also concluded that MRAC has large overshoot compared to IMC. MRAC method shows some improvement in its response but proposed IMC have much faster time response and give satisfactory and improved results.

In future a suitable advanced controller for first order system can be implemented for the control of spherical tank. The performance of that controller can be compared with MRAC and IMC PID to prove its effectiveness in nonlinear system control.

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