International Journal of Advance Research in Science and Engineering
Vol. No.4, Issue 11, November 2015
www.ijarse.com
IJARSE
ISSN 2319 - 8354

STUDY ON STABILITY BEHAVIOR OF NANOBEAM WITH AXIAL FORCE USING NONLOCAL ELASTICITY THEORY

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ABSTRACT

This paper presents the analytical solution for the free vibration of Nanobeam along with axial compression force based on nonlocal elasticity theory. Classical theory is inherently size dependent while nonlocal theory provides an effective tool to assess the small scale effects in nanobeam. The governing equation for free vibration of nanobeam with axial compression force is derived using Euler beam theory and by introducing Eringen's nonlocal elasticity theory. The effects of nonlocal nanoscale parameter and the axial compression force on the vibrational behavior of nanobeam are studied. The analytical results and the available solutions are compared and the frequencies for all the boundary conditions are in excellent agreement with the existing results.

Keywords: Axial Compression, Frequency, Nanobeam, Nonlocal Elasticity theory, Vibration

I. INTRODUCTION

Nanotubes and nanobeams play key roles in many engineering devices at nanometer scale, such as micro- or nanoelectromechanical systems (MEMS or NEMS). Although classical theories of linear and nonlinear vibration of strings and beams at macroscales are well established, the vibration behavior of structures at nanoscale which is significantly size dependent is far from being well understood. In an effort to extend the classical theory for size dependent Nano mechanics, the vibration of a nanobeam along with axial force is investigated herein based on the nonlocal elastic stress theory.

Mingwu Li[2], obtained the analytical solutions for the vibration of a beam with axial force subjected to generalized support motion and compared with the finite element solutions. YS Li, P Ma and W Wang [3] studied the bending, buckling, and free vibration of magnetoelectroelastic nanobeam based on nonlocal theory and Timoshenko beam theory. Yi-Ze Wang, Feng-Ming Li [4] studied the nonlinear primary resonance of nanobeam with the axial initial load with the influence of small scale effect, axial initial load, mode number, and the ratio of the length to the diameter. Reza Nazemnezhad, Shahrokh Hosseini-Hashemi, Hossein Rokni [5] investigated the nonlinear free vibration analysis of simply-supported nanoscale beams incorporating surface effects using the nonlocal elasticity within the frame work of Euler-Bernoulli beam theory. Huu-Tai Thai [6] proposed nonlocal shear deformation beam theory for bending, buckling, and vibration of nanobeam using the nonlocal differential constitutive relations of Eringen. LI Cheng, LIM C. W., YU Ji Lin & ZENG Qing Chuan

[7] conducted a study on the transverse vibration of a simply supported nanobeam with an initial axial tension based on the nonlocal stress field theory with a nonlocal size parameter. C. Li, C. W. Lim, J. L. Yu and Q. C. Zeng [8] studied the analytical solutions for the transverse vibration of simply supported nanobeams subjected to an initial axial force based on nonlocal elasticity theory. Pin Lu, H.P. Lee, C. Lu, P.Q. Zhang [8] conducted a study on the wave and vibration properties of single- or multi-walled carbon nanotubes. C Li, C W Lim and J L Yu [9] investigated the natural frequency and stability for the transverse vibrations of a nanobeam subjected to a variable initial axial force, including axial tension and axial compression, based on nonlocal elasticity theory. S. Naguleswaran [10] studied the transverse vibration of uniform Euler–Bernoulli beams under linearly varying fully tensile, partly tensile or fully compressive axial force distribution. The authors of this paper are of the opinion that the studies hereto reported are not sufficient to thoroughly understand the effect of small scale parameter and the limitations of classical elastic theory in dealing with the length scale of nanobeams. The main objective of this paper is carry out undamped free flexural vibration of the nanobeam along with axial compression force under arbitrary boundary conditions by using Galerkin's method and to study the effect of the nonlocal parameter on the vibrational frequencies.

II. NONLOCAL ELASTICITY

In the analysis of macro beams, the classical theory of elasticity is used. But when the small scale/ nano scale (e.g. CNT) is taken into account, the classical theory does not hold good. Hence the Nonlocal elasticity theory which was proposed by Eringen[1] is adopted to account for small scale effect in elasticity by assuming the stress at a reference point to be a functional of the strain field at every point in the body. Since the classical theory of elasticity (Hooke's Law) excludes the long range interactions. The nonlocal theory holds good for these effects.

According to Eringen[1] , the nonlocal stress tensor σ at point X is expressed as

$$\sigma = \int_V K(|\mathbf{x}' - \mathbf{x}|, \tau)\mathbf{t}(\mathbf{x}')d\mathbf{x}',$$

Use of integral constitutive relations is relatively more difficult in computation than using algebraic or differential constitutive relations. Realizing this fact, Eringen[1] proposed an equivalent differential model as

$$\sigma_{xx} - \mu\frac{\partial^2\sigma_{xx}}{\partial x^2} = E\varepsilon_{xx}$$

$\sigma_{xx}, \varepsilon_{xx}$ And E are a nonlocal stress, a normal strain and Young's modulus, respectively of small scale length.

$\mu \; = {e_0}^2 l^2$ is the function of material constant,

e_0 = Material constant

l = internal characteristic lengths (such as the lattice spacing).

In general, $e_0 a$ iscalled the nonlocal parameter which is a factor to be incorporated while considering the effect of small length scale.

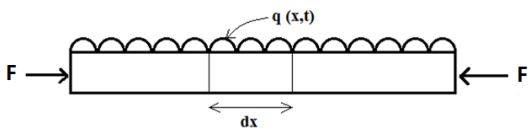
III. GALERKIN'S METHOD

The Galerkin's method constructs an approximate solution of the given problem. Each basic function must satisfy an admissibility condition appropriate for the problem. The basic functions can be chosen to be weak solutions of the Galerkin's integral representation associated with the given problem. The following steps are performed in Galerkin's method:

1. Assume an approximate solution which satisfies the boundary conditions and substitute it in the governing equation. This will result in a residue/ error.
2. Multiply the residual of the governing equation by a weighting function or the assumed solution and set the domain integral equal to zero.

IV. EQUATION OF MOTION

The governing equation for an undamped free flexural vibration of a nanobeam along with axial compression force based on the assumptions of Euler- Bernoulli beam theory is derived as follows.



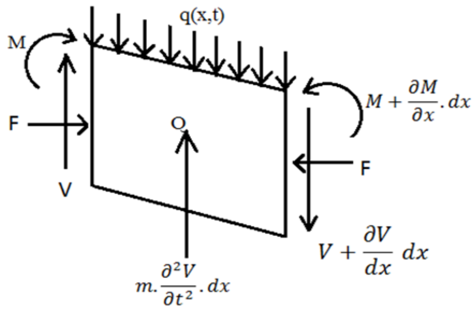


Fig.1 Equilibrium Condition of a Differential Element of a Nanobeam Subjected to a Uniformly Distributed Load.

From Fig 2, Considering force equilibrium in vertical direction,

$$\frac{dv}{dx} = -\,q(x,t) + m\frac{\partial^2 v}{\partial x^2} \qquad (1)$$

Taking moment about the center,

$$v = \frac{\partial M}{\partial x} - F\frac{\partial v}{\partial x} \qquad (2)$$

Substituting (2) in (1), we get

$$\frac{d^2 M}{dx^2} - F\frac{\partial^2 v}{\partial x^2} - m\frac{\partial^2 v}{\partial t^2} = -q(x,t) \qquad (3)$$

Since, Moment can be written as,

$$M = \int \sigma \; dA\, dy \qquad (4.1)$$

And the nonlocal constitutive law can be written as,

$$\sigma(x) - (e_0 a)^2 \frac{\partial^2 \sigma(x)}{\partial x^2} = E\,\varepsilon(x) \tag{4.2}$$

Substituting (4.2) in (4.1)

$$\frac{\partial^2 M}{\partial x^{\ 2}} = 1/(e_0 a)^2 \left[M + EI\frac{\partial^2 v}{\partial x^2}\right] \tag{5}$$

Deriving $\frac{\partial^2 M}{\partial x^{\ 2}}$ from (3) and equating to (5),

$$\mathrm{M} = \mathrm{F}\,(\mathrm{e_0 a})^2 \frac{\partial^2 \mathrm{v}}{\partial \mathrm{x}^2} + \mathrm{m}(\mathrm{e_0 a})^2 \frac{\partial^2 \mathrm{v}}{\partial \mathrm{t}^2} - (\mathrm{e_0 a})^2\, \mathrm{q(x,t)} - \mathrm{EI}\frac{\partial^2 \mathrm{v}}{\partial \mathrm{x}^2} \tag{6}$$

Substituting (6) in (2)

$$\mathrm{v} = -\mathrm{EI}\frac{\partial^3 \mathrm{v}}{\partial \mathrm{x}^3} + \mathrm{F}\,(\mathrm{e_0 a})^2 \frac{\partial^3 \mathrm{v}}{\partial \mathrm{x}^3} + \mathrm{m}\,(\mathrm{e_0 a})^2 \frac{\partial^3 \mathrm{v}}{\partial \mathrm{x}\,\partial \mathrm{t}^2} - (\mathrm{e_0 a})^2 \frac{\partial \mathrm{q}}{\partial \mathrm{x}} - \mathrm{F}\frac{\partial \mathrm{v}}{\partial \mathrm{x}} \tag{7}$$

Substituting (7) in (1) and for free flexural vibration put q(x) =0, we get

$$\mathrm{EI}\,\frac{\partial^4 \mathrm{v}}{\partial \mathrm{x}^4} + \mathrm{F}\,\frac{\partial^2}{\partial \mathrm{x}^2}\left[\mathrm{v} - (\mathrm{e_0 a})^2 \frac{\partial^2 \mathrm{v}}{\partial \mathrm{x}^2}\right] + \mathrm{m}\,\frac{\partial^2}{\partial \mathrm{t}^2}\left[\mathrm{v} - (\mathrm{e_0 a})^2 \frac{\partial^2 \mathrm{v}}{\partial \mathrm{x}^2}\right] = 0 \tag{8}$$

Let the nanobeam vibrate harmonically, so put v(x) $=\emptyset(x)\,\sin pt$, differentiating and substituting in (8) we can get the following solution,

$$\emptyset^{IV} + g^2\,\emptyset^{II} - a^4\,\emptyset = 0 \tag{9}$$

Where,

$$\mathrm{g}^2 = \left[\left(\frac{\mathrm{F}}{\mathrm{EI}} + \frac{\mathrm{m}}{\mathrm{EI}}(\mathrm{e_0 a})^2\,\mathrm{p}^2\right)/\left(1 - \frac{\mathrm{F}}{\mathrm{EI}}(\mathrm{e_0 a})^2\right)\right]$$

$$\mathrm{a}^4 = \left[\frac{\mathrm{m}}{\mathrm{EI}}\,\mathrm{p}^2/\left(1 - \frac{\mathrm{F}}{\mathrm{EI}}(\mathrm{e_0 a})^2\right)\right]$$

V. SOLUTIONS FOR VARIOUS SUPPORT CONDITIONS

5.1. Simply Supported End Conditions

The function assuming for this end condition should satisfy the boundary conditions,

$$\emptyset(\mathrm{x}) = \sin\frac{\mathrm{n\pi x}}{\mathrm{l}} \tag{10}$$

By differentiating (10) and substituting in (9) and by using Galerkin's method we can get the frequency equation as,

$$p^2 = \frac{\frac{n^4\pi^4}{l^4}\left(1 - \frac{F}{EI}(e_0 a)^2\right) - \frac{n^2\pi^2}{l^2}\frac{F}{EI}}{\left[\frac{m}{EI} + \frac{m}{EI}(e_0 a)^2 \frac{n^2\pi^2}{l^2}\right]} \tag{11}$$

5.2 Clamped Clamped End Conditions

The function satisfying the boundary condition of this end condition is,

$$\emptyset(\mathrm{x}) = 1 - \cos\frac{2\mathrm{n\pi x}}{\mathrm{l}} \tag{12}$$

By differentiating (12) and substituting in (9) and by using Galerkin's method we can get the frequency equation as,

$$p^2 = \frac{\left(\frac{2^4 n^4 \pi^4}{l^4}\right)\left(1 - \frac{F}{EI}(e_0 a)^2\right) - \left(\frac{2^2 n^2 \pi^2}{l^2}\right)\frac{F}{EI}}{\left(3\frac{m}{EI} + \frac{m}{EI}(e_0 a)^2\left(\frac{2^2 n^2 \pi^2}{l^2}\right)\right)} \tag{13}$$

IJARSE
ISSN 2319 - 8354

5.3 Clamped Hinged End Conditions

The function satisfying the boundary condition of this end condition is,

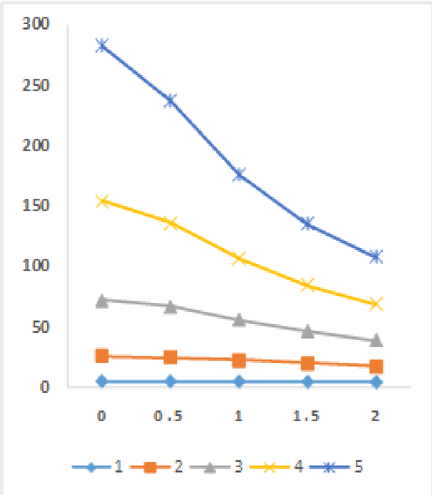
$$\emptyset(x) = \sin\frac{n\pi x}{l}\left(1 - \cos\frac{2n\pi x}{l}\right) \tag{14}$$

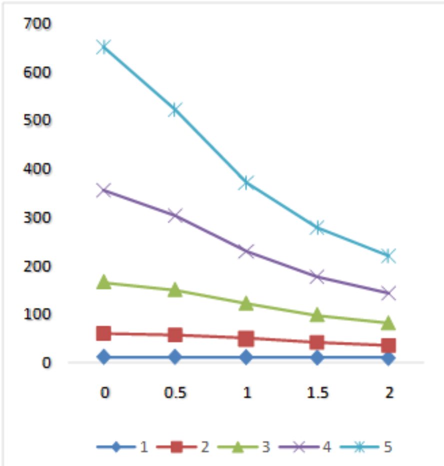
By differentiating (14) and substituting in (9) and by using Galerkin's method we can get the frequency equation as,

$$p^2 = \frac{45\left(\frac{n^4\pi^4}{l^4}\right)\left(1-\frac{F}{EI}(e_0 a)^2\right) - 9\left(\frac{n^2\pi^2}{l^2}\right)\left(\frac{F}{EI}\right)}{5\frac{m}{EI} + 9\frac{m}{EI}(e_0 a)^2\left(\frac{n^2\pi^2}{l^2}\right)} \tag{15}$$

VI. ANALYTICAL RESULTS

Undamped free flexural vibration analysis of nanobeams along with axial compression force with different nonlocal parameters and different axial compression forces has been done using Galerkin's method. The results are tabulated and graphically depicted for the various boundary conditions.





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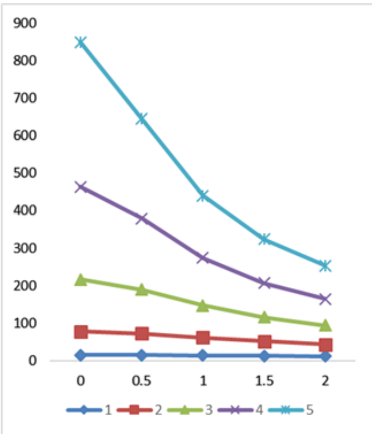

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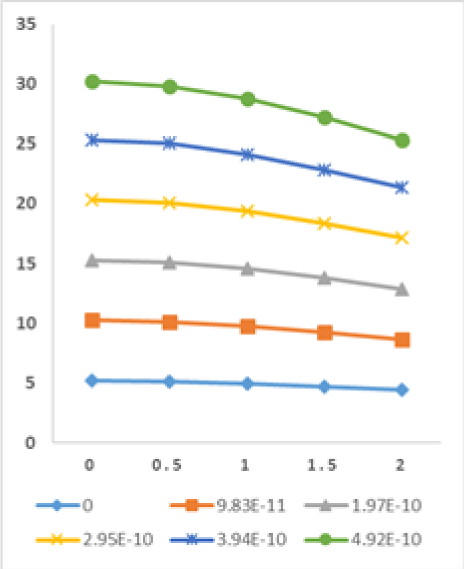
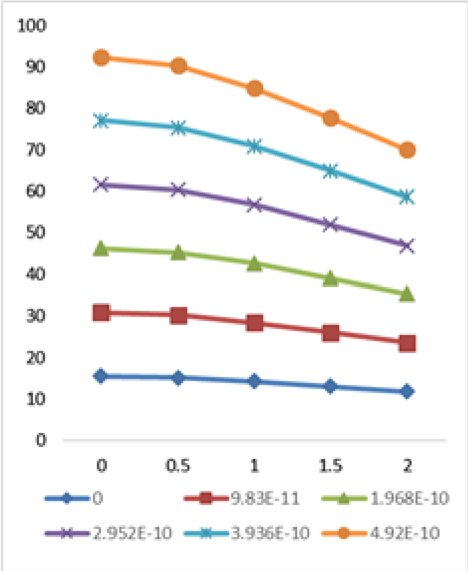
Fig 2: Effect of nonlocal parameter on the vibrational frequency for the increasing mode numbers for various end conditions





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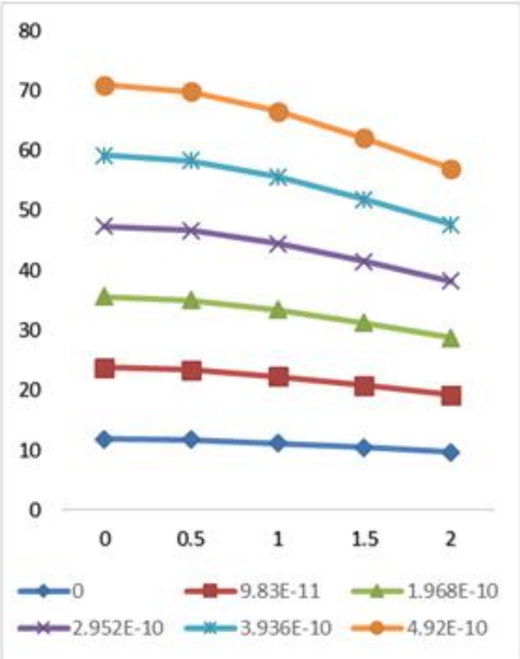

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Fig 3: Effect of nonlocal parameter on the first mode of vibration frequency for the increasing axial compression for various end condition

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IJARSE
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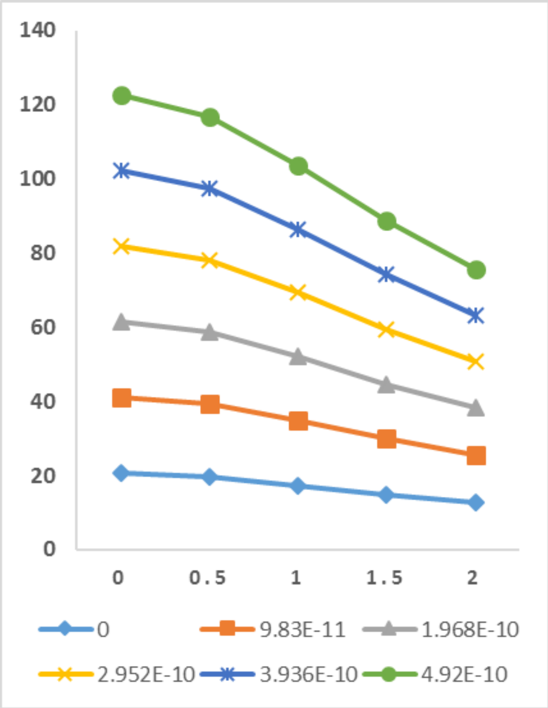

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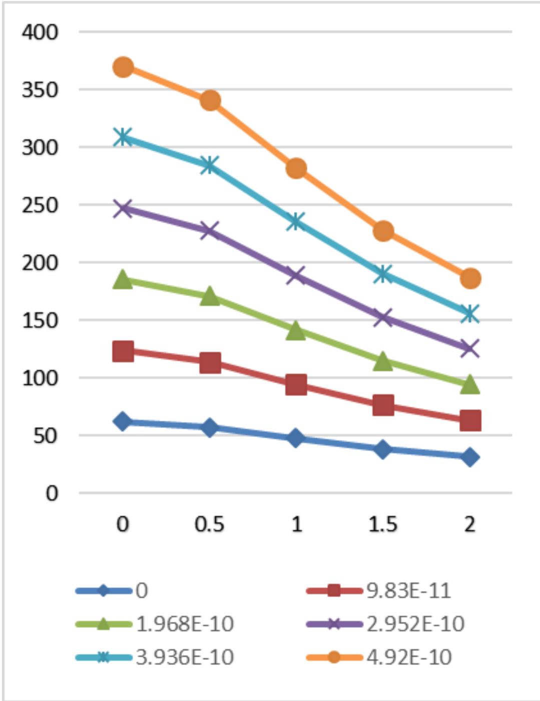

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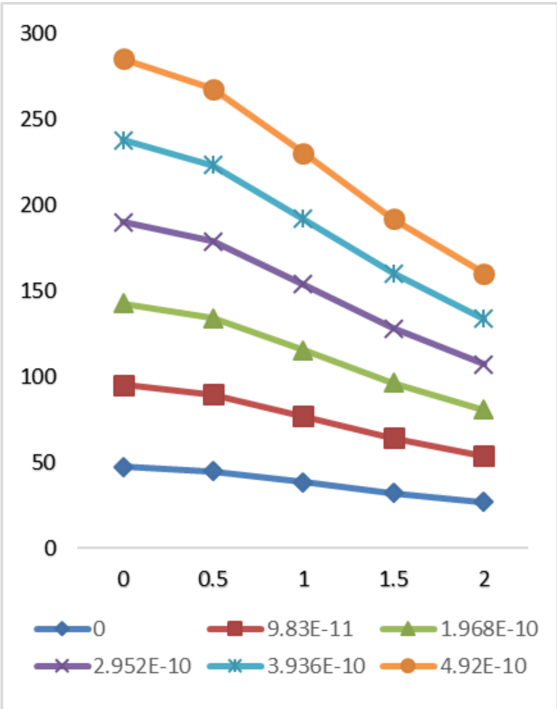

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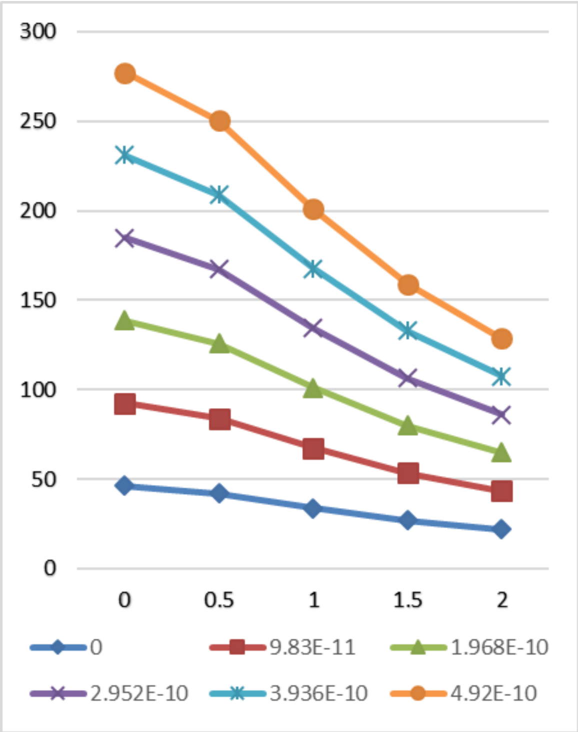
Fig 4: Effect of nonlocal parameter on the second mode of vibrational frequency for various axial compression forces for various end conditions



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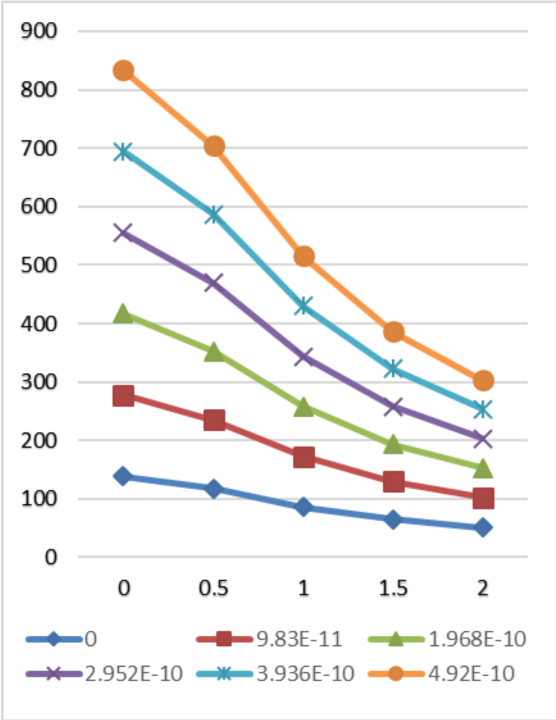

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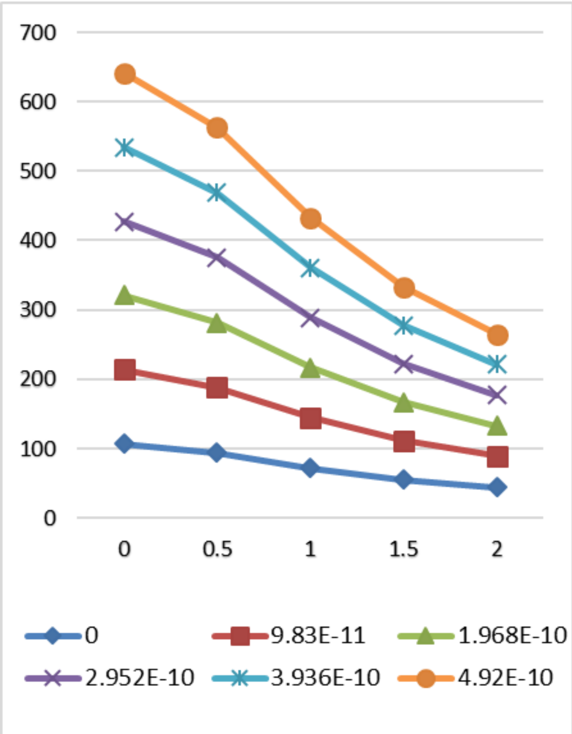

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Fig 5: Effect of nonlocal parameter on the third mode of vibrational frequency for various axial compression forces for various end conditions

VII. DISCUSSIONS

The influence of nonlocal nanoscale parameter, mode numbers and axial compression forces on the vibrational frequency for various boundary conditions is discussed. The variation of frequency for increasing mode numbers under the influence of nonlocal parameter e0a for three boundary conditions is shown in Fig. 2. According to the graph, it is seen that the increase in the nonlocal parameter e0a for various mode numbers such 1, 2,3,4,5 shows that there is a decrease in the frequency of nanobeam. The variation of frequency for various axial compression forces under the influence of nonlocal parameter for fundamental mode for three boundary conditions is shown in Fig 3. According to the graph it is seen that the increase in the axial compression forces decreases the frequency of nanobeam. Hence it is known that the increase in e0a decreases the frequency of nanobeam. The variation of frequency for various axial compression forces under the influence of nonlocal parameter for second and third mode for three boundary conditions is shown in Fig 4, 5. From the graph, it is shown that the increase in the e0a shows a gradual decrease in the frequency and also increase in the axial force decreases the frequency. Comparing to the frequency of first mode, the rate of decrease has increased for the second mode.

VIII. CONCLUSION

- The effect of nonlocal parameter for various mode numbers decreases the frequency of nanobeam.
- The frequency decreases for increasing axial compression force under the influence of nonlocal effect.
- The nonlocal parameter causes the frequency to decrease for the increasing axial compression force for both first, second and third mode.
- The increase in axial force degrades the stiffness and consequently leads to decrease in flexural frequency

IX. ACKNOWLEDGEMENT

The authors are thankful to the Management of Karunya University for providing necessary facilities to carry out the work reported in this paper.

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