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EXPONENTIAL B-SPLINE COLLOCATION METHOD FOR THE NUMERICAL SOLUTION OF ONE-SPACE DIMENSIONAL NONLINEAR WAVE EQUATION WITH STRONG STABILITY PRESERVING TIME INTEGRATION

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ABSTRACT

In this paper, a new numerical method is presented to approximate the solution of second order one-dimensional nonlinear wave equation. The method is based on collocation of exponential B-splines. Exponential B-splines are applied for spatial variable and derivatives. The method produces two systems of first order ordinary differential equations. We solve these systems using strong stability preserving methods. Numerical experiments are presented to illustrate the accuracy and efficiency of the proposed method.

Keywords: Exponential B-Spline Method, SSP-HBT54 Method, SSP-RK54 Method, Tri-Diagonal Solver, Wave Equation

I. INTRODUCTION

We consider the following one-space dimensional nonlinear hyperbolic partial differential equation:

$$u_{tt} = u_{xx} + g(x,t,u,u_x,u_t), a < x < b, t > 0 \tag{1}$$

subject to the initial conditions:

$$u(x,0) = g_1(x), u_t(x,0) = g_2(x), a \le x \le b \tag{2}$$

and the boundary conditions:

$$u(a,t) = f_1(t), u(b,t) = f_2(t), t \ge 0 \tag{3}$$

The numerical solution of second order one space dimensional nonlinear wave equation is of great importance in various fields of sciences. Many researchers have studied various numerical techniques for the solution of linear and non-linear wave equations. Gao, Chi [1] presented unconditionally stable schemes for one dimensional linear hyperbolic equation. Mohanty et al [2]-[6] proposed several methods based on uniform and variable mesh for the solution of nonlinear hyperbolic partial differential equations. Many methods based on

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polynomial slpines have been developed for the solution of equation (1). Recently, Mittal and Bhatia [7] presented modified cubic B-spline Differential Quadrature Method for the solution of (1). Not much work based on exponential splines has been done. However, it is being stated by McCartin [8], [9] that the exponential splines are more general splines. McCartin further stated that cubic splines many times exhibit unwanted oscillations in the form of overshoots and/or extraneous inflection points and that exponential splines can remedy this situation for appropriately chosen tension parameters. Reza Mohammadi used exponential B-spline for solving Convection-Diffusion equations in [10]. We, in this paper, present the collocation method based on exponential B-spline basis functions to solve some benchmark nonlinear wave equations. Equation (1) is converted into a system of partial differential equations and then exponential B-spline collocation method is used to discretize the equations spatially which leads to formulation of two systems of first order ordinary differential equations which are then solved by SSP-RK54 [11] and SSP-HBT54 [12] methods respectively.

The outline of the paper is as follows: In section 2, we discuss exponential B-spline collocation method. In section 3, we apply this method to nonlinear hyperbolic wave equations. Numerical experiments are illustrated in section 4 and finally concluding remarks are given in section 5.

II. EXPONENTIAL B-SPLINE COLLOCATION METHOD

In exponential B-splines collocation method the approximate solution can be written as a linear combination of exponential B-spline basis functions for the approximation space under consideration. We consider a mesh $a = x_0 < x_1 < \ldots < x_{N-1} < x_N = b$ as a uniform partition of the solution domain $[a, b]$ by knots x_l with spacing $h = x_l - x_{l-1} = \frac{b-a}{N}$ for $l = 1(1)N$.

The exponential B-splines $B_l(x)$ at the above defined knots together with additional knots x_{-1}, x_{N+1} are given by:

$$B_l(x) = \begin{cases} a\left((x_{l-2} - x) - \frac{1}{p}\left(\sinh\left(p(x_{l-2} - x)\right)\right)\right), x \in [x_{l-2}, x_{l-1}) \\ b_1 + b_2(x_l - x) + b_3 \exp\left(p(x_l - x)\right) + b_4 \exp\left(-p(x_l - x)\right), x \in [x_{l-1}, x_l) \\ b_1 + b_2(x - x_l) + b_3 \exp\left(p(x - x_l)\right) + b_4 \exp\left(-p(x - x_l)\right), x \in [x_l, x_{l+1}) \\ a\left((x - x_{l+2}) - \frac{1}{p}\left(\sinh\left(p(x - x_{l+2})\right)\right)\right), x \in [x_{l+1}, x_{l+2}) \\ 0, \quad \text{otherwise} \end{cases} \tag{4}$$

where

$$a = \frac{p}{2(phc - s)}, b_1 = \frac{phc}{(phc - s)}, b_2 = \frac{p}{2}\left[\frac{c(c-1) + s^2}{(phc - s)(1 - c)}\right],$$

$$b_3 = \frac{1}{4}\left[\frac{\exp(-ph)\,(1 - c) + s(\exp(-ph) - 1)}{(phc - s)(1 - c)}\right],$$

$$b_4 = \frac{1}{4}\left[\frac{\exp(ph)\,(c - 1) + s(\exp(ph) - 1)}{(phc - s)(1 - c)}\right], s = \sinh(ph), c = \cosh(ph)$$

where p is a free parameter. Additional knots are required to define all the exponential splines. The set $\{B_{-1}, B_0, B_1, \ldots, B_{N-1}, B_N, B_{N+1}\}$ forms a basis for functions defined over the region $[a, b]$. Each basis

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function $B_l(x)$ is twice continuously differentiable. The values of $B_l(x), B_l'(x), B_l''(x)$ at knots are tabulated in Table 1.

Table 1: Values of exponential B-spline and its derivatives at different knots

x	x_{l-2}	x_{l-1}	x_l	x_{l+1}	x_{l+2}
$B_l(x)$	0	$\frac{s-ph}{2(phc-s)}$	1	$\frac{s-ph}{2(phc-s)}$	0
$B_l'(x)$	0	$\frac{p(c-1)}{2(phc-s)}$	0	$-\frac{p(c-1)}{2(phc-s)}$	0
$B_l''(x)$	0	$\frac{p^2s}{2(phc-s)}$	$-\frac{p^2s}{(phc-s)}$	$\frac{p^2s}{2(phc-s)}$	0

In the collocation method with exponential B-splines, an approximate solution $U(x,t)$ to the analytical solution $u(x,t)$ can be written in the form:

$$U(x,t) = \sum_{l=-1}^{l=N+1} c_l(t) B_l(x) \tag{5}$$

where $c_l(t)$ are unknown quantities to be determined from the boundary conditions and collocation form of the differential equation (1). In order to eliminate the coefficients $c_{-1}(t)$ and $c_{N+1}(t)$, we redefine the exponential B-spline basis functions as:

$$\left.\begin{aligned} \tilde{B}_0(x) &= B_0(x) + 2B_{-1}(x), && \text{for } l = 0, \\ \tilde{B}_1(x) &= B_1(x) - B_{-1}(x), && \text{for } l = 1, \\ \tilde{B}_l(x) &= B_l(x), && \text{for } l = 2(1)N-2, \\ \tilde{B}_{N-1}(x) &= B_{N-1}(x) - B_{N+1}(x), && \text{for } l = N-1, \\ \tilde{B}_N(x) &= B_N(x) + 2B_{N+1}(x), && \text{for } l = N. \end{aligned}\right\} \tag{6}$$

Then, the approximate solution $U(x,t)$ can be rewritten as the linear combination of redefined exponential B-spline basis functions (6) as:

$$U(x,t) = \sum_{l=0}^{l=N} c_l(t) \tilde{B}_l(x) \tag{7}$$

From equation (7) and Table 1, the approximate values of $U(x,t)$ and its first and second order derivatives are determined in terms of the time parameters c_l as follows:

$$\left.\begin{aligned} U(x_l,t) &= m_1 c_{l-1} + c_l + m_1 c_{l+1}, \\ U'(x_l,t) &= m_2(c_{l+1} - c_{l-1}), \\ U''(x_l,t) &= m_3(c_{l-1} - 2c_l + c_{l+1}) \end{aligned}\right\} \tag{8}$$

where

$$m_1 = \frac{s-ph}{2(phc-s)}, m_2 = \frac{p(c-1)}{2(phc-s)}, m_3 = \frac{p^2s}{2(phc-s)}$$

III. NUMERICAL METHOD

We first split equation (1) into system of equations as follows:

$$\left.\begin{array}{c} u_t = v, \\ v_t = u_{xx} + g(x,t,u,u_x,v) \end{array}\right\} \tag{9}$$

Then using (7), the approximate values of $U_t(x,t)$ and $U_x(x,t)$ can be written as:

$$U_t(x,t) = \sum_{l=0}^{N} \dot{c}_l(t)\tilde{B}_l(x) \tag{10}$$

$$U_x(x,t) = \sum_{l=0}^{N} c_l(t)\tilde{B}_l{}'(x) \tag{11}$$

where $\dot{c}_l(t)$ is the derivative of $c_l(t)$ with respect to t and $\tilde{B}_l{}'(x)$ is the derivative of $\tilde{B}_l(x)$ with respect to x.

Evaluation at the boundary knots: Imposing boundary conditions and using the redefined basis functions (6) and Table 1 in (10), we can write system (9) at the boundary knots as:

$$U_t(x_0,t) = (1+2m_1)\dot{c}_0 = f_1'(t), \quad \text{for } l = 0 \tag{12a}$$

$$\dot{v}_0(x_0,t) = f_1''(t), \quad \text{for } l = 0 \tag{12b}$$

and

$$U_t(x_N,t) = (1+2m_1)\dot{c}_N = f_2'(t), \quad \text{for } l = N \tag{13a}$$

$$\dot{v}_N(x_0,t) = f_2''(t), \quad \text{for } l = N \tag{13b}$$

Evaluation at the internal knots: Using the redefined basis functions (6) and Table 1 in (10) and (11), we can write system (9) at the interior knots $l = 1(1)N-1$ as

$$\left.\begin{array}{c} U_t(x_l,t) = v(x_l,t), \\ \dot{v}(x_l,t) = \sum_{l=0}^{N} c_l(t)\tilde{B}_l''(x_l) + g\left(x_l, t, \sum_{l=0}^{N} c_l\tilde{B}_l(x_l) \;, \sum_{l=0}^{N} c_l\tilde{B}_l{}'(x_l), v(x_l,t)\right) \end{array}\right\} \tag{14}$$

Finally, using the definition of basis functions (6) and Table 1, equation (14) can be written as the following systems of ordinary differential equations:

$$\left.\begin{array}{c} m_1\dot{c}_{l-1} + \dot{c}_l + m_1\dot{c}_{l+1} = v(x_l,t)\,, \\ \dot{v}(x_l,t) = m_3(c_{l-1} - 2c_l + c_{l+1}) + \\ g\left(x_l, t, (m_1c_{l-1} + c_l + m_1c_{l+1}), m_2(c_{l+1} - c_{l-1}), v(x_l,t)\right) \end{array}\right\} \tag{15}$$

which in matrix form can be written as:

$$\boldsymbol{A}\dot{\boldsymbol{c}} = \boldsymbol{F} \tag{16}$$

$$\dot{\boldsymbol{v}} = \boldsymbol{G} \tag{17}$$

where, $\boldsymbol{A} = \begin{bmatrix} 1 & m_1 & \cdots & \cdots & 0 \\ m_1 & 1 & m_1 & \cdots & 0 \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \ddots \\ & & m_1 & 1 & m_1 \\ 0 & & & m_1 & 1 \end{bmatrix}, \dot{\boldsymbol{c}} = \begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \vdots \\ \dot{c}_{N-2} \\ \dot{c}_{N-1} \end{bmatrix}, \boldsymbol{F} = \begin{bmatrix} v_1 - m_1\dot{c}_0 \\ v_2 \\ \vdots \\ v_{N-2} \\ v_{N-1} - m_1\dot{c}_N \end{bmatrix}, \dot{\boldsymbol{v}} = \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \vdots \\ \dot{v}_{N-2} \\ \dot{v}_{N-1} \end{bmatrix}, \boldsymbol{G} = \begin{bmatrix} G_1 \\ G_2 \\ \vdots \\ G_{N-2} \\ G_{N-1} \end{bmatrix},$

where, $G_l = m_3(c_{l-1} - 2c_l + c_{l+1}) + g(x_l, t, (m_1c_{l-1} + c_l + m_1c_{l+1}), m_2(c_{l+1} - c_{l-1}), v_l)$

and $\dot{v}_l$ denotes $\dot{v}(x_l,t)$ for $l = 1(1)N-1$.

To compute the solution at the required knots, the vector $\boldsymbol{c}$ is to be determined at each time level. We solve equation (12a) and (13a) for c_0 and c_N respectively by SSP-RK54 method, then at each time level $t > 0$, $\dot{\boldsymbol{c}}$ is evaluated from (16) by using tri-diagonal solver. Then the obtained system of equations along with the system (17) gives first order ordinary differential equations. We solve the former system for $\boldsymbol{c}$ by SSP-RK54 method and the latter for $\boldsymbol{v}$ by SSP-HBT54 method. Consequently, on using (8), the approximate solution $U(x,t)$ is obtained.

To initiate the computation we need initial vectors, $\boldsymbol{c}^0$ and $\boldsymbol{v}^0$ which can be determined by using initial conditions (2):

$U(x_i,0) = g_1(x_i)$, $i = 0,1,\ldots,N$, which in matrix form can be written as:

$$\boldsymbol{A}\boldsymbol{c}^0 = \boldsymbol{F} \tag{18}$$

where,

$$\boldsymbol{A} = \begin{bmatrix} 1 & m_1 & \dots & \dots & 0 \\ m_1 & 1 & m_1 & \dots & 0 \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \ddots \\ & & m_1 & 1 & m_1 \\ 0 & & & m_1 & 1 \end{bmatrix}, \boldsymbol{c}^0 = \begin{bmatrix} c_1^0 \\ c_2^0 \\ \vdots \\ c_{N-2}^0 \\ c_{N-1}^0 \end{bmatrix}, \boldsymbol{F} = \begin{bmatrix} g_1(x_1) - m_1 c_0^0 \\ g_1(x_2) \\ \vdots \\ g_1(x_{N-2}) \\ g_1(x_{N-1}) - m_1 c_N^0 \end{bmatrix} \text{ and } c_0^0 = \frac{g_1(x_0)}{1+2m_1}, c_N^0 = \frac{g_1(x_N)}{1+2m_1}.$$

Now, $\boldsymbol{A}$ is a tri diagonal matrix, hence, equation (18) can be solved for $\boldsymbol{c}^0$ by tri-diagonal solver.

Similarly, second initial condition gives

$$U_t(x_i,0) = g_2(x_i), \ i = 0,1,\ldots,N,$$

i.e. we have,

$$v(x_i,0) = g_2(x_i), i = 0,1,\ldots,N. \tag{19}$$

Hence, from (19), initial vector $\boldsymbol{v}^0$ can be calculated.

IV. NUMERICAL EXPERIMENTS

In this section, we present the numerical results of present method on one linear and four nonlinear wave equations. We also compare obtained results with the results obtained by existing methods. For all the problems we choose $p = 1$. The accuracy of the presented method is measured using L_2 errors, maximum absolute errors (MAE) and root mean square errors (RMSE).

$$L_2 = \|u - U\|_2 = \sqrt{h \sum_{i=0}^{N} |u_i - U_i|^2},$$

$$\text{MAE} = \|u - U\|_\infty = \max_i |u_i - U_i|,$$

$$\text{RMSE} = \sqrt{\frac{\sum_{i=0}^{N} |u_i - U_i|^2}{N+1}}.$$

where u and U denote the exact and approximate solutions respectively.

Example 1. (Wave equation in polar coordinates)

$$u_{tt} = u_{rr} + \frac{\alpha}{r} u_r - \frac{\alpha}{r^2} u + \left(3 + \frac{\alpha}{r^2}\right) e^{-2t} \cosh(r) - \frac{\alpha}{r} e^{-2t} \sinh(r), 0 < r < 1, t > 0$$

This equation represents one-dimensional wave equation in cylindrical and spherical coordinates for $\alpha = 1$ and 2 respectively. The analytical solution is $u(r,t) = e^{-2t}\cosh(r)$. The initial and boundary conditions can be obtained using analytical solution. The maximum absolute errors obtained at different time levels using the proposed method are given in Table 2 for $\alpha = 1, 2$. A comparison between analytical and numerical solution upto $t = 1$ for $\alpha = 2$ is done by plotting space time graphs which are given in Fig. 1 and Fig. 2. It is clear from the Table and graphs that our method is efficient in approximating the solution of wave equation in polar coordinates.

Table 2: MAE error for example 1 at t=2 with $\Delta t = 0.0001,\ h = 0.04$

t	$\alpha = 1$		$\alpha = 2$	
	MAE	CPU time(in sec)	MAE	CPU time(in sec)
0.25	4.3651e-04	1.13	4.7525e-04	1.13
0.50	5.3182e-04	2.19	6.3619e-04	2.17
0.75	6.0372e-04	3.22	9.5102e-04	3.23
1	5.3498e-04	4.31	9.9524e-04	4.24

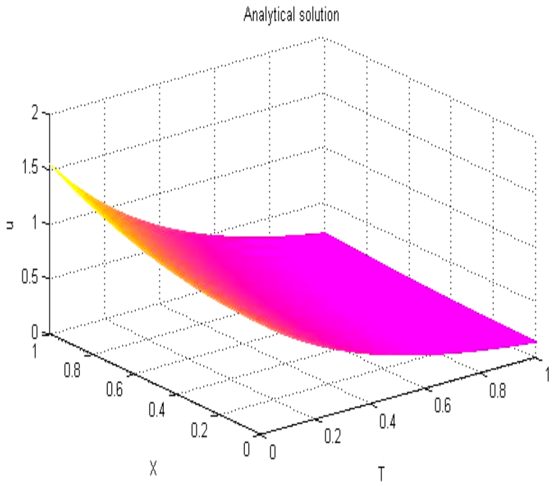

Figure 1: Analytical solution of example 1 for $\alpha = 2$

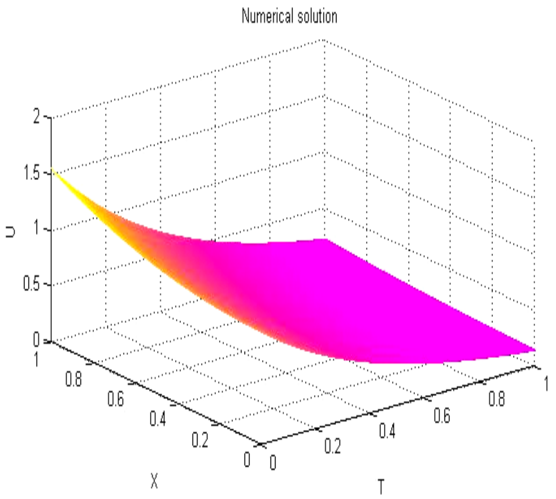

Figure 2: Numerical Solution of Example 1 for $\alpha = 2$

Example 2. (Van der Pol type nonlinear wave equation)

$$u_{tt} = u_{xx} + \gamma(u^2 - 1)u_t + \left(\pi^2 + \gamma^2 e^{-2\gamma t}\sin^2(\pi x)\right)e^{-\gamma t}\sin(\pi x), 0 < x < 1, t > 0$$

The analytical solution is $u(x,t) = e^{-\gamma t}\sin(\pi x)$. The maximum absolute errors obtained at $t = 2$ for $\Delta t = 0.0001$ and different values of h are given in Table 3. A comparison between analytical solution and numerical solution upto $t = 1$ for $\gamma = 3$, $h = 0.05$ and $\Delta t = 0.0001$ can be done by studying Fig. 3 and Fig. 4. It is evident from the figures that our method is efficient in approximating solution of Van der pol type nonlinear wave equation.

Table 3: MAE error for example 2 at $t = 2$ with $\Delta t = 0.0001$.

h	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$
$\frac{1}{8}$	3.6000e-03	1.7000e-03	7.2618e-04
$\frac{1}{16}$	9.2084e-04	4.3797e-04	1.8142e-04
$\frac{1}{32}$	2.3184e-04	9.7609e-05	4.4266e-05
$\frac{1}{64}$	5.8628e-05	2.7616e-05	9.9387e-06

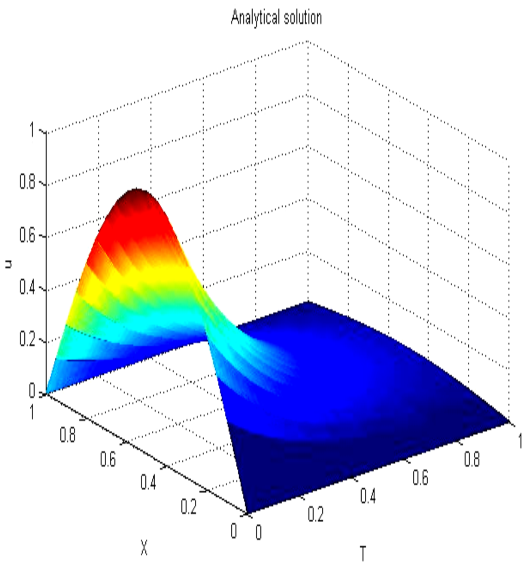

Figure 3: Analytical Solution of Example 2 for $\gamma = 3$, $h = 0.05$, $\Delta t = 0.0001$

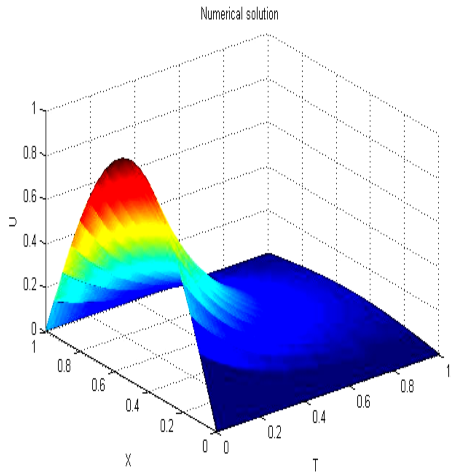

Figure 4: Numerical Solution of Example 2 for $\gamma = 3$, $h = 0.05$, $\Delta t = 0.0001$

Example 3. (Dissipative non-linear wave equation)

$$u_{tt} = u_{xx} - 2u\,u_t + (\pi^2 - 1 - 2\sin(\pi x)\sin(t))\sin(\pi x)\cos(t)\;,0 < x < 1, t > 0.$$

The analytical solution is $u(x,t) = \sin(\pi x)\cos(t)$. The maximum absolute errors and root mean square errors at different times t and $h = 0.05$, $\Delta t = 0.01$ are tabulated in Table 4. The results obtained are compared with the results obtained by Mittal and Bhatia [7]. Our results are in good agreement with the results obtained in [7]. Space-time graphs of analytical and numerical solutions are given in Fig. 5 and Fig. 6 respectively, which also confirm the accuracy of the method.

Table 4: Errors for Example 3 for h =0.05,$\Delta t = 0.01$

t	Our Method		Mittal and Bhatia [7]	
	RMSE	MAE	RMSE	MAE
1	2.0000e-03	2.8000e-03	3.046e-03	4.274e-03
2	1.9000e-03	2.6000e-03	3.251e-03	4.625e-03
3	2.8396e-05	3.8175e-05	5.737e-05	9.782e-05

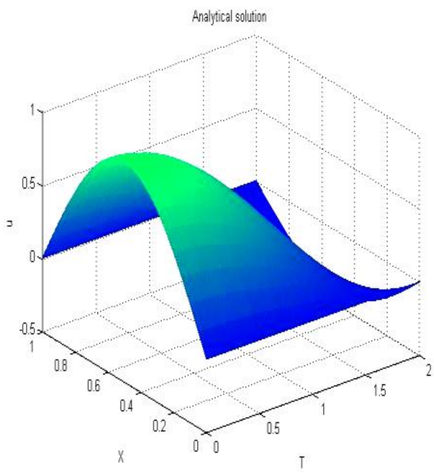

Figure 5: Analytical Solution of Example 3 upto $t = 2$ for $h = 0.05, \Delta t = 0.01.$

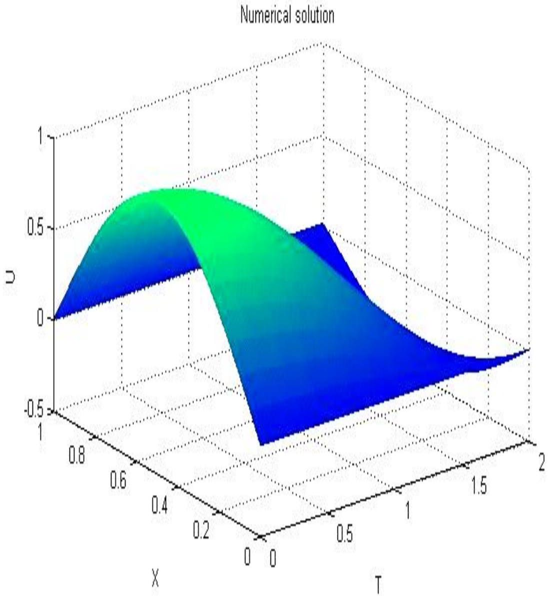

Figure 6: Numerical solution of example 3 upto $t = 2$ for $h = 0.05, \Delta t = 0.01$

Example 4. (Non-linear wave equation)

$$u_{tt} = u_{xx} + \gamma u(u_x + u_t) + (x^2 \sinh(t) - 2\sinh(t) - \gamma x^2 \sinh(t)(2x \sinh(t) + x^2 \cosh(t)), 0 < x < 1, t > 0,$$

The analytical solution is $u(x,t) = x^2 \sinh(t)$. We report the maximum absolute errors obtained at $t = 1$ for $\Delta t = 0.001$ in Table 5. The calculations are carried out for different values of γ and h. Space-time graphs of analytical and numerical solution are also plotted in Fig. 7 and Fig. 8.

Table 5: MAE for Example 4 at t=1 with Δt=0.001

h	$\gamma = 1$	$\gamma = 5$	$\gamma = 10$
$\frac{1}{8}$	7.1000e-03	1.3400e-02	3.5400e-02
$\frac{1}{16}$	1.9000e-03	3.7000e-03	9.8000e-03
$\frac{1}{32}$	6.6791e-04	1.2000e-03	3.3000e-03
$\frac{1}{64}$	3.6757e-04	6.3917e-04	1.7000e-03

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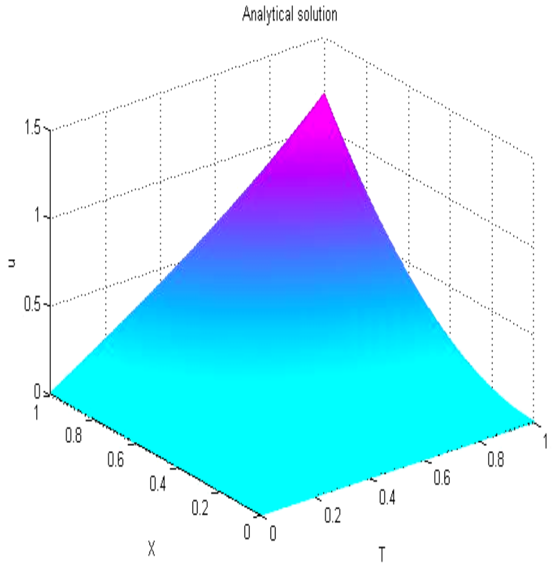

Figure 7: Analytical Solution of Example 4 for $\gamma = 10, h = \frac{1}{64}, \Delta t = 0.001$

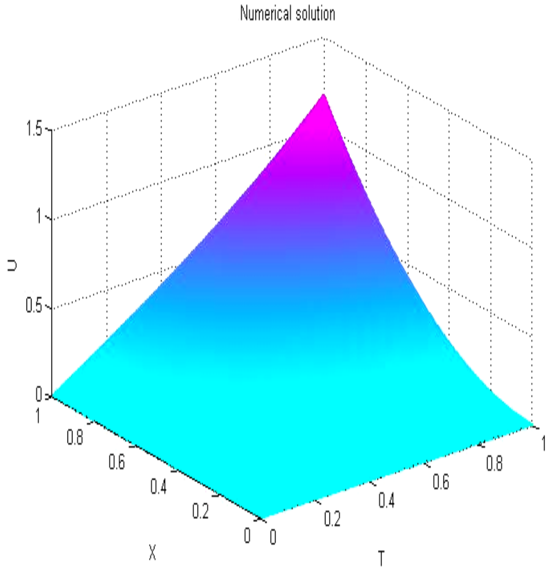

Figure 8: Numerical Solution of Example 4 for $\gamma = 10, h = \frac{1}{64}, \Delta t = 0.001$

Example 5. Consider the following Sine-Gordan equation:

$u_{tt} = u_{xx} - \sin(u)\ , -1 < x < 1, t > 0$

The analytical solution is $u(x,t) = 4\ \arctan(t\ \mathrm{sech}(x))$. This example is solved at different time levels for $\Delta t = 0.001$ and $h = 0.02, 0.04$. The results obtained are compared with the results obtained by Dehghan and Shokri [13]. It is evident from the Table 6 that our results are in good agreement with the results obtained by Dehghan and Shokri [13]. Moreover, the numerical solution obtained for $\Delta t = 0.001$ and $h = 0.02$ is compared with the analytical solution in Fig. 9 and 10.

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Table 6: Errors Calculated for Example 5 for $\Delta t = 0.001$

	Proposed Method				Dehghan and Shokri [13]	
t	$h = 0.02$		$h = 0.04$		$h = 0.04$	
	L_2	MAE	L_2	MAE	L_2	MAE
.25	3.0413e-06	1.0875e-05	1.0875e-05	5.3761e-06	3.91e-05	5.89e-06
.50	1.2125e-05	4.5317e-05	4.5317e-05	1.4216e-05	1.30e-04	2.01e-05
.75	2.7560e-05	1.0236e-04	1.0236e-04	3.5439e-05	2.35e-04	3.63e-05
1	5.4593e-05	2.0372e-04	2.0372e-04	6.0377e-05	3.27e-04	5.07e-05

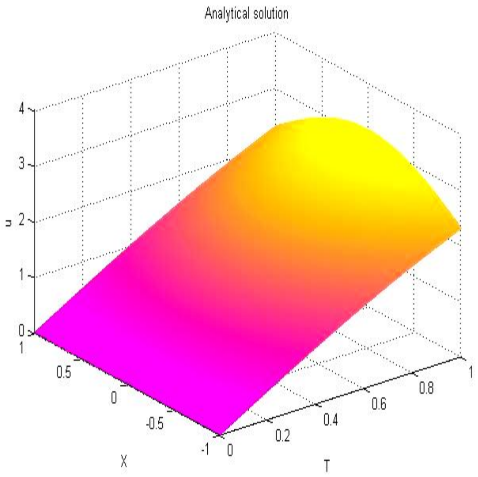

Figure 9: Analytical Solution of Example 5 for $\Delta t = 0.001$ and $h = 0.02$

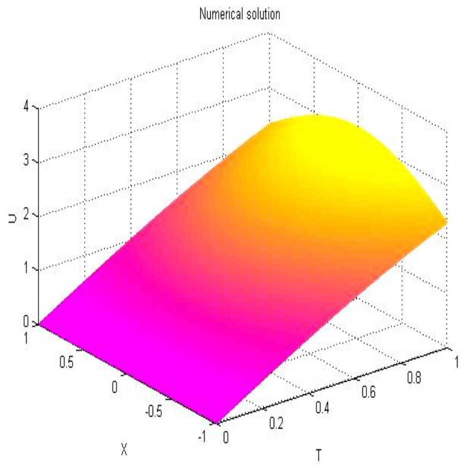

Figure 10: Numerical Solution of Example 5 for $\Delta t = 0.001$ and $h = 0.02$

V. CONCLUSION

In this paper, an exponential B-spline collocation method has been proposed to solve second order one dimensional nonlinear wave equation. The second order problem is first converted into two first order partial differential equations. Then, exponential B-spline collocation method is applied to convert these equations into two systems of first order ordinary differential equations which are then solved by strong stability preserving

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five stage, fourth order Runge-Kutta method and Hermite-Birkhoff-Taylor method respectively. This choice of methods gives better results compared to the results obtained by using one of them only. The main advantage of this method is that because of its simplicity, it is easy to be applied to any linear or nonlinear problem available in literature and gives accurate results.

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