International Journal of Advance Research in Science and Engineering
Vol. No.4, Issue 11, November 2015
www.ijarse.com
IJARSE
ISSN 2319 - 8354

STUDY ON DYNAMIC BEHAVIOUR OF NANOBAR USING NONLOCAL ELASTICITY THEORY

K.Ajeetha[1], M.G.Rajendran[2]

[1] *PG Student,* [2] *Professor, School of Civil Engineering, Karunya University, Coimbatore, Tamilnadu, (India)*

ABSTRACT

In this paper, undamped free axial vibration of nanobar is investigated using Eringen's nonlocal elasticity theory. The governing equation for undamped free axial vibration of nanobars is derived using Euler Bernoulli beam theory and by introducing Eringen's nonlocal elasticity theory. The solutions are obtained for clamped-clamped, clamped-free, and the bar with concentrated end mass boundary conditions. The effect of nonlocal parameter on the free axial vibrational frequencies of nanobar is studied. The analytical results and the available solutions are compared and the frequencies for all the boundary conditions are in excellent agreement with the existing results.

Keywords: Nanobars, Nonlocal Elasticity Theory, Axial Vibration, Natural Frequency

I. INTRODUCTION

Carbon nanotubes are discovered by Iijima [1] in 1991. Carbon nanotubes are the strongest and stiffest materials in terms of tensile strength and elastic modulus. The studies related with nanostructures have shown that CNTs have good electrical properties and high mechanical strength, so they can be used for nanoelectronics, nanodevices and nanocomposites [2].

C. M. Wang et al. [3] reviews recent research studies on the buckling of carbon nanotubes. The structure and properties of carbon nanotubes are introduced. The various buckling behaviours exhibited by carbon nanotubes are also presented. It also found that CNTs have the remarkable flexibility and stability under external loading. S.Adhikari et al. [4] studied the Free and forced axial vibrations of damped nonlocal rods using dynamic finite element method. Helong wu [5] investigated the free vibration and elastic buckling of sandwich beams with a stiff core and functionally graded carbon nanotube reinforced composite (FG-CNTRC) face sheets within the framework of Timoshenko beam theory. Metin Aydogdu [6] developed the Nonlocal elastic rod model and applied it to investigate the small scale effect on axial vibration of the nanorods. [7] In this study generalized non local beam theory is proposed to study bending, buckling free vibrations of nanobeams. Nonlocal constitutive equations of Eringen are used in the formulations. J.N. Reddy, et al. [8] developed non- linear finite element models based on Eringen's [9] non-local differential model to obtain numerical results for static bending of nanobeam Structures. Hassan Raffaele Barretta [10] proposed the variational formulation of the nonlocal elastostatic problem to study the small-scale effects in nanorods. Chawis Thongyothee et al. [11] investigated the free vibration analysis of single-walled carbon nanotubes including the effect of small length scale based on the nonlocal elasticity theory. Q.Wang, K.M.Liew [12] investigated The Scale effects on static deformation of micro and nanorods or nanotubes. The nonlocal Euler-Bernoulli beam theory and Timoshenko beam theories are used. As studied earlier, the nanobars have varied engineering applications as components in nano devices, nano- composites, electrical and electromechanical instruments due to their good electrical properties and high mechanical strength. During operation often these components may be subjected to external loads, which may have an impact on their dynamics characteristics. The response characteristics of nanostructures are very different from other micro/ macro structures due to the inherent size effect. Hence studies on the vibrational behavior of nanobars must be carried out for their rational design in nano devices The authors of this paper are of the opinion that the studies here to reported are not sufficient to thoroughly understand the effect of small scale parameter and the limitations of classical elastic theory in dealing with the length scale of nanobars. The main objective of this paper is to carry out undamped free axial vibration of the nanobars under arbitrary boundary conditions using nonlocal elasticity theory and to study the effect of the nonlocal parameter on the vibrational frequencies.

II. NONLOCAL ELASTICITY THEORY

In the analysis of macro beams, the classical theory of elasticity is used. But when the small scale/ nanoscale (e.g. CNT) is taken into account, the classical theory does not hold good. Hence the Nonlocal elasticity theory which was proposed by Eringen is adopted to account for small scale effect in elasticity by assuming the stress at a reference point to be a functional of the strain field at every point in the body [9]. In this way, the internal size scale could be considered in the constitutive equations simply as a material parameter. The application of nonlocal elasticity, in micro and nano materials, has received much attention among the nanotechnology community recently. This important length scale effect is used in vibration, buckling and bending of CNTs studies. The application of nonlocal elasticity is recommend in revealing scale effects for nano-materials like CNTs.

ccording to Eringen, the nonlocal stress tensor σ at point X is expressed as

$$\sigma = \int_V K(|\mathbf{x}' - \mathbf{x}|, \tau)\mathbf{t}(\mathbf{x}')d\mathbf{x}',$$

Use of integral constitutive relations is relatively more difficult in computation than using algebraic or differential constitutive relations. Realizing this fact Eringen proposed an equivalent differential model as

$$(1 - \mu_0{}^2\nabla^2)\bar{\sigma} = \sigma, \mu_0 = \tau^2.l^2 = e_0{}^2.a^2$$

$\mu = e_0{}^2 a^2$ is the function of material constant,

$e_0 =$ material constant

a = internal characteristic lengths (such as the lattice spacing).

In general, $e_0 a$ is called the nonlocal parameter which is a factor to consider the effect of small length scale.

International Journal of Advance Research in Science and Engineering
Vol. No.4, Issue 11, November 2015
www.ijarse.com
IJARSE
ISSN 2319 - 8354

III. FORMULATION OF GOVERNING EQUATION

The governing equation for an undamped free axial vibration of a nanobar using nonlocal elasticity theory is derived on the assumptions of the Euler Bernoulli beam theory.

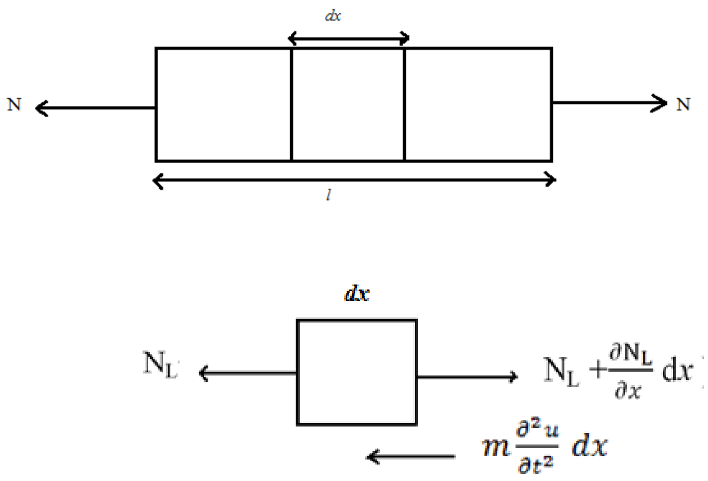


Fig.1 Equilibrium Condition of a Differential Element of a Nanobar Subjected to the Axial Force.

Force equilibrium along horizontal direction,

$$-N_L + \left(N_L + \frac{\partial N_L}{\partial x} dx \right) - m \frac{\partial^2 u}{\partial t^2} dx = 0$$

$$\frac{\partial N_L}{\partial x} = m \frac{\partial^2 u}{\partial t^2} \quad (1)$$

Nonlocal constitutive law

$$\left[1 - (e_0 a)^2 \frac{\partial^2}{\partial x^2}\right] \tau_{xx} = E\varepsilon \quad (2)$$

For local,

$$N_L = \int \sigma_{xx} \, dA \quad (3)$$

For nonlocal,

$$N = \int \tau_{xx} \, dA \quad (4)$$

Integrating (2) w.r.to area,

$$\int \left[\left\{1 - (e_0 a)^2 \frac{\partial^2}{\partial x^2}\right\}\right] \tau_{xx} \, dA = \int E\varepsilon \, dA$$

$$\left[1 - (e_0 a)^2 \frac{\partial^2}{\partial x^2}\right] N = N_L \quad (5)$$

Diff (5) w.r. to *x*,

$$\left[1 - (e_0 a)^2 \frac{\partial^2}{\partial x^2}\right] \frac{\partial}{\partial x} N = \frac{\partial^2 u}{\partial x^2} EA \quad (6)$$

Substitute (1) in (6), we get

$$\left[1 - (e_0 a)^2 \frac{\partial^2}{\partial x^2}\right] m \frac{\partial^2 u}{\partial t^2} = EA \frac{\partial^2 u}{\partial x^2}$$

Governing equation for the axial vibration of the nanobars

$$EA\,\frac{\partial^2 u}{\partial x^2} = m\frac{\partial^2 u}{\partial t^2} - \; m(e_0 a)^2\frac{\partial^4 u}{\partial x^2\,\partial t^2} \tag{7}$$

Solution for the harmonic vibration

Let $u(x,t) = \emptyset(x)\sin\omega t$ (8)

Substitute (8) into (7)

$$\frac{-m\omega^2 l^2}{EAl^2}\emptyset + \frac{m(e_0 a)^2\omega^2 l^2}{EAl^2}\emptyset^{II} \;=\; \emptyset^{II}$$

Let $k^2 \;=\; \frac{m\omega^2 l^2}{EA}$ (9)

k = dimensionless frequency parameter

$$\emptyset^{II} + \frac{\frac{k^2\emptyset}{l}}{\left[1-\frac{(e_0 a)^2 k^2}{l^2}\right]} = 0$$

$$\beta^2 \;=\; \frac{\frac{k^2}{l^2}}{\left[1-\frac{(e_0 a)^2 k^2}{l^2}\right]} \tag{10}$$

$$\emptyset^{II} + \beta^2\emptyset = 0$$

The solution is $\emptyset(x) \;= A\cos\beta x + B\sin\beta x$ (11)

IV. ANALYSIS OF NANOBAR WITH VARIOUS BOUNDARY CONDITIONS

4.1 Clamped – Clamped

The solution is $\emptyset(x) \;= A\cos\beta x + B\sin\beta x$

Boundary conditions

(i) $\emptyset(0) = 0$

A=0 (12)

(ii) $\emptyset(l) = 0$

$B\sin\beta l \;= 0$ (13)

To satisfy (13), β can be chosen as $\beta = n\pi$

$\beta^2 = (n\pi)^2$ (14)

Substituting (10) in (14) ,we get

$$\frac{k^2 l^2/_{l^2}}{\left[1-\frac{(e_0 a)^2 k^2}{l^2}\right]} \;=\; (n\pi)^2$$

solving we get,

$$k^2 = \frac{(n\pi)^2}{1+\frac{(e_0 a)^2(n\pi)^2}{l^2}}$$

$$\omega^2 = \frac{(n\pi)^2 EA}{[l^2 + (n\pi)^2 (e_0 a)^2] m}$$

4.2 Clamped Free

Boundary conditions

(i) $\emptyset(0) = 0$

$A = 0$ (15)

(ii) $\frac{d^3\emptyset}{dx^3}(l) = 0$

$\cos\beta l = 0$ (16)

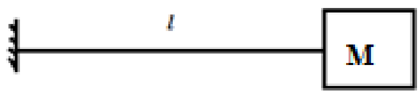
To satisfy (16), choose $\beta = \frac{(2n-1)\pi}{2}$ where $n = 1,2,3 \ldots$

Solving we get

$$k^2 = \frac{\left[\frac{(2n-1)\pi}{2}\right]^2}{1 + \frac{(e_0 a)^2}{l^2}\left[\frac{(2n-1)\pi}{2}\right]^2}$$

$$\omega^2 = \frac{\left[\frac{(2n-1)\pi}{2}\right]^2 EA}{\left[l^2 + (e_0 a)^2 \left[\frac{(2n-1)\pi}{2}\right]^2\right] m}$$

4.3 Bar with Free Concentrated End Mass



The solution is $u(x,t) = (A\cos\beta x + B\sin\beta x)\sin\omega t$

Boundary conditions

(i) $\emptyset(0) = 0$

$A = 0$

$\emptyset(x) = B\sin\beta x$

(ii) At $x = l$

$$EA\frac{\partial u}{\partial x} = -m\frac{\partial^2 u}{\partial t^2}$$

$$EA\frac{\partial u}{\partial x} = m\ \sin\beta l\,\omega^2 \sin\omega t$$

$$\tan\beta l = \frac{EA\beta}{m\omega^2}$$

IJARSE
ISSN 2319 - 8354

If m is very small compared to the beam mass (m = ρA)

$\tan \beta l = \propto$ (if m = 0)

Choosing $\beta l = \left[\frac{(2n-1)\pi}{2}\right]$, where n =1,3,5 ....

When n=1

$$\beta^2 = \frac{\pi^2}{4l^2} = \frac{m\omega^2}{EA-(m\omega^2)(e_0 a)^2}$$

$$\frac{EA\pi^2}{4l^2} - \frac{\pi^2}{4l^2}(m\omega^2)(e_0 a)^2 = m\omega^2$$

$$\omega^2 = \frac{\pi^2 E / 4l^2\rho}{\left[1 + \frac{\pi^2}{4l^2}(e_0 a)^2\right]}$$

V. RESULTS AND DISCUSSIONS

Frequency analysis has been done for nanobars of various spans with different nonlocal parameters, lengths and mode numbers. The results are graphically depicted for various boundary conditions**.**

5.1 Results

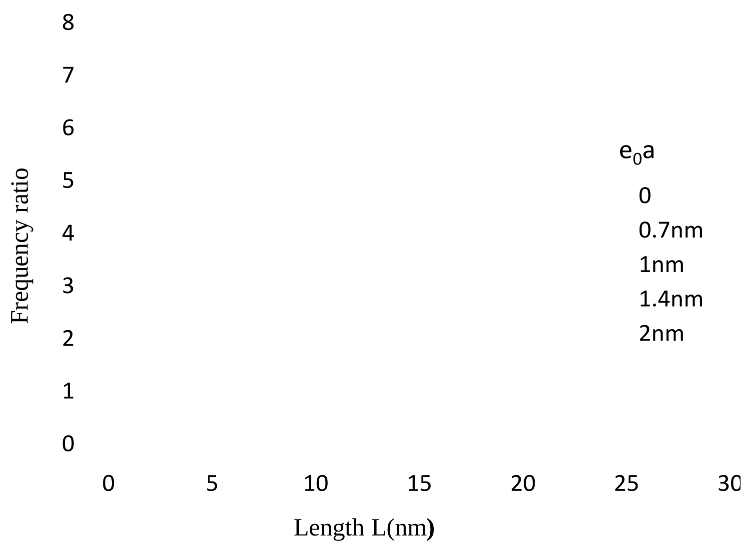


Fig 2. Small Scale Effect on Clamped-Clamped Nanobar at Different Scale Coefficients for Fundamental Frequency

International Journal of Advance Research in Science and Engineering
Vol. No.4, Issue 11, November 2015
www.ijarse.com
IJARSE
ISSN 2319 - 8354

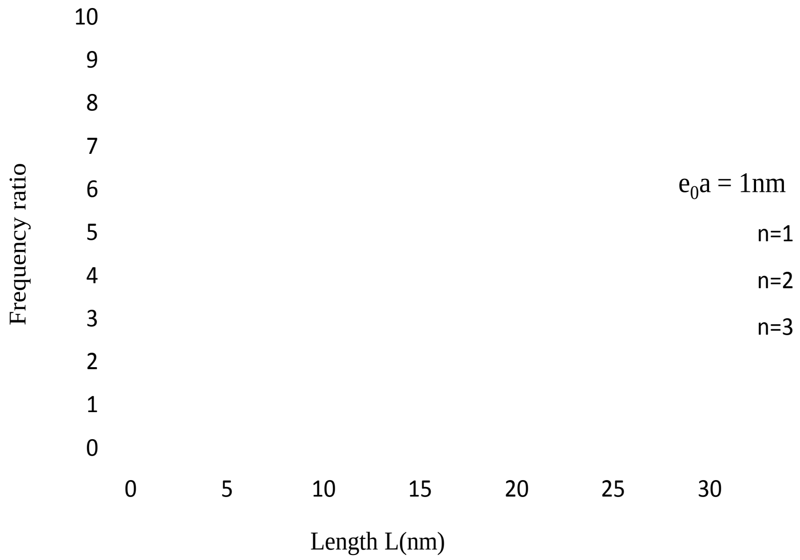


Fig 3.Small Scale Effect on Clamped-Clamped Nanobar for First Three Frequencies

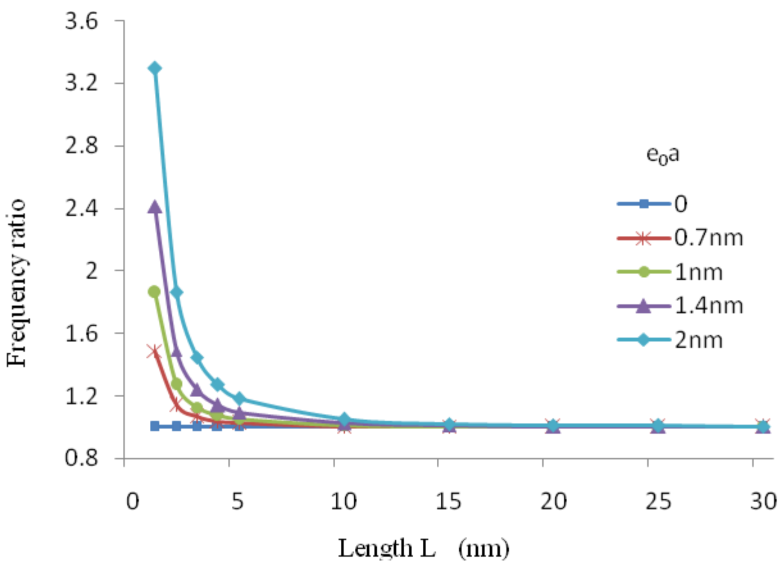


Fig 4.Small Scale Effect on Clamped-Free Nanobars at Different Scale Coefficients for Fundamental Frequency

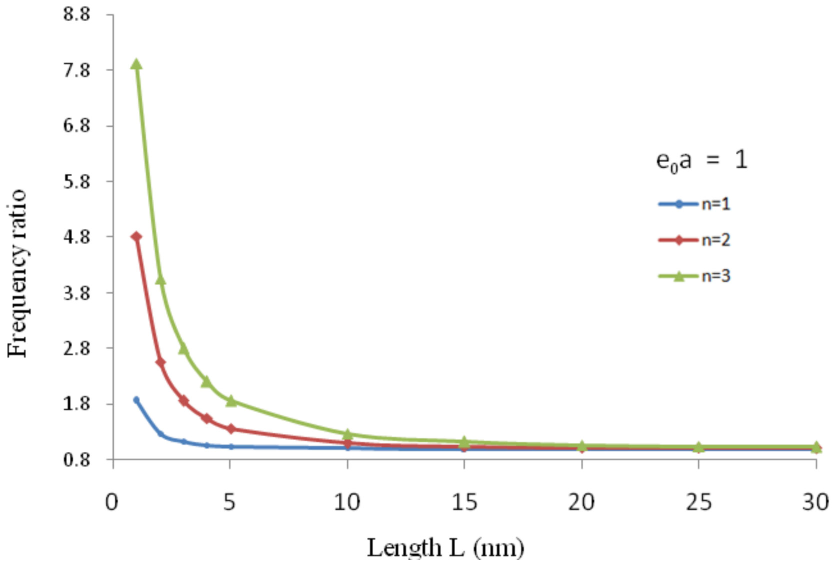


Fig 5. Small Scale Effect on Clamped-Free Nanobar for First Three Frequencies

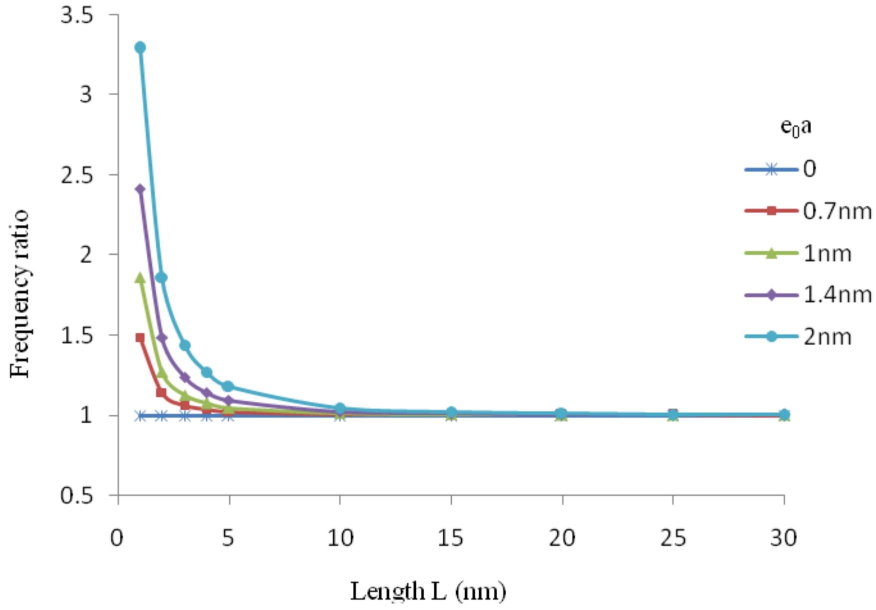


Fig 6. Small Scale Effect on Bar with Concentrated End Mass at Different Scale Coefficients for Fundamental Frequency.

IJARSE
ISSN 2319 - 8354

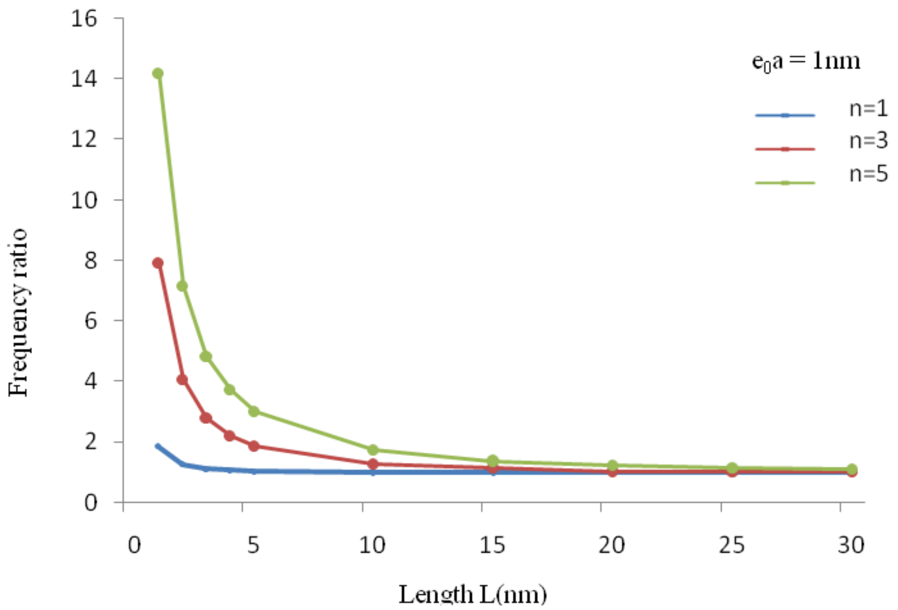


Fig 7. Small Scale Effect on Bar with Concentrated End Mass for Three Frequencies.

5.2 Discussions

The ratio of local frequency to nonlocal frequency is discussed for various boundary conditions for different scale coefficients, lengths and mode numbers to illustrate the influence of small length scale on the undamped axial vibration of nanobars. Variation of fundamental frequency parameter with length of rod is given for different scale coefficients e_0a for three boundary conditions considered in Figs. 2, 4, 6. From these figures it is evident that, nonlocal solution of the frequency is smaller than the classical result due to the effect of small length scale. Increasing the nonlocal parameter decreases the frequency (i.e. increases frequency ratio).The result may be interpreted as increasing the nonlocal parameter for fixed L leads to a decrease in the stiffness of structure. For L ≥ 20nm all results converge to the local frequency. Frequency ratio decreases with the increase of the rod length L. It means nonlocal effects are lost after a certain length. $e_0a = 0$ corresponds to classical solution where ratio of classical frequency to nonlocal frequency equals to unity.

In figures 3, 5, 7, the three frequency parameters are depicted for $e_0a = 1$ nm for C–C, C–F, Bar with free concentrated end mass boundary conditions, respectively. Effect of small length scale is higher for higher modes. Nonlocal effects are lost almost at L = 10 for n = l, but same effects disappear at nearly L = 25 for n = 3. This is because of small wavelength effects for higher modes.

VI. CONCLUSIONS

The above analytical investigations lead to the conclusion that the effect of the increase in nonlocal parameter is to decrease the frequency of axial vibration in nanobars. The nonlocal effects are very minimal at L=10 nm and disappear for greater lengths. The followings are noteworthy

- In the case of nanobars under undamped axial vibration, the nonlocal solution of frequency is smaller than the classical result.
- The frequencies of nanobars decreases with increase in nonlocal parameter.

- Frequency Ratio decreases with increase in the bar length which means that the nonlocal effects are lost after a certain length.
- Ratio of classical frequency to nonlocal frequency tends to unity when nonlocal parameter tends to zero.
- Effect of small length scale is higher for higher modes.

VII. ACKNOWLEDGEMENT

The authors are thankful to the Management of Karunya University for providing necessary facilities to carry out the work reported in this paper.

REFERENCES

[1] Iijima.S., “Helical microtubules of graphitic carbon”,Nature, 354, 1991, 56-58

[2] Ball.P., “Roll up for revolution”, Nature,414,2001,142-144

[3] C.M.Wang et al., “Recent Studies on Buckling of Carbon Nanotubes”, Journal of applied mechanics reviews, 2010, Vol. 63

[4] S.Adhikari ,T.Murmu, M.A Mccarthy, “Dynamic finite element analysis of axially vibrating nonlocal rods”, Journal of finite elements in analysis and design, 2013, vol 63

[5] Helong Wu., “Free Vibration and Buckling Analysis of Sandwich Beams with Functionally Graded Carbon Nanotube-Reinforced Composite Face Sheets” International Journal of Structural Stability and Dynamics Vol. 15,2015, No. 7

[6] Metin Aydogdu., “Axial vibration of the nanorods with the nonlocal continuum rod model”, Physica E 41, 2009, 861–864

[7] Metin Aydogdu, “A general nonlocal beam theory:its application to nanobeam bending,buckling and vibration” Journal of physica E 41, 2009

[8] J.N. Reddy, Samiel-Borgi and JaniRomanoff, “ Non-linear analysis of functionally graded microbeams using Eringen's non-local differential model”, International Journal of Non-Linear Mechanics 67, 2014, 308–318

[9] A. Cemal Eringen , “On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves”, Journal of applied physics 54, 1983, 4703.

[10] Raffaele Barretta ,Francescco Marotti,Marina Diaco, “Small-scale effects in nanorods” Journal of Acta Mech 225, (2014)

[11] Chawis Thongyothee, Somchai Chucheepsakuland Teng Li, “Nonlocal Elasticity Theory for Free Vibration of Single-walled Carbon Nanotubes”**.** Advanced Materials Research, 2013, Vol. 747

[12] Q.Wang,K.M.Liew, “Application of nonlocal continuum mechanics to static analysis of micro and nano structures” Journal of physics letter A ,363, 2007

[13] Devesh Kumar,christian Heinrich,anthony M.Wass, “Buckling analysis of carbon nanotubes modelled using nonlocal continuum theories” journal of applied physics, 2008