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T -COLORING ON SIERPINSKI NETWORK

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ABSTRACT

The Remarkable growth of different modes of communication provides the reason for many real life problems. The allocation of radio frequencies to radio transmitter network is one such problem. This paper exhibits an existing network called Sierpinski network, and concept of $\boldsymbol{T}$-coloring *and we discoverthe generalizations of the* $\boldsymbol{T}$-colorings *of Sierpinski network with their variations in parameters like size of the graph, color of the graph and the set* $\boldsymbol{T}$.

Keywords*: *Networks, T–coloring, Sierpinski Network.

I. INTRODUCTION

Graph theory is a vast and old, but its exploration in proving techniques is unique in mathematics, and its results have continuous applications in many areas of the computing, social and natural science. The paper published by Leonard Euler on the Konigsberg Bridge in 1736 is considered as the first paper in graph theory.

Graphs and networks are seen around us in our daily life which includes **technological networks** like Internet, power grids, telephone networks, transportation networks, etc., **social networks** like social graphs, affiliation networks, etc., **information networks** like World Wide Web, citation graphs, patent networks and **biological networks** like biochemical networks, neural networks, food webs, etc. Graphs provide a structural model that makes it possible to analyze and understand separate systems acting together.

Graph coloring is a unique way of graph labeling; it is an assignment to elements of graphs called “colors”subject to certain conditions. The coloring of a graph is labeling colors to vertices, edges, or faces of a plane graph. In graph theory, there are several dozen of graph coloring models described in the literature of which most of them deal with vertex-coloring and edge-coloring, but some of them are defined only for vertex-coloring or edge-coloring. In types of graph coloring, T-coloring[1]is considered the most important in practical point of view and has numerous applications that have many open problems[2] observing different types of$T$-coloring.

T **-**colorings of graphs were first introduced by Hale in connection with frequency constrained channel assignment problem[3]. The instance of T-coloring is just a pair(G, T), where G is a graph or network, and T is a subset of natural numbers. In the channel assignment problem[4], several transmitters and a forbidden set T (called T-*set*) of non-negative integers containing 0, are given. We assign a non-negative integer channel to each transmitter under a constraint for two transmitters where the difference of their channels does not fall within the given T-set.

In this paper, the succeeding section analyzesbriefly thesurvey of literature on how T-colorings came into existence in real life. The following third and fourth sections deals with an introduction on the Sierpinski network, its construction and few properties,T-coloring of Sierpinski network and its generalizations respectively. Finally, we conclude that the T-coloring of Sierpinski network has a bound.

II. SURVEY OF LITERATURE

In the year 1980, Hale proposed a graph model for the$T$-coloring problem and generalized the problem with respect to graph labeling and introduced the concept of$T$-coloring[5]. In a graph model, the transmitters are represented by the vertices of a graph and the interference between two transmitters represent the edges respectively. Also, he called any two interfering transmitters as close transmitters. A frequency assignment[6][7] is the function that assigns labels to the vertices of graph and T is the set of disallowed separations.

Generalized T-coloring problem by Hale is as follows. If $d(0) > d(1) > \cdots > d(m) > 0$are rational numbers and $\{0\} = T(0) \subset T(1) \subset \cdots \subset T(m)$ are finite subsets of Z^+. Let V be a finite subset of the vertex set of the graph and let $R = \{(T(i), d(i): i = 0,1, \dots m\}$ be a set of frequency distance constraints. If $f: V \to Z^+$satisfying the constraint $|f(u) - f(v)|$ is not an element of $T(i)$ whenever u, v and $d(u, v) \leq d(i)$, for $i = 0, 1, 2, \dots m$ then f is called a feasible assignment for V and R. In other words, any assignment in which$|f(u) - f(v)|$ is an element of $T(i)$ is prohibited. Later it became known as T-coloring of graphs.

In 1988, Roberts proposed an advancement of the channel assignment problem[8]. They considered two levelinterference namely major and minor and classified transmitters as very close and close transmitters. If the interference between two transmitters is major, then these two transmitters are known as very close transmitters and if the interference is minor then these two transmitters are known as close transmitters. Roberts proposed that close transmitters must receive different channels and very close transmitters must receive channels that are at least two apart.

III. SIERPINSKI NETWORK

The Sierpinski triangle which is also called Sierpinski gasket, is a mathematically generated pattern named after the Polish Mathematician Waclaw Sierpinski in 1915. The Sierpinski network[9]of dimension n, is denoted as $S(n)$.This network is a fractal that can be constructed by a recursive procedure; at each step an equilateral triangle is divided into four new triangles, of which three are kept for further iterations. The construction of sierpinski Gasket graphs $S(n)$ is naturally defined for a finite number of iterations.The construction of sierpinski network of dimension 3 is as follows:

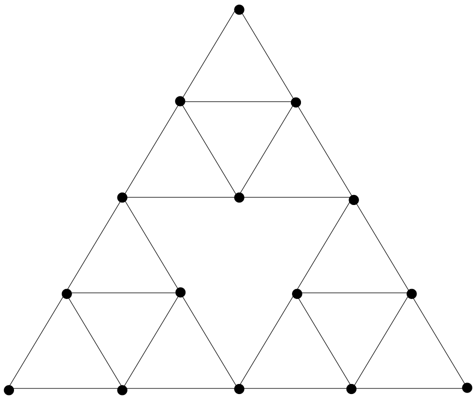


Fig 1: Sierpinski Network $S(3)$

$S(3)$ denotes the Sierpinski network of dimension 3. Consider $S(3)$, it has three copies of the same network of dimension 2 as in Fig:1,one located on the top and other two are found to the left and right of the centred inverted triangle. Similarly $S(2)$ contains three copies $S(1)$ as shown in Fig:2, where $S(1)$ is just a triangle.

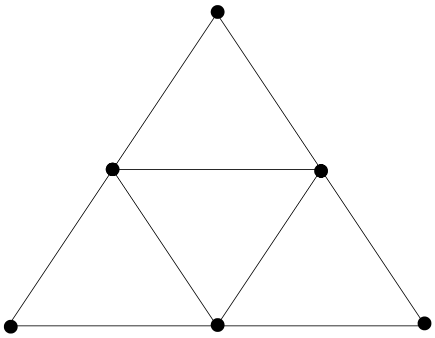


Fig 2: Sierpinski Network $S(2)$

Hence we have the generalized Sierpinskinetwork $S(n)$ to have three copies of $S(n-1)$.Also we express the Sierpinski network of dimension 1 as follows:

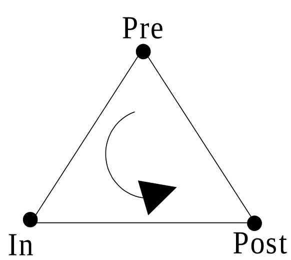


Fig 3: Pre-order Traversal

The Pre-vertex is labeled first, followed by in-order and then post-order. Suppose when the In-vertex or the post-vertex is labeled first (on the top), the order is preserved. That is, the order Pre-In-Post is never changed along the same direction. Diagrammatic representation is done as follows:

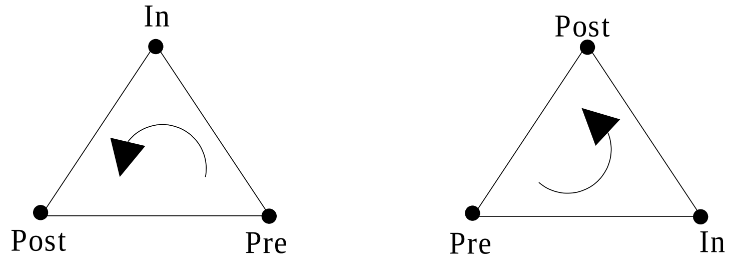


Fig 4: In-order and Post-order Traversals

This can be stated in an algorithm[10][11] as follows:

Algorithm preOrder (root)

Traverse a binary tree in node-left-right sequence.

Pre-root is the entry node of a tree or subtree

Post each node has been processed in order

If (root is not null)

  process (root)

  preOrder (leftSubtree)

  preOrder (rightSubtree)

End if

EndpreOrder

The algorithm[12] for the in-order and post-order vertices are the similar to that of pre-vertex.

IV. T -COLORING OF SEIRPINKI NETWORK

The generalized definition of T-coloring is framed as follows, Let $G = (V, E)$ be a graph and T is a set of non-negative integers including 0 such that $T = \{0, s, 2s, \dots ks\}$ where $s, k \geq 1$. A T-coloring of G is a function $f: V(G) \rightarrow I$ which assigns a positive integer to each vertex u of G so that if u and v are joined by an edge of$G$, then$|f(u) - f(v)| \notin T$. If $T = \{0\}$ then the T-coloring reduces to an ordinary vertex coloring. Given T and$G$, the T-*chromatic number*$\chi T(G)$is the minimum order among all possible T-colorings of$G$, the T-*span* $spT(G)$is the minimum span among all possible T-colorings of$G$.

For instance, consider the case of a complete graph[13] on three vertices and the set T= {0,1,4,5}. We can color the vertices “greedily,” using the lowest acceptable positive integer for each. Here the set T is chosen at random. In that case, we would color the first vertex 1, the second 3, and the third 9. Another T-coloring would be constructed using the integers 1, 4, 7. These two T-colorings are comparable in terms of the number of colors (channels) used. However, the second is better in terms of the separation between the largest and smallest colors used. This separation is called the span of the T-coloring.

Now we check the T-coloring of the Sierpinski network $S(n), n \geq 1$ through the following theorem.

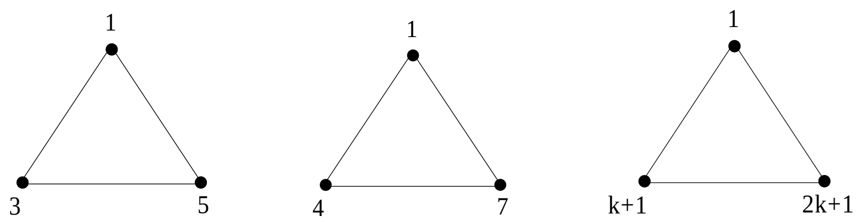
Theorem:

Let G be the Sierpinski network $S(n), n \geq 1$ then the T-coloring of G satisfies the condition,$T(G) \geq n - 1$.

Proof

Now let us see the Sierpinski network of dimension 1.We assign the first label 1, next we label the In-vertex the value 3 and the last is assigned the value 5 for $T = \{0,1\}$. Hence, we similarly obtain the T-coloring with varying range of the set T.

Considering $S(1)$,



$T = \{0,1\}T = \{0,1,2\}$ $\quad T = \{0,1,2 \ldots k - 1\}$

Fig 5: T-coloring of $S(1)$

Since $S(2)$ has three copies of $S(1)$ and $S(3)$ has three copies of $S(2)$, then we can say that $S(3)$ has nine copies of $S(1)$ and so on for higher dimensions of this network.

Let us consider Sierpinski network of dimension 2 and we check for variation in the set T.

Claim: $|f(u) - f(v)| \notin T$ whenever $u, v \in E(G)$.

i.e., we have to prove that T-coloring $T(G) \geq k - 1$.

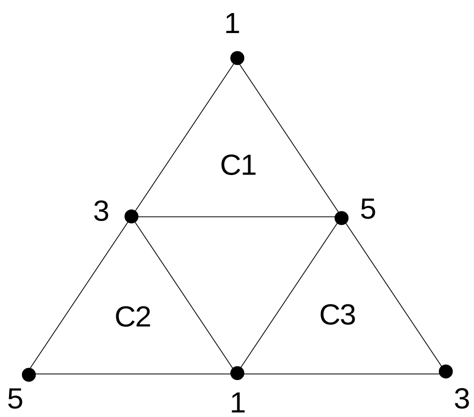
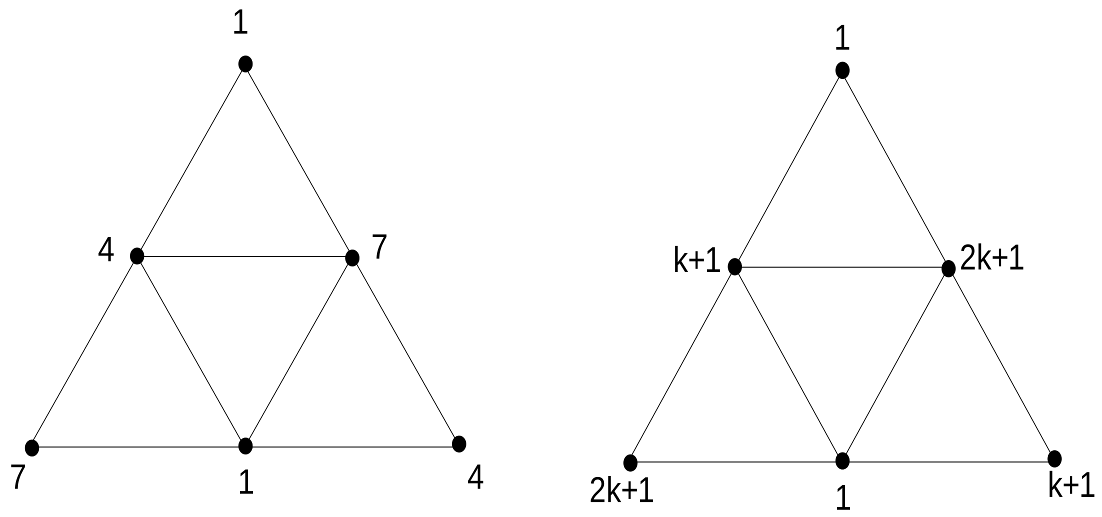


Fig 6: T-coloring of $S(2)$ with $T = \{0, 1\}$

In the above diagram $S(2)$, we see three copies of $S(1)$ and they are named as $C1, C2, C3$ respectively. We notice that $C1$ is labeled exactly as of $S(1)$ and the left base of $C1$ gets the label "3" which it is taken as the Pre-vertex of $C2$ and the order follows. Similarly, the right base of $C1$ has label "5" which is assumed to be the Pre-vertex of $C3$ and we complete labeling the graph by following the order for each $S(1)$ in the network $S(2)$.

Similarly, we can find the labeling for different range of the set T and the generalization of Sierpinskinetwork of dimension $n = 2$ is given below:



$\boldsymbol{T = \{0, 1, 2\}}$ $\boldsymbol{T = \{0, 1, 2 \ldots k - 1\}}$

$\boldsymbol{Fig}$ 7: $\boldsymbol{T}$-coloring of $\boldsymbol{S(2)}$

Consider network $S(3)$, it has nine copies of $S(1)$ and is labeled as above by considering all the $S(1)$'sof $S(3)$to be $C1, C2, \ldots C9$ such that the order of each $Ci, i = 1,2, \ldots 9$ is preserved. The generalized Sierpinski network of dimension $n = 3$ for $T = \{0,1,2,3, \ldots k - 1\}$is as follows:

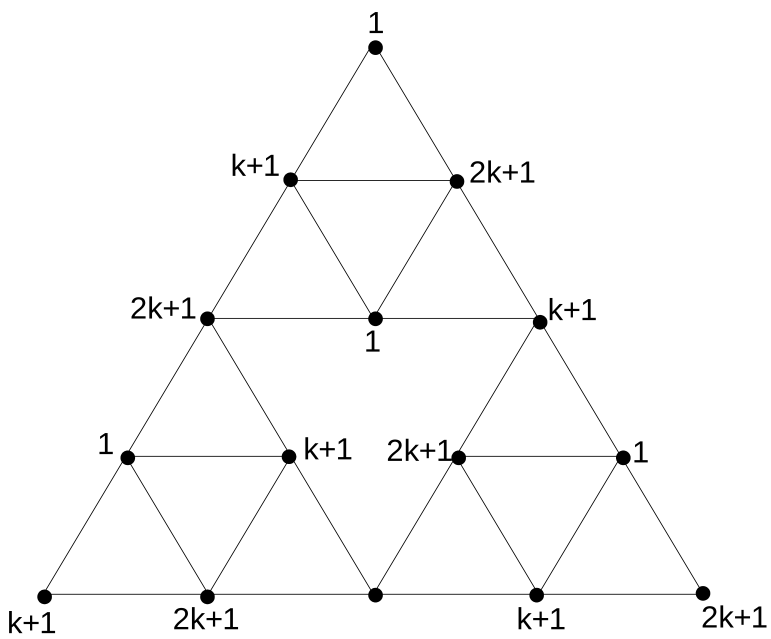


Fig 8: Generalized $\boldsymbol{T}$-coloring of $\boldsymbol{S(3)}$

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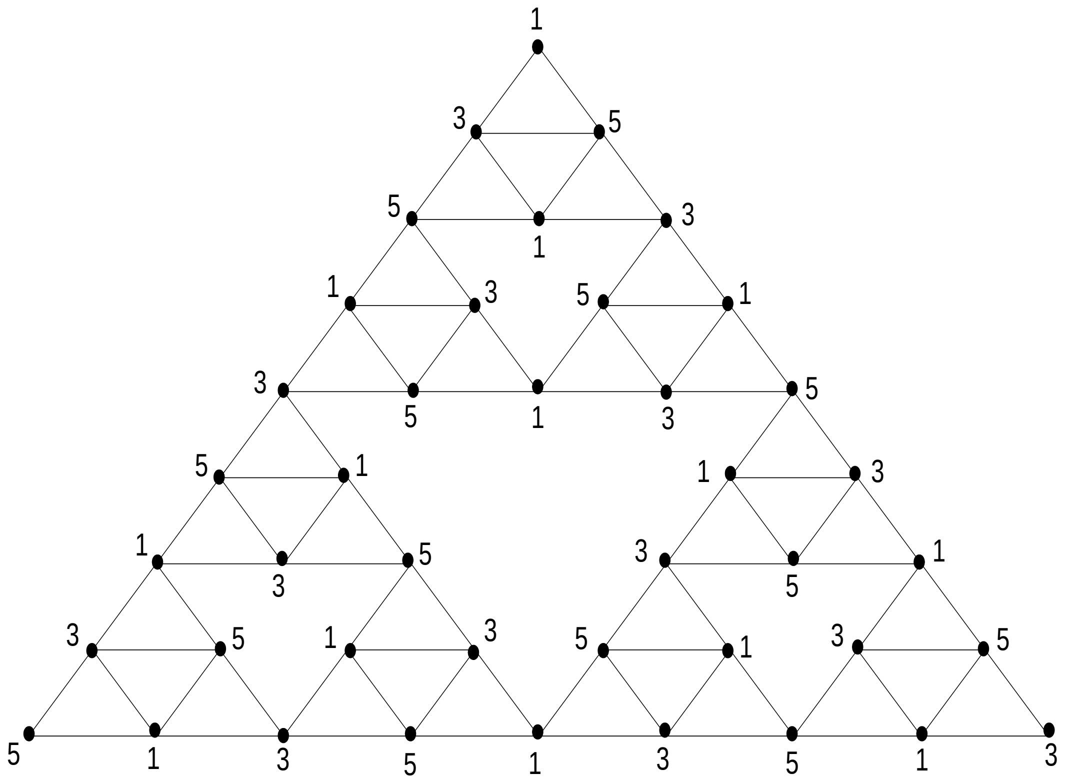

Fig 9: T-coloring of $S(4)$ with $T = \{0, 1\}$

Now we define a mapping $f: V(G) \to I$ for $S(1)$since the whole network is based upon it and is defined by,

$$f(Pre) = 1 f(In) = k + 1 f(Post) = 2k + 1$$

Thus for varied values of set T, we have found the T-coloring of this network to be greater than or equal to $k - 1$. That is $T(G) \geq k - 1$.

Hence the proof.

V. CONCLUSION

In this paper, we have examined about T-coloring, and we have interpreted this into the Sierpinski Network and have generalized the results obtained with variations in parameters like the set T, the size and the dimension n. Also, we have used algorithms to generalize the higher dimensions.

The T-coloring problem blows up to numerous problems that can be solved efficiently for many choices of T sets. The T-coloring of any graph can be done for different values of the set T like, namely, the set T can be taken for odd or even values.T-coloring can also be expanded to sequences of primes, prime odds, powers, cubes, prime powers orrandom numbers of any kind.

The generalization of T-coloring can be expanded to certain networks like honeycomb network, butterfly network, Hexagon network will have a wide range of applications in the field of communication and media. The same networks with varied range of set T may create drastic changes in communication assignment problems, task assignment problems, Mobile or Radio Frequency assignment problems, etc.

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