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STUDY ON THE DYNAMIC BEHAVIOUR OF NANOBEAM USING NONLOCAL ELASTICITY THEORY

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ABSTRACT

This paper investigates the flexural vibration of nanobeam based on Eringen's nonlocal elasticity theory. The governing equation for free flexural vibration of nanobeam using Euler- Bernoulli Beam theory has been developed to study the effect of the small- scale parameter on the vibrational frequency. The small- scale parameter is taken into consideration by using Eringen's nonlocal elasticity theory. The analytical solutions are obtained for simply supported, clamped- clamped and clamped- hinged end conditions using Galerkin's method of weighted residual. The effect of the nonlocal parameter on the free flexural vibration frequencies is studied. The results and the available solutions are compared and the frequencies for all boundary conditions are found to be in excellent agreement with existing results.

Keywords: Nanobeams, Natural Frequency, Nonlocal Elasticity Theory, Nonlocal Nanoscale.

I. INTRODUCTION

There has been spectacular development in nanotechnology recently. Nanobeams have varied Engineering applications as components in nano devices, nano- composites, electrical and electromechanical instruments due to their good electrical properties and high mechanical strength. During operation often these components may be subjected to external loads, which may have an impact on their dynamics characteristics. The response characteristics of nanostructures are very different from other micro/ macro structures due to the inherent size effect. Hence studies on the vibrational behavior of nanobeams must be carried out for their rational design in nano devices.

Farzad Ebrahimi and Parisa Nasirzadeh[2] studied the small- scale effect on the transverse vibrational behavior of single- walled carbon nanotubes based on the assumptions of Timoshenko beam theory. Fehmi Najar, et al [3] studied the effect of the small- scale parameter on the static and dynamic responses of a Nano-actuator subjected to D.C voltage. J.N. Reddy, et al [4] developed non- linear finite element models based on Eringen's [1] non-local differential model to obtain numerical results for static bending of nano beam structure. Hassan Kananipour, et al [5] conducted a study on the dynamic analysis of nanobeam in polar coordinate system using Differential quadrature method (DQM). Milad Hemmatnezhad and Reza Ansari [6] investigated the effects of the small scale parameter and thermal effect on the vibration characteristics of double-walled carbon nanotubes modelled using Timoshenko beam theory J.V. Ara´ujo dos Santosa and J.N. Reddy [7] studied the influence of rotary inertia and non-local parameter on the fundamental and higher natural frequencies of a Timoshenko beam. S. Narendar, et al [8] studied the vibrational behavior of micro/ nano bars by employing strain gradient theory. C.M.C. Roque, et al [9] conducted a study on the bending, buckling and free vibrational response of a simply supported Timoshenko nanobeam by employing Eringen's non-local elasticity theory. Maziar Janghorban [10] studied the static and free vibration analysis of carbon nano- wires with rectangular cross-section based on Timoshenko beam theory using Differential Quadrature method. Payam Soltani, et al [11] analysed the effect of waviness on the transverse vibration of single-walled carbon nanotubes. The authors of this paper are of the opinion that the studies hereto reported are not sufficient to thoroughly understand the effect of small scale parameter and the limitations of classical elastic theory in dealing with the length scale of nanobeams.The main objecive of this paper is carry out undamped free flexural vibration of the nanobeam under arbitrary boundary conditions by using Galerkin's method and to study the effect of the nonlocal parameter on the vibrational frequencies.

1.1 Nonlocal elasticity theory

As for physical interpretation, the nonlocal theory incorporates long range interactions between points in a continuum model. Such long range interactions occur between charged atoms or molecules in a solid. Long range forces may also be considered to propagate along fibers or laminae in a composite material. The classical theory of elasticity (Hooke's law) excludes these effects. In order to remedy this situation, nonlocal elasticity theories were proposed employing the granular nature of materials. The nonlocal elasticity theory is concerned with material bodies whose behavior at any interior point depends on the state of all other points in the body. In the theory of nonlocal elasticity according to A.Cernal Eringen [1], the stress at a reference point x is considered to be a functional of the strain field at every point x' in the body. Thus, the non-local stress tensor $\bar{\sigma}$ at point x is expressed as:

$$\overline{\sigma} = \int_{\Omega} K(|\mathbf{x}' - \mathbf{x}|, \tau)\sigma(\mathbf{x}')\, d\mathbf{x}'$$

Use of this integral constitutive relation is relatively more difficult in computation than using algebraic or differential constitutive relations. Realizing this fact, Eringen(1) proposed an equivalent differential model as

$$(1 - {\mu_0}^2\nabla^2)\bar{\sigma} = \sigma, \mu_0 = \tau^2.l^2 = {e_0}^2.a^2$$

$\bar{\sigma}, \varepsilon_{xx}$ and E are nonlocal stress, a normal strain and Young's modulus, respectively of small length scale.

$\mu \ = {e_0}^2 a^2$ is the function of material constant

e_0 =Material constant

a = internal characteristic lengths (such as the lattice spacing).

$e_0 a$ is called the nonlocal parameter, which is the factor to be incorporated while considering the effect of small length scale.

1.2 Galerkin's Method

The Galerkin's method constructs an approximate solution of the given problem. Each basic function must satisfy an admissibility condition appropriate for the problem. The basic functions can be chosen to be weak solutions of the Galerkin's integral representation associated with the given problem. The following steps are performed in Galerkin's method:

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1. Assume an approximate solution which satisfies the boundary conditions and substitute it in the governing equation. This will result in a residue/ error.
2. Multiply the residual of the governing equation by a weighting function or the assumed solution and set the domain integral equal to zero.

II. FORMULATION OF GOVERNING EQUATION

The governing equation for an undamped free flexural vibration of a nanobeam based on the assumptions of Euler- Bernoulli beam theory is derived as follows.

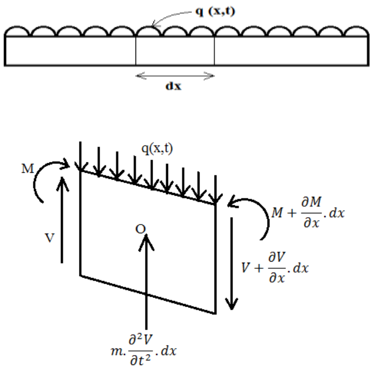


Fig.1 Equilibrium condition of a differential element of a nanobeam subjected to a uniformly distributed load.

Force equilibrium in vertical direction:

$$\frac{\partial V}{\partial x} = m.\frac{\partial^2 V}{\partial t^2} - q(x,t) \quad (1)$$

Taking moment about O and equating to zero

$$V = \frac{\partial M}{\partial x} \quad (2)$$

Substituting (2) into (1)

$$\frac{\partial^2 M}{\partial x^2} - m.\frac{\partial^2 V}{\partial t^2} = -q(x,t) \quad (3)$$

$$\text{M}=\int \bar{\sigma}\,.\,y.\,dA \quad (4a)$$

$$\varepsilon_x = -y.\frac{\partial^2 V}{\partial x^2} \quad (4b)$$

Non- local constitutive law:

$$\bar{\sigma}(x) - (e_0 a)^2.\frac{\partial^2 \sigma}{\partial x^2} = E.\varepsilon_x \quad (5)$$

Substituting (5) into (4),

$$\frac{\partial^2 M}{\partial x^2} = 1/(e_0 a)^2.[M + EI.\partial^2 V/\partial x^2] \quad (6)$$

From (3),

$$\frac{\partial^2 M}{\partial x^2} = -q(x,t) + m.\frac{\partial^2 V}{\partial t^2}$$

Equating (3) and (6)

$$M = -(e_0 a)^2.q(x,t) + m.(e_0 a)^2.\frac{\partial^2 V}{\partial t^2} - EI.\partial^2 V/\partial x^2 \quad (7)$$

Substitute (7) into (2),

$$V = -(e_0 a)^2.\frac{\partial q(x,t)}{\partial x} + m.(e_0 a)^2.\frac{\partial^3 V}{\partial x.\partial t^2} - EI.\partial^3 V/\partial x^3 \quad (8)$$

Substituting (8) into (1),

$$EI.\frac{\partial^4 V}{\partial x^4} + m.\frac{\partial^2}{\partial t^2}[V - (e_0 a)^2.\frac{\partial^2 V}{\partial x^2}] + \left[\frac{(e_0 a)^2 \partial^2}{\partial x^2} - 1\right].q(x,t) = 0 \quad (9)$$

For free vibration, q=0. Hence we get,

$$EI.\frac{\partial^4 V}{\partial x^4} + m.\frac{\partial^2 V}{\partial t^2} - m.(e_0 a)^2.\frac{\partial^4 V}{\partial x^2.\partial t^2} = 0 \quad (10)$$

Put $V(x,t) = \emptyset(x).Y(t)$

$$EI.\emptyset^{IV}.Y + m.\emptyset.\ddot{Y} - m.(e_0 a)^2.\ddot{Y}.\emptyset^{II} = 0 \quad (11)$$

Put Y=sin*pt*

Substituting in (11) we get,

$$\emptyset^{IV} - m.\emptyset.\frac{p^2}{EI} + m.\frac{(e_0 a)^2}{EI}.p^2.\emptyset^{II} = 0 \quad (12)$$

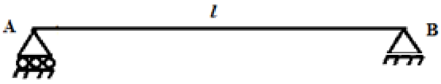
Put $g^2 = (e_0 a)^2.m.\frac{p^2}{EI}$ and $a^4 = m.\frac{p^2}{EI}$

$$\emptyset^{IV} + g^2.\emptyset^{II} - a^4.\emptyset = 0 \quad (13)$$

III. ANALYSIS OF NANO BEAM WITH VARIOUS BOUNDARY CONDITIONS

3.1 Simply Supported End Conditions

Boundary conditions are as follows:



$\emptyset(0) = 0$

$\emptyset^{II}(0) = 0$

$\emptyset(l) = 0$

$\emptyset^{II}(l) = 0$

The function satisfying these boundary conditions is $\emptyset(x) = \sin\frac{n\pi x}{l}$

Substituting in (13) we get,

$$(\frac{n\pi}{l})^4.sin\frac{n\pi x}{l} - g^2(\frac{n\pi}{l})^2.sin\frac{n\pi x}{l} - a^4.sin\frac{n\pi x}{l} = R \quad (14)$$

Where R= Residue or error arising due to the assumed solution. To make the error zero, we use Galerkin's method of weighted residual.

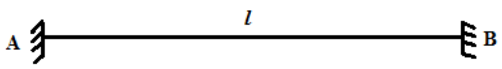
$$\int_0^l \{[(\frac{n\pi}{l})^4.sin\frac{n\pi x}{l} - g^2\left(\frac{n\pi}{l}\right)^2.sin\frac{n\pi x}{l} - a^4.sin\frac{n\pi x}{l}\right].\left[sin\frac{n\pi x}{l}\right]\}.dx = 0$$

$$(\frac{n\pi}{l})^4 - m.\frac{p^2}{EI}[1 + (e_0 a)^2.(\frac{n\pi}{l})^2] = 0$$

$$p^2 = \frac{(\frac{n\pi}{l})^4.EI}{m.[1+(e_0 a)^2.(\frac{n\pi}{l})^2]} \quad (15)$$

3.2 Clamped- Clamped end Conditions

Boundary conditions are as follows:



$\emptyset(0) = 0$

$\emptyset^I(0) = 0$

$\emptyset(l) = 0$

$\emptyset^I(l) = 0$

The function satisfying these boundary conditions is $\emptyset(x) = [1 - \cos\frac{2n\pi x}{l}]$

Substituting in (13) we get,

$$[-(\frac{2\pi}{l})^4.cos\frac{2\pi x}{l}] + [g^2.(\frac{2\pi}{l})^2.cos\frac{2\pi x}{l}] - \left[a^4\left(1 - cos\frac{2\pi x}{l}\right)\right] = R \quad (16)$$

Using Galerkin's method, we get,

$$m.\frac{p^2}{EI}.\left(\frac{3l}{2}\right) + (e_0 a)^2.m.\frac{p^2}{EI}.(\frac{2\pi}{l})^2.\frac{l}{2} = (\frac{2\pi}{l})^4.\frac{l}{2}$$

$$p^2 = \frac{(\frac{2n\pi}{l})^4.EI}{m.[3+(e_0 a)^2.(\frac{2n\pi}{l})^2]} \quad (17)$$

3.3 Clamped- Hinged end Conditions

Boundary conditions are as follows:

$\emptyset(0) = 0$

$\emptyset^I(0) = 0$

$\emptyset(l) = 0$

$\emptyset^{II}(l) = 0$

The function satisfying these boundary conditions is

$$\emptyset(x) = \sin\left(\frac{n\pi x}{l}\right).\left[1 - \cos\left(\frac{2n\pi x}{l}\right)\right]$$

Simplifying this function we get, $\emptyset(x) = 3.\sin\left(\frac{n\pi x}{l}\right) - \sin\left(\frac{3n\pi x}{l}\right)$

Substituting in (13) we get,

$$3.\left(\frac{n\pi}{l}\right)^4.\sin\left(\frac{n\pi x}{l}\right)-\left(\frac{3n\pi}{l}\right)^4.\sin\left(\frac{3n\pi x}{l}\right)-g^2.3.\left(\frac{n\pi}{l}\right)^2.\sin\left(\frac{n\pi x}{l}\right)$$

$$+g^2.\left(\frac{3n\pi}{l}\right)^2.\sin\left(\frac{3n\pi x}{l}\right)-a^4.3.\sin\left(\frac{n\pi x}{l}\right)+a^4.\sin\left(\frac{3n\pi x}{l}\right)=R \quad (18)$$

Using Galerkin's method we get,

$$\int_0^l[3.\left(\frac{n\pi}{l}\right)^4.\sin\left(\frac{n\pi x}{l}\right)-\left(\frac{3n\pi}{l}\right)^4.\sin\left(\frac{3n\pi x}{l}\right)-g^2.3.\left(\frac{n\pi}{l}\right)^2.\sin\left(\frac{n\pi x}{l}\right)+g^2.\left(\frac{3n\pi}{l}\right)^2.\sin\left(\frac{3n\pi x}{l}\right)-a^4.3.\sin\left(\frac{n\pi x}{l}\right)+a^4.\sin\left(\frac{3n\pi x}{l}\right)]$$

$$[\,3.\sin\left(\frac{n\pi x}{l}\right)-\sin\left(\frac{3n\pi x}{l}\right)].dx=0$$

$$45.\left(\frac{n\pi}{l}\right)^4=9.(e_0a)^2.m.\frac{p^2}{EI}.\left(\frac{n\pi}{l}\right)^2+5.m.\frac{p^2}{EI}$$

$$p^2=\frac{45.\left(\frac{n\pi}{l}\right)^4.EI}{m.[5+9.(e_0a)^2.\left(\frac{n\pi}{l}\right)^2]} \quad (19)$$

IV RESULTS AND DISCUSSIONS

Undamped free flexural vibration analysis of nanobeams of various spans with different nonlocal parameters has been done using Galerkin's method. The results are tabulated and graphically depicted for the various boundary conditions**.**

4.1 Simply Supported End Condition

Table 1: Small scale effect on the fundamental frequency for different length scales for simply supported end conditions of nanobeam

e_oa	Frequency 'p'			
	L= 5 nm	L= 10 nm	L= 15 nm	L= 20 nm
0	6.570E+11	1.642E+11	0.730E+11	0.410E+11
0.50E-9	6.268E+11	1.622E+11	0.726E+11	0.409E+11
1.00E-9	5.563E+11	1.567E+11	0.714E+11	0.405E+11
1.50E-9	4.781E+11	1.485E+11	0.696E+11	0.399E+11
2.00E-9	4.091E+11	1.390E+11	0.673E+11	0.391E+11
2.50E-9	3.528E+11	1.291E+11	0.646E+11	0.382E+11
3.00E-9	3.079E+11	1.195E+11	0.618E+11	0.371E+11
3.50E-9	2.719E+11	1.105E+11	0.588E+11	0.359E+11
4.00E-9	2.429E+11	1.022E+11	0.559E+11	0.347E+11
4.50E-9	2.190E+11	0.948E+11	0.531E+11	0.335E+11
5.00E-9	1.992E+11	0.882E+11	0.504E+11	0.322E+11

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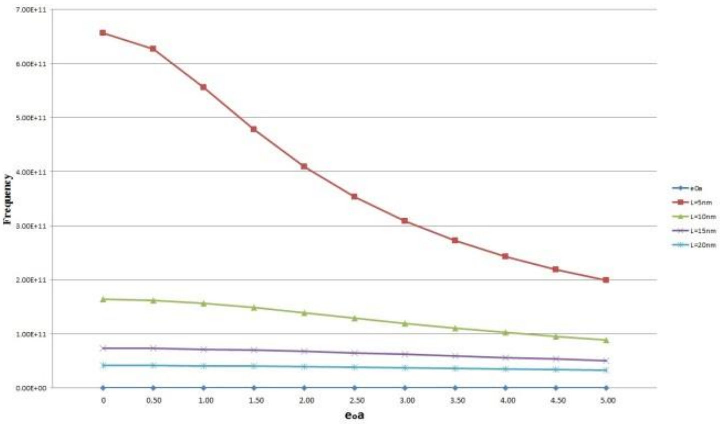

Fig.2- Small scale effect on the fundamental frequency for different length scales for simply supported end conditions of nanobeam

From this graph it is evident that the non-local fundamental frequency decreases with increasing values of the non-local parameter.

4.2 Clamped- Clamped end Condition

Table 2: Small scale effect on the fundamental frequency for different length scales for clamped- clamped end conditions of nanobeam

E	Frequency ‘p’			
	L= 5 nm	L= 10 nm	L= 15 nm	L= 20 nm
0	1.517E+12	3.793E+11	1.686E+11	0.948E+11
0.50E-9	1.426E+12	3.732E+11	1.673E+11	0.944E+11
1.00E-9	1.228E+12	3.566E+11	1.638E+11	0.933E+11
1.50E-9	1.026E+12	3.332E+11	1.584E+11	0.915E+11
2.00E-9	8.610E+11	3.070E+11	1.517E+11	0.891E+11
2.50E-9	7.326E+11	2.810E+11	1.442E+11	0.863E+11
3.00E-9	6.334E+11	2.566E+11	1.364E+11	0.833E+11
3.50E-9	5.560E+11	2.347E+11	1.286E+11	0.800E+11
4.00E-9	4.943E+11	2.152E+11	1.211E+11	0.767E+11
4.50E-9	4.443E+11	1.981E+11	1.140E+11	0.734E+11
5.00E-9	4.032E+11	1.831E+11	1.074E+11	0.702E+11

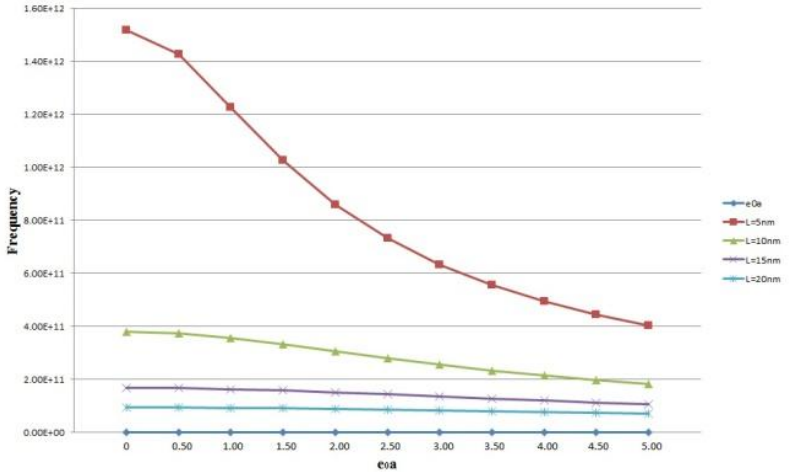

Figure 3: Small scale effect on the fundamental frequency for different length scales for clamped- clamped end conditions of nanobeam

4.3 Clamped- Hinged End Condition

Table 3: Small scale effect on the fundamental frequency for different length scales for clamped- hinged end conditions of nanobeam

eoa	Frequency 'p'			
	L= 5 nm	L= 10 nm	L= 15 nm	L= 20 nm
0	1.971E+12	4.927E+11	2.190E+11	1.231E+11
0.50E-9	1.816E+12	4.821E+11	2.168E+11	1.225E+11
1.00E-9	1.507E+12	4.541E+11	2.108E+11	1.205E+11
1.50E-9	1.222E+12	4.165E+11	2.018E+11	1.174E+11
2.00E-9	1.005E+12	3.767E+11	1.909E+11	1.135E+11
2.50E-9	8.450E+11	3.392E+11	1.792E+11	1.089E+11
3.00E-9	7.248E+11	3.056E+11	1.674E+11	1.041E+11
3.50E-9	6.327E+11	2.765E+11	1.561E+11	0.991E+11
4.00E-9	5.604E+11	2.513E+11	1.455E+11	0.941E+11
4.50E-9	5.024E+11	2.298E+11	1.358E+11	0.893E+11
5.00E-9	4.550E+11	2.112E+11	1.270E+11	0.848E+11

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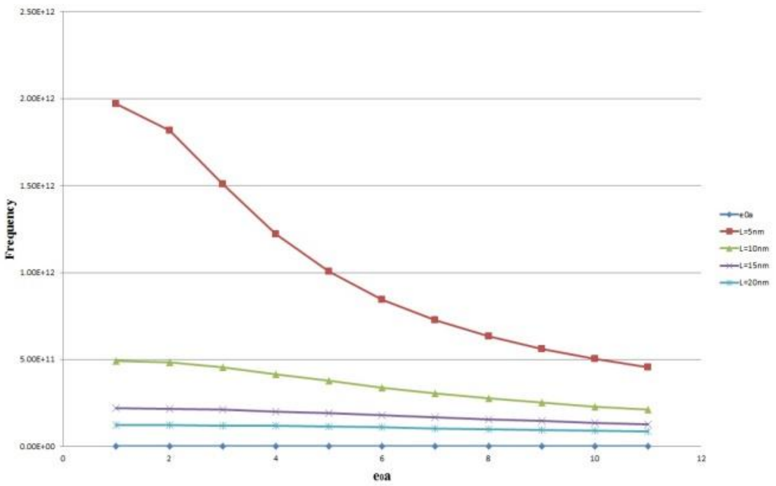

Figure 4: Small scale effect on the fundamental frequency for different length scales for clamped- hinged end conditions of nanobeam

4.4 Discussions

The variation of fundamental frequency for various boundary conditions for different values of the nonlocal parameter, and length illustrates the influence of the small scale parameter on the free flexural vibrational behavior of nanobeams. Variation of fundamental frequency with length of the nanobeam for different e_0a values for three boundary conditions is considered in figures 2, 3 and 4. According to these figures it is seen that, nonlocal solution of the frequency is smaller than the classical (local) result due to the effect of small length scale. Furthermore, increasing the nonlocal parameter decreases the frequency. The result may be interpreted as increasing the nonlocal parameter for fixed 'L' leads to a decrease in the stiffness of structure. Approximately, for L ≥ 20nm all results converge to the local frequency. Thus it is confirmed that the effect of the nonlocal parameter is to decrease the frequency. Also frequency decreases with the increase of the beam length L. This is because the wavelength gets larger with increase in beam length. It means nonlocal effects are lost after a certain length. This implies that as the length of the nanobeam increases, the nonlocal effect decreases and will finally be lost after reaching a certain length value. The effect of the nonlocal parameter in decreasing the frequency is more predominant in clamped- clamped end condition when compared with the other two boundary conditions.

V. CONCLUSIONS

The above analytical investigations lead to the conclusion that the effect of the increase in nonlocal parameter is to decrease the frequency of flexural vibration in nanobeams. The nonlocal effects are very minimal at L=10 nm and disappear for greater lengths. The following are noteworthy:

1) In the case of nanobeams, under free flexural vibration, the nonlocal solution of frequency is smaller than the classical result.
2) The frequency of nanobeams decreases with increase in nonlocal parameter.

3) Frequency decreases with increase in the beam length which means that the nonlocal effects are lost after a certain length. The nonlocal frequency becomes equal to the classical frequency when the nonlocal parameter becomes zero.

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