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QUASI γ-NORMAL SPACES IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce the concept of $\pi g\gamma$-closed sets as weak form of πg-closed sets. We also introduce the notion of quasi γ-normal spaces and by using $\pi g\gamma$-closed sets, we obtain a characterization and preservation theorems for quasi γ-normal spaces. Further we show that this property is a topological property and it is a hereditary property only with respect to closed domain subspaces. The relationships among normal, π-normal, quasi-normal, mildly-normal, p-normal, πp-normal, quasi p-normal, mildly p-normal, γ-normal, $\pi\gamma$-normal, quasi γ-normal, mildly γ-normal are investigated.

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Keywords: ***π-open, γ-open, and $\pi g\gamma$-closed sets; $\pi g\gamma$-closed, almost $\pi g\gamma$-closed, $\pi g\gamma$-continuous and almost $\pi g\gamma$-continuous functions; quasi γ-normal spaces.***

I INTRODUCTION

The notion of quasi normal space was introduced by Zaitsev [**14**]. Levine [**6**] initiated the investigation of g-closed sets in topological spaces. Singal and Singal [**11**] introduced the notion of mildly normal spaces which are weaker than quasi-normal spaces. Nour [**9**] introduced the notion of p-normal spaces and obtained their properties. Lal and Rahman [**5**] have further studied notions of quasi normal and mildly normal spaces. Dontchev and Noiri [**1**] introduced the notion of πg-closed sets as a weak form of g-closed sets due to Levine [**6**]. By using πg-closed sets, Dontchev and Noiri [**1**] obtained a new characterization of quasi normal spaces. Kalantan [**4**] introduced a weaker version of normality called π-normality and proved that π-normality is a property which lies between normality and almost normality. Ekici [**2**] introduced a new class of normal spaces, called γ-normal spaces and the relationships among s-normal, p-normal and γ-normal spaces are investigated. Thabit and Kamarulhaili [**13**] introduced a weaker version of p-normality called πp-normality, which lies between p-normality and almost p-normality. Recenty, Thabit and Kamarulhaili [**12**] introduced a weaker form of p-normality ca1led quasi p-normality, which lies between πp-normality and mild p-normality.

II PRELIMINARIES

Throughout this paper, spaces (X, τ), (Y, σ), and (Z, γ) (or simply X, Y and Z) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. The closure of A and interior of A are denoted by cl(A) and int(A) respectively. A subset A is said to be **regular open** (resp. **regular closed**) if A = int(cl(A)) (resp. A = cl(int(A)). The finite union of regular open sets is said to be **π-open**. The complement of a π-open set is said to be **π-closed.** A subset A is said to **γ-open [3]** if $A \subset cl(int(A)) \cup int(cl(A))$. The complement of a γ-open set is said to be **γ-closed** [**3**]. The intersection of all γ-closed sets containing A is called **γ-closure** [**3**] of A, and is denoted by γcl(A). The **γ-interior** [**3**] of A, denoted by γint(A), is defined as union of all γ-open sets contained in A.

2.1 Definition. A subset A of a space X is said to be

(1) generalized closed (briefly **g-closed**) **[6]** if $cl(A) \subset U$ whenever $A \subset U$ and $U \in \tau$.

(2) πg-closed [**1**] if $cl(A) \subset U$ whenever $A \subset U$ and U is π–open in X.

(3) generalized γ-closed [**2**] if $\gamma cl(A) \subset U$ whenever $A \subset U$ and $U \in \tau$.

(4) πgγ-closed if $\gamma cl(A) \subset U$ whenever $A \subset U$ and U is π–open in X.

(5) g-open (resp**. πg-open**, **gγ-open**, **πgγ-open**) if the complement of A is g-closed (resp. πg-closed, gγ-closed, πgγ–closed).

III QUASI γ -NORMAL SPACES

3.1 Definition. A space X is said to be γ**-normal** [**2**] (resp. p-normal [**9, 10]**) if for every pair of disjoint closed subsets A, B of X, there exist disjoint γ-open (resp. p-open) sets U, V of X such that $A \subset U$ and $B \subset V$.

3.2 Definition. A space X is said to be **$\pi\gamma$–normal** (resp. **π-normal** [**4**], **πp-normal** [**13**]) if for every pair of disjoint closed subsets A, B of X, one of which is π-closed, there exist disjoint γ-open (resp. open, p-open) sets U, V of X such that $A \subset U$ and $B \subset V$.

3.3 Definition. A space X is said to be **quasi γ-normal** (resp. **quasi-normal** [**14**], **quasi p-normal [12]**) if for every pair of disjoint π-closed subsets H, K of X, there exist disjoint γ-open (resp. open, p-open) sets U, V of X such that $H \subset U$ and $K \subset V$.

3.4 Definition. A space X is said to be **mildly γ-normal** (resp. **mildly-normal** [**11**], **mildly p-normal [7]**) if for every pair of disjoint regular closed subsets H, K of X, there exist disjoint γ-open (resp. open, p-open) sets U, V of X such that $H \subset U$ and $K \subset V$.

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By the definitions stated above, we have the following diagram:

$$\begin{array}{ccccccc}
\text{normality} & \Rightarrow & \pi\text{-normalily} & \Rightarrow & \text{quasi-normality} & \Rightarrow & \text{mild-normality} \\
\Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\
\text{p-normality} & \Rightarrow & \pi\text{p-normalily} & \Rightarrow & \text{quasi p-normality} & \Rightarrow & \text{mild p-normality} \\
\Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\
\gamma\text{-normality} & \Rightarrow & \pi\gamma\text{-normalily} & \Rightarrow & \text{quasi } \gamma\text{-normality} & \Rightarrow & \text{mild } \gamma\text{-normality}
\end{array}$$

Where none of the implications is reversible as can be seen from the following examples:

3.5 Example. Let $X = \{a, b, c\}$ and $\tau = \{\varnothing, \{a\}, \{a, b\}, \{a, c\}, X\}$. Then the space X is πp-normal as well as $\pi\gamma$-normal but not p-normal.

3.6 Example. Let $X = \{a, b, c\}$ and $\tau = \{\varnothing, \{a\}, \{b\}, \{a, b\}, X\}$. Then the space X is p-normal as well as πp-normal.

3.7 Example. Let $X = \{a, b, c, d\}$ and $\tau = \{\varnothing, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$. Then the space X is γ-normal as well as $\pi\gamma$-normal but not p-normal.

3.8 Example. Let $X = \{a, b, c, d\}$ and $\tau = \{\varnothing, \{b, d\}, \{a, b, d\}, \{b, c, d\}, X\}$. Then the space X is γ-normal but not normal.

3.9 Example. Let $X = \{a, b, c, d\}$ and $\tau = \{\varnothing, \{a\}, \{c\}, \{a, c\}, \{a, b, d\}, \{b, c, d\}, X\}$. The pair of disjoint closed subsets of X are $A = \{a\}$ and $B = \{c\}$. Also $U = \{a, b\}$ and $V = \{c, d\}$ are p-open sets such that $A \subset U$ and $B \subset V$. Hence the space X is p-normal as well as γ-normal but not normal.

3.10. Example. Let $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, \{e\}, \{a, b\}, \{c, d\}, \{a, b, e\}, \{c, d, e\}, \{a, b, c, d\}, X\}$. The pair of disjoint π-closed subsets of X are $A = \{a, b\}$ and $B = \{c, d\}$. Also $U = \{a, b, e\}$ and $V = \{c, d\}$ are γ-open sets such that $A \subset U$ and $B \subset V$. Hence the space X is quasi γ-normal but not quasi normal, since U and V are not open sets.

3.11. Example. Let $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c, d, e\}, \{b, c, d, e\}, X\}$. The pair of disjoint π-closed subsets of X are $A = \{a\}$ and $B = \{b\}$. Also $U = \{a, c\}$ and $V = \{b, d\}$ are p-open sets such that $A \subset U$ and $B \subset V$. Hence the space X is quasi p-normal as well as quasi γ-normal but not quasi normal, since U and V are not open sets.

3.12 Theorem. For a space X, the following are equivalent:

(a) X is quasi γ-normal.

(b) For every pair of π-open subsets U and V of X whose union is X, there exist γ-closed subsets G and H of X such that $G \subset U$, $H \subset V$ and $G \cup H = X$.

(c) For any π-closed set A and every π-open set B in X such that $A \subset B$, there exists a γ-open subset U of X such that $A \subset U \subset \gamma cl(U) \subset B$.

(d) For every pair of disjoint π-closed subsets A and B of X, there exist γ-open subsets U and V of X such that $A \subset U$, $B \subset V$ and $\gamma cl(U) \cap \gamma cl(V) = \varnothing$.

Proof. (a) $\Rightarrow$ (b), (b) $\Rightarrow$ (c), (c) $\Rightarrow$ (d) and (d) $\Rightarrow$ (a).

(a)$\Rightarrow$ (b). Let U and V be any π-open subsets of a quasi γ-normal space X such that $U \cup V = X$. Then, $X - U$ and $X - V$ are disjoint π-closed subsets of X. By quasi γ-normality of X, there exist disjoint γ-open subsets U_1 and V_1 of X such that $X - U \subset U_1$ and $X - V \subset V_1$. Let $G = X - U_1$ and $H = X - V_1$. Then, G and H are γ-closed subsets of X such that $G \subset U$, $H \subset V$ and $G \cup H = X$.

(b) $\Rightarrow$ (c). Let A be a π-closed and B is a π-open subsets of X such that $A \subset B$. Then, $X - A$ and B are π-open subsets of X such that $(X - A) \cup B = X$. Then, by part (b), there exist γ-closed sets G and H of X such that $G \subset (X - A)$, $H \subset B$ and $G \cup H = X$. Then, $A \subset (X - G)$, $(X - B) \subset (X - H)$ and $(X - G) \cap (X - H) = \varnothing$. Let $U = X - G$ and $V = (X - H)$. Then U and V are disjoint γ-open sets such that $A \subset U \subset X - V \subset B$. Since $X - V$ is γ-closed, then we have $\gamma cl(U) \subset (X - V)$. Thus, $A \subset U \subset \gamma cl(U) \subset B$.

(c) $\Rightarrow$ (d). Let A and B be any disjoint π-closed subset of X. Then $A \subset X - B$, where $X - B$ is π-open. By the part (c), there exists a γ-open subset U of X such that $A \subset U \subset \gamma cl(U) \subset X - B$. Let $V = X - \gamma cl(U)$. Then, V is a γ-open subset of X. Thus, we obtain $A \subset U$, $B \subset V$ and $\gamma cl(U) \cap \gamma cl(V) = \varnothing$.

(d) $\Rightarrow$ (a). It is obvious.

3.13 Proposition. Let $f : X \to Y$ be a function, then:

(a) The image of γ-open subset under an open continuous function is γ-open.

(b) The inverse image of γ-open (resp. γ-closed) subset under an open continuous function is γ-open (resp. γ-closed) subset.

(c) The image of γ-closed subset under an open and a closed continuous surjective function is γ-open.

3.14 Theorem. The image of a quasi γ-normal space under an open continuous injective function is a quasi γ-normal.

Proof. Let X be a quasi γ-normal space and let $f : X \to Y$ be an open continuous injective function. We need to show that f(X) is a quasi γ-normal. Let A and B be any two disjoint π-closed sets in f(X). Since the inverse image of a π-closed set under an open continuous function is a π-closed. Then, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint π-closed sets in X. By quasi γ-normality of X, there exist γ-open subsets U and V of X such that $f^{-1}(A) \subset U$, $f^{-1}(B) \subset V$ and $U \cap V = \varnothing$. Since f is an open continuous injective

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function, we have $A \subset f(U)$, $B \subset f(V)$ and $f(U) \cap f(V) = \varnothing$. By **Proposition 3.13**, we obtain f(U) and f(V) are disjoint γ-open sets in f(X) such that $A \subset f(U)$ and $B \subset f(V)$. Hence f(X) is quasi γ-normal.
From the above theorem, we have the following corollary.

3.15 Corollary. Quasi γ-normality is a topological property.

The following lemma helps us to show that quasi γ-normality is a hereditary with respect to closed domain subspaces.

3.16 Lemma. Let M be a closed domain subspace of a space X. If A is a γ-open set in X, then $A \cap M$ is γ-open set in M.

3.17 Theorem. Quasi γ-normality is a hereditary with respect to closed domain subspaces.
Proof. Let M be a closed domain subspace of a quasi γ-normal space X. Let A and B be any disjoint π-closed sets in M. Since M is a closed domain subspace of X, then we have A and B be any disjoint π-closed sets of X. By quasi γ-normality of X, there exist disjoint γ-open subsets U and V of X such that $A \subset U$ and $B \subset V$. By the **Lemma 3.16,** we obtain $U \cap M$ and $V \cap M$ are disjoint γ-open sets in M such that $A \subset U \cap M$ and $B \subset V \cap M$. Hence, M is quasi γ-normal subspace.

IV PRESERVATION THEOREMS

The following result is useful for giving some other characterizations of quasi γ-normal spaces.
4.1 Lemma. A subset A of a space X is $\pi g\gamma$-open if and only if $F \subset \gamma int(A)$ whenever $F \subset A$ and F is π-closed.
4.2 Theorem. For a space X, the following are equivalent:
(a) X is quasi γ-normal.
(b) For any disjoint π-closed sets H and K, there exist disjoint $g\gamma$-open sets U and V such that $H \subset U$ and $K \subset V$
(c) For any disjoint π-closed sets H and K, there exist disjoint $\pi g\gamma$-open sets U and V such that $H \subset U$ and $K \subset V$.
(d) For any π-closed set H and any π-open set V containing H, there exists a $g\gamma$-open set U of X such that $H \subset U \subset \gamma cl(U) \subset V$.
(e) For any π-closed set H and any π-open set V containing H, there exists a $\pi g\gamma$-open set U of X such that $H \subset U \subset \gamma cl(U) \subset V$.
Proof. (a) $\Rightarrow$ (b), (b) $\Rightarrow$ (c), (c) $\Rightarrow$ (d), (d) $\Rightarrow$ (e) and (e) $\Rightarrow$ (a).

(a) $\Rightarrow$ (b). Let X be quasi γ-normal space. Let H, K be disjoint π-closed sets of X. By assumption, there exist disjoint γ-open sets U, V such that $H \subset U$ and $K \subset V$. Since every γ-open set is gγ-open, so U and V are gγ-open sets such that $H \subset U$ and $K \subset V$.

(b) $\Rightarrow$ (c). Let H, K be two disjoint π-closed sets. By assumption, there exist disjoint gγ-open sets U and V such that $H \subset U$ and $K \subset V$. Since gγ-open set is πgγ-open, so U and V are πgγ-open sets such that $H \subset U$ and $K \subset V$.

(c) $\Rightarrow$ (d). Let H be any π-closed set and V be any π-open set containing H. By assumption, there exist disjoint πgγ-open sets U and W such that $H \subset U$ and $X - V \subset W$. By **Lemma 4.1**, we get $X - V \subset \gamma int(W)$ and $\gamma cl(U) \cap \gamma int(W) = \varnothing$. Hence $H \subset U \subset \gamma cl(U) \subset X - \gamma int(W) \subset V$.

(d) $\Rightarrow$ (e). Let H be any π-closed set and V be any π-open set containing H. By assumption, there exist gγ-open set U of X such that $H \subset U \subset \gamma cl(U) \subset V$. Since, every g$\gamma$-open set is πgγ-open, there exists πgγ-open sets U of X such that $H \subset U \subset \gamma cl(U) \subset V$.

(e) $\Rightarrow$ (a). Let H, K be any two disjoint π-closed sets of X. Then $H \subset X - K$ and $X - K$ is π-open. By assumption, there exists πgγ-open set G of X such that $H \subset G \subset \gamma cl(G) \subset X - K$. Put $U = \gamma int(G)$, $V = X - \gamma cl(G)$. Then U and V are disjoint γ-open sets of X such that $H \subset U$ and $K \subset V$.

4.3 Definition. A function $f : X \to Y$ is said to be

(a) **γ-closed** [**2**] (resp. **πgγ-closed**) if f(F) is γ-closed (resp. πgγ-closed) in Y for every closed set F of X.

(b) **rc-preserving** [**8**] (resp. **almost closed** [**11**], **almost γ-closed**, **almost πgγ-closed**) if f(F) is regular closed (resp. closed, γ-closed, πgγ-closed) in Y for every $F \in RC(X)$.

(c) **π-continuous** [**1**] (resp. **almost π-continuous** [**1**]) if $f^{-1}(F)$ is π-closed in X for every closed (resp. regular closed) set F of Y.

(d) **almost continuous** [**11**] if $f^{-1}(V)$ is open in X for every regular open set V of Y.

(e) **πgγ-continuous** (resp. **almost πgγ-continuous**) if $f^{-1}(F)$ is πgγ-closed in X for every closed (resp. regular closed) set F of Y.

4.4 Theorem. If $f : X \to Y$ is an almost π-continuous and πgγ-closed function, then f(A) is πgγ-closed in Y for every πgγ-closed set A of X.

Proof. Let A be any πgγ-closed set of X and V be any π-open set of Y containing f(A). Since f is almost π-continuous, $f^{-1}(V)$ is π-open in X and $A \subset f^{-1}(V)$. Therefore, we have $\gamma cl(A) \subset f^{-1}(V)$ and hence $f(\gamma cl(A)) \subset V$. Since f is πgγ-closed, $f(\gamma cl(A))$ is πgγ-closed in Y and hence we obtain $\gamma cl(f(A)) \subset \gamma cl(f(\gamma cl(A))) \subset V$. Hence f(A) is πgγ-closed in Y.

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4.5 Theorem. A surjection $f : X \rightarrow Y$ is almost $\pi g\gamma$-closed if and only if for each subset S of Y and each $U \in RO(X)$ containing $f^{-1}(S)$, there exists a $\pi g\gamma$-open set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.

Proof. Necessity. Suppose that f is almost $\pi g\gamma$-closed. Let S be a subset of Y and $U \in RO(X)$ containing $f^{-1}(S)$. If $V = Y - f(X - U)$, then V is a $\pi g\gamma$-open set of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.

Sufficiency. Let F be any regular closed set of X. Then $f^{-1}(Y - f(F)) \subset (X - F)$ and $(X-F) \in RO(X)$. There exists a $\pi g\gamma$-open set V of Y such that $Y - f(F) \subset V$ and $f^{-1}(V) \subset (X - F)$. Therefore, we have $f(F) \supset (Y - V)$ and $F \subset X - f^{-1}(V) \subset f^{-1}(Y - V)$. Hence we obtain $f(F) = Y - V$ and f(F) is $\pi g\gamma$-closed in Y, which shows that f is almost $\pi g\gamma$-closed.

4.6 Theorem. If $f : X \rightarrow Y$ is an almost $\pi g\gamma$-continuous, rc-preserving injection and Y is quasi γ-normal then X is quasi γ-normal.

Proof. Let A and B be any disjoint π-closed sets of X. Since f is an rc-preserving injection, f(A) and f(B) are disjoint π-closed sets of Y. Since Y is quasi γ-normal, there exist disjoint γ-open sets U and V of Y such that $f(A) \subset U$ and $f(B) \subset V$.

Now if $G = int(cl(U))$ and $H = int(cl(V))$. Then G and H are regular open sets such that $f(A) \subset G$ and $f(B) \subset H$. Since f is almost $\pi g\gamma$-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint $\pi g\gamma$-open sets containing A and B which shows that X is quasi γ-normal.

4.7 Theorem. If $f : X \rightarrow Y$ is π-continuous, almost γ-closed surjection and X is quasi γ-normal space then Y is γ-normal .

Proof. Let A and B be any two disjoint closed sets of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint π-closed sets of X. Since X is quasi γ-normal, there exist disjoint γ-open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$.

Let $G = int(cl(U))$ and $H = int(cl(V))$. Then G and H are disjoint regular open sets of X such that $f^{-1}(A) \subset G$ and $f^{-1}(B) \subset H$. Now, we set $K = Y - f(X-G)$ and $L = Y- f(X-H)$. Then K and L are γ-open sets of Y such that $A \subset K$, $B \subset L$, $f^{-1}(K) \subset G$ and $f^{-1}(L) \subset H$. Since G and H are disjoint, K and L are disjoint. Since K and L are γ-open and we obtain $A \subset \gamma int(K)$, $B \subset \gamma int(L)$ and $\gamma int(K) \cap \gamma int(L) = \varnothing$. Therefore, Y is γ-normal.

4.8 Theorem. Let $f : X \rightarrow Y$ be an almost π-continuous and almost $\pi g\gamma$-closed surjection. If X is quasi γ-normal space then Y is quasi γ-normal.

Proof. Let A and B be any disjoint π-closed sets of Y. Since f is almost π-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint π-closed sets of X. Since X is quasi γ-normal, there exist disjoint γ-open sets U and V of X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$.

Put $G = int(cl(U))$ and $H = int(cl(V))$. Then G and H are disjoint regular open sets of X such that $f^{-1}(A) \subset G$ and $f^{-1}(B) \subset H$. By **Theorem 4.5,** there exist gγ-open sets K and L of Y such that $A \subset K$, $B \subset L$, $f^{-1}(K) \subset G$ and $f^{-1}(L) \subset H$. Since G and H are disjoint, so are K and L by **Lemma 4.1,** $A \subset \gamma int(K)$, $B \subset \gamma int(L)$ and $\gamma int(K) \cap \gamma Int(L) = \emptyset$. Therefore, Y is quasi γ-normal.

4.9 Corollary. If $f : X \to Y$ is an almost continuous and almost closed surjection and X is a normal space, then Y is quasi γ-normal.

Proof. Since every almost closed function is almost πgγ-closed by **Theorem 4.8,** Y is quasi γ-normal.

V CONCLUSION

We introduced a weaker version of γ-normality called quasi γ-normality. We proved that it is a topological property and a hereditary property with respect to closed domain subspaces. We gave some characterizations and preservation theorems of quasi γ-normal spaces. Some counterexamples were given and some basic properties were presented. The relationships among normal, π-normal, quasi-normal, mild-normality, p-normal, πp-normal, quasi p-normal, mild p-normal, γ-normal, πγ-normal, quasi γ-normal, mild γ-normal are investigated.

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