

# MATHEMATICAL MODELING OF HOUSEHOLD WASTEWATER TREATMENT BY DUCKWEED BATCH REACTOR

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## ABSTRACT

Wastewater treatment is an essential criterion for the water quality management. The study emphasized on mathematical modeling of wastewater treatment using mixed population of different duckweed species (*Spirodela polyrhiza*, *lemna minor* and *wolffia arrhiza*) were adopted in order to investigate and later to develop kinetic models. The four reactors with different withdrawal ports were used to carry out batch experiments. The initial concentration and the final concentration of the wastewater were analysed for two sets of data, continuous set and discrete set, based on specified detention time of 10 days and 6 days respectively. The kinetic coefficients were obtained by the least squares method. The first order kinetic coefficients of the models were modified by the inverse technique in order to account for the uncertainties in the discrete experiments in the validation process.

**Keywords:** Duckweeds, Kinetic Coefficients, Mathematical Modeling, Reactors, Wastewater

## I. INTRODUCTION

The main concern in water quality management is to provide suitable treatment which reduces the concentration of the pollutant present in the wastewater by employing physical, chemical and biological treatment to the wastewater [1]. Now a day's researchers are trying to find alternative techniques to convert and to treat the wastewater for secondary purpose. Different kinds of microorganisms and aquatic plants are utilized in biological treatment of wastewater. The research work was mainly concentrated on floating aquatic macrophytes in wastewater treatment due to their multiple mechanisms of pollutant removal and economical cost of treatment.

Many authors have conducted the research on utilization of the macrophytes with the development of the various models the biochemical phenomenon carried out experiments in two mini ponds [2]. previous researchers developed a mathematical model to predict for pulp and paper industry and distillery wastewater by using water hyacinth which fair accurate model was observed [3],[4] on comparison with duckweeds which



has more advantage than water hyacinth have been used as a phytoremediators in the wastewater treatment [4][5]. Thus there is a need for the research for the household wastewater treatment by the utilization of aquatic macrophytes. Thus the main objective was to develop the mathematical model of phytoremediation potential of the duckweed for household wastewater treatment.

## II. MATHEMATICAL MODEL

Mathematical modeling of first order kinetics for duckweed batch reactor represents the natural systems is represented by the set of the mathematical equations.

Let C be the phytoremediation of the duckweed plants at time t=0 from the initial day of the experiment. The rate of change c with respect to t from starting time upto the time when plants attain the equilibrium is directly proportional to C

The mathematical equation is represented by

$$\frac{dc}{dt} \propto C$$

$$\frac{dc}{dt} = \mu C \quad \text{-----}$$

-(1)

Where  $\mu$  is proportionality constant = k first order kinetic constant

The above equation represents the first order linear differential equation. The solution to the above equation is given by method of separation of variables integrating with setting up the limits from time t=0 starting of the time of the day, c will be maximum thus

$$\int_{c_0}^c \frac{dc}{c} = \mu \int_0^T dt$$

$$\ln(c) - \ln(c_0) = \mu T - 0$$

$$\ln(c/c_0) = \mu T$$

$$\text{or } C = C_0 * \exp(\pm \mu T) \quad \text{-----}$$

(2)

where C= Concentration of the pollutant at time = T in days

## III EXPERIMENTAL SET UP

The experimental setup includes the plastic cans of cylindrical type of 20L capacity were selected as a reactor. The four cans were provided with the ports made at the different depths which were designated as reactor number –R1 R2, R3,R4. The duckweeds were collected from the kukkarahalli lake region (of mysore) in one set of experiment Spirodela polyrhiza and lemna was collected. Wolffia arrhiza was found upon another time of collection from the lake. After collection, the duckweeds were primarily washed and separated the unwanted leaves and insects present in it. later it was cultured in the laboratory under the presence of normal sunlight for its adaptation to new environmental conditions the wastewater were collected from the inlet of the Activated sludge process further it was diluted in the ratio of 2:1 and is filled into the reactor the duckweed plants were transferred into the sieve net which were weighed initially and further placed on the surface of the duckweed

reactor that were placed in the laboratory near to the window region which are essential for the growth of duckweeds in wastewater since the entire treatment is of a batch process, suitable detention time and collection of the samples at particular intervals were adopted. The experimental run were of two types discrete run and continuous run set .

### 3.1 Sample Collection

In the discrete run of experiments, the samples were collected from the reactor after specified treatment period. The reactors were stirred intermittently once in 48 hours in order to ensure homogenous mixing condition. Each set of samples were collected on individual basis 1, 2, 3, 4, 5, 6 days of experimental run further it was subjected to analysis. In continuous run of the experiments the samples were collected about every 48 hr interval upon with the stirring action the detention time ranged from 2 to 10 days with every 2 day interval.

### 3.2 Data Collection

The experimental observation, graphical representation and the validation of the models with respect to the various Physico-chemical parameters of the data were generated. The data that were collected from both discrete and continuous experiment are orderly arranged which include the physicochemical parameters such as pH, BOD, COD, NH<sub>3</sub>, Nitrate nitrogen, phosphate, TDS.

The data obtained from the continuous experiment were used as reference in the mathematical modeling to estimate the kinetic coefficients. The validation is completed for the discrete run.

## IV. RESULTS AND DISCUSSION

The continuous and discrete set of data were collected at depths 0.3m, 0.25m, 0.16m and 0.08m measured respectively with respect to the bottom of the reactors from reactor R1, R2, R3 and R4. The first order models are commonly used to predict water quality decay or constituents along with associated factors. The first order kinetic constant were determined by least squares method to collect the data thus the value of the  $\mu$  in the equation were determined by least squares method.

$$X = \ln(C/C_0) = -kt \text{ and } Y = t \tag{3}$$

Thus equation (3) was used in the result analysis and interpretation of the model.

**Table 1 Average rate of the kinetic coefficients for different physico – chemical parameters**

Reactor NO	Depth (m)	TDS	BOD	COD	NO <sub>3</sub>	NH <sub>3</sub>	phosphate
R1	0.305	-0.005	-0.2630	-0.271	-0.163	-0.153	-0.235
R2	0.25	-0.02	-0.2170	-0.235	-0.183	-0.154	-0.221
R3	0.162	-0.02	-0.1900	-0.199	-0.145	-0.105	-0.206
R4	0.08	-0.014	-0.1190	-0.193	-0.147	-0.113	-0.186
	Kinetic coefficients(/d)	<b>-0.013</b>	<b>-0.1907</b>	<b>-0.221</b>	<b>-0.15167</b>	<b>-0.1237</b>	<b>-0.209</b>

By observing the values of the kinetic coefficients in table 1 the trend of the physico –chemical parameters are enumerated based upon the observation and the deviation of the data values the modification of the kinetic values using equation (3) were put in order to maintain minimization of uncertainties. Further it has been modified by the use of the inverse technique with TDS as 0.018/d , 0.48/d for BOD ,0.495 for COD, 0.34/d for phosphate, 0.3/d for nitrate nitrogen, 0.16/d which were used for the validation of the model was carried out with discrete data, such that the observed points would be coming closure to the mathematical model bringing down the error to minimum extent.

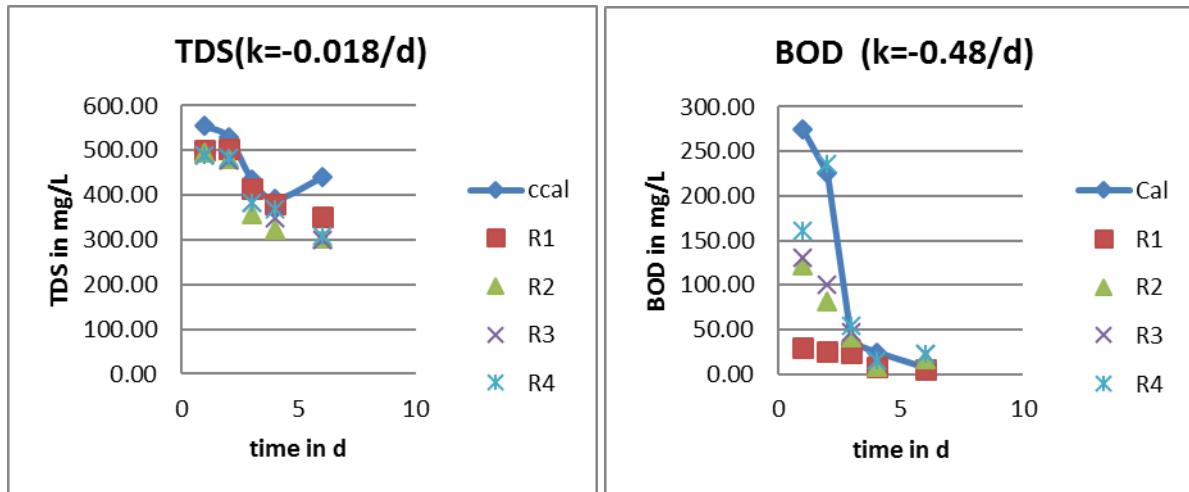


Figure1. Experimental and observed values for (a)TDS and (b) BOD in mg/L vs time .

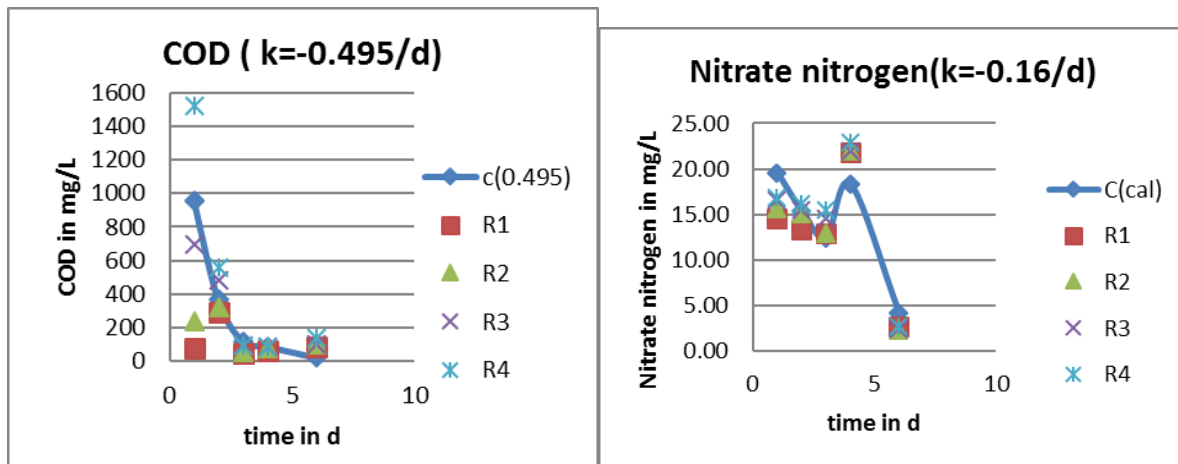
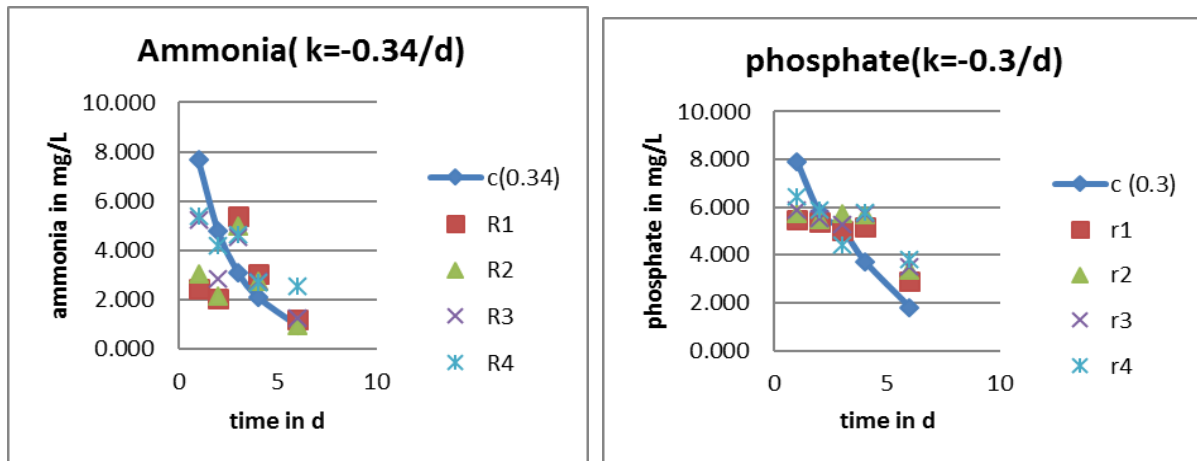


Figure2. Experimental and observed values for (a) COD and (b) Nitrate nitrogen in mg/L vs time.



**Figure 3. Experimental and observed values for (a) ammonia and (b) phosphate in mg/L vs time.**

The exponential curve fit was plotted to all the data set in order to check for the data deviation. The TDS values were almost following the normal trend. Fig 1b the BOD graph showed that as increase in time the correlation is approaching to a higher degree. This could be due to the acclimatization of the microbes under the above uncertainties. With reference to Fig 2b The reasons are COD removal were suddenly decreased for 1 and 2 day time period and while the 3, 4 and 6 days of detention time which showed the lesser rate of COD removal which indicates the presence of slowly degradable organics or non biodegradable compounds with respected to the variation in the input of the wastewater over their respective period of time which represents the points near to the standard line. The variation of the nitrate nitrogen curve in Fig 2b on the fourth day time interval was high due to the presence of high initial concentration .in the input to the reactor compared to the other values. From fig 3a we can infer that though the input concentration is not varied much with discrete and continuous but the variation in the curve were due to uncertainties hence  $K=0.34/d$  was adopted for the validation of the model for ammonia. From fig 3b, the average  $k$  model is tend to be very efficient however  $k=-0.3/d$  is found for phosphate by the application of inverse technique [6]. Along with the kinetic model the phytoremediation potential were also with the acute agreement.

## V. CONCLUSION

The behavioral trend observed to be same in both the experimental analysis with respect to physico-chemical parameters. Most of the parameters were observed to be removed at the faster rate near the surface compared to the bottom of the reactors. The rate of reaction were found to increase with respect to time in all the parameters. More or less overall discrete experiment trend has followed the mathematical model. First order kinetic values were modified through inverse technique in order to tune the model parameter due to existence of uncertainties. The modified values of  $k(-)$  are as follows  $0.0146/d$  for TDS,  $0.48/d$  for BOD,  $0.495/d$  for COD,  $0.16/d$  for nitrate nitrogen,  $0.34/d$  for ammonia nitrogen,  $0.3/d$  for phosphate. Kinetic model validated with a fair degree of correlation to discrete data. Model can be used to predict the behavior within the experimental range. Further the model can be extended up for other physicochemical and biological parameters

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