



ULTRA WIDE BAND ENERGY DETECTION USING COMPRESSED SENSING BASED NBI DETECTION

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ABSTRACT

Now a day, ultra-wideband (UWB) energy detectors are developed enormously with compressed sensing (CS) theory in environment of multipath fading. Wideband communication is sensitive to narrowband interference (NBI), so it is necessary for efficient UWB energy detector to mitigate NBI-affected measurements without harming samples containing important information. According to the traditional sampling theorem, UWB requires huge bandwidth for short range communication with little utilization. To avoid this wastage of frequency band, CS process uses sub-Nyquist rate and provides compressed version of received signal. In this paper, reconstruction-based energy detector is presented which is robust to NBI. In this article, notch out method is employed at the detector for the mitigation of NBI-affected measurements. Energy detection of the UWB detector before adding NBI and after mitigating NBI is compared. Experimental results show that the presented energy detector is robust to NBI due to superior performance of the notch out method.

Keywords: *Compressed sensing, Energy detector, Narrowband interference, ultra-wideband Communication field*

I. INTRODUCTION

Digital communication is evident of growth in applications which consists signals of very high bandwidth. To receive these signals, reducing the sampling rate is the challenge. Compressive sampling gives reduced and efficient sampling than the traditional sampling rate. The ultra-wideband (UWB), impulse-radio (IR) signals are most attractive because of their unique properties such as fine time resolution, high user capacity as well as low probability of interception and detection [1], [2]. But, the big problems in employment of IR-UWB is power consumption in sensitivity of wide band signals towards narrow band interference (NBI) and analog to digital converter (ADC).

According to the FCC, UWB signals are defined as signals having a fractional bandwidth greater than 20% or signals having an absolute bandwidth greater than 0.5 GHz [3]. There are two techniques to generate a UWB signal. One is carrier-less and other is carrier based. Earlier technique uses spreading schemes like frequency hopping or direct sequence which makes architecture complex due to presence of mixer and other circuitry. The



next technique is also known as impulse radio (IR) this technique uses transmission of short pulses in time domain which occupies the complete frequency band. Its transceiver is simpler than the earlier technique of UWB signal generation. Also, the transmit power in IR-UWB can be decreased by transmitting same information over multiple frames, with each frame transmitting at a very low power.

According to Shannon-Nyquist-Whittaker-Kotelnikov sampling theorem [4]-[5], a band-limited signal $x(t)$ can be recovered fully from its sampled version $x(iT)$ only if $T \leq 1/(2F_{max})$. In other words, sampling rate should be equal to or greater than twice the maximum frequency of the signal so that signal is reconstructed completely. But the signals with large bandwidth like UWB signals which is having 3.1-10 GHz bandwidth carry less information, so that they are sparse in nature. If those signals are sampled at traditional sampling rate, ADC can be overburdened and it requires lots of power [6]-[7], so they need to be sampled according to the amount of information contained in the signal. It can be achieved by CS theory which is proposed by D. L. Donoho and E. J. Candes in [8]-[9]. According to CS theory, the sparse signal can be recovered properly with lower than the traditional sampling rate. The measurement matrix and reconstruction algorithm play important role for efficient performance of CS theory. In this paper, we use CS to reduce receiver sampling rate and implementation complexity as well as, digital notch to eliminate narrow band interference (NBI) effect.

Our contribution:

- We show here, the NBI symbol affects severely when added with UWB symbol by using energy detection equation provided in [10].
- We use the digital notch proposed in [11], to eliminate the NBI affected measurements from the compressed version of the received vector.
- By using energy detection and bit error probability equations for Gaussian distributed channel provided in [10], we compare the detector before adding NBI and after notching out NBI.

II. LITERATURE SURVEY

The field of UWB communication employed with CS theory is under huge development. In [12] the receiver which is proposed for IR-UWB communication using CS is characterized by bursty traffic and severe power constraints. The receiver can acquire and track the channel response in any of the environmental conditions and severe inter-symbol interference. The receiver proposed in [12] is further extended in [11] using notch out method for NBI mitigation. The CS theory reconstructs the sparse signal and also provides the generalized likelihood ratio test (GLRT) detector for I-UWB [13]. The GLRT detector of [13] is again developed with matching pursuit (MP) algorithm for pilot assisted IR-UWB detection in [14]. The IR-UWB detector proposed in [14] to suppress NBI using subspace detection which also further extended in [15]. The signal can be theoretically sub-sampled by projection matrix according to CS theory, but the multiplication of matrix and signal needs already sampled received signal. The random matrix is not realizable using hardware also under-sampling is uncontrollable. In CS measuring projection stage, as described in [16] these problems can be solved by replacing random matrix with analog to information converter (AIC). But, this method does not guarantee the precise reconstruction of sampled signal.



Novel differential detection method is proposed which exploits CS framework and optimization problem is formulated to jointly reconstruct the sparse signal and differentially encoded data in [17]. The differential detection method proposed in [17] is further extended in [18] for multiple symbols using generalized likelihood ratio tests. In [19]-[20], for CS based UWB communication methods for channel estimation are provided. In [21], time delay estimation is provided. In [10] there are two types of CS based UWB energy detectors proposed one is direct compressed energy detection and other is reconstruction based energy detector. TABLE 2.1 shows the difference between both the detectors [10].

From the bellow table, we can say that both the detectors are important in different situations with their own applications. But these energy detectors are very sensitive to the NBI because of large bandwidth symbol. The NBI affected IR-UWB measurements can be mitigated using method proposed in either [11] or [15].

Table 2.1 Comparison of direct compressed and reconstruction based UWB energy detector [10].

Features	Direct compressed energy detector	Reconstruction based energy detector
Type of samples	Compressed samples	Reconstructed samples
Timing information	Cannot be relaxed	Can be relaxed
Measurement process	Identical	Independent
Theoretical BEP	Requires orthogonal measurement matrix	Requires random measurement matrix

TABLE 2.2 comparisons of NBI mitigation methods

Features	[15]	[11]
Pulsing rate	independent	Low
Timing issue	Robust	Requires perfect timing
Discrete cosine transform (DCT)	Do not require	Requires
Domain of CS ensembles	Fourier	Time

III. METHODOLOGY

Compressed sensing

For conversion of the analog signal into digital with the traditional method, first step is to sample it and after that compress by eliminating zero or near to zero valued samples. For sampling the complete signal in this process large power consumption is required. But the compressed sensing unifies both, the compression and sampling processes so it is called as compressive sampling [8]-[9]. Compressed sensing (CS) is a signal processing technique for efficiently acquiring and reconstructing a signal, by finding solutions to underdetermined linear systems.

For sampling the complete signal in this process

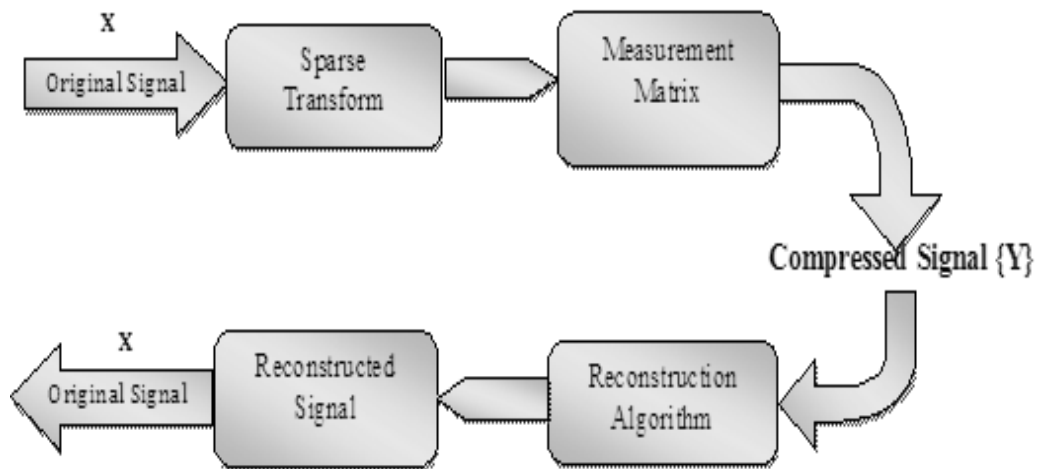


Fig. 1 Process of compressed sensing theory

As shown in Fig. 1, in the process of CS, reconstruction algorithm and measurement matrix have important role in reconstructing and compressing the signal respectively. The linear system to be passed through the CS process can be considered as, $Y = \phi X \dots (1)$ where, X is an $N \times 1$ vector of optimization variables, Y is an $M \times 1$ vector of compressed measurements as $M < N$ and ϕ is an $M \times N$ dimensional measurement matrix. Here, sparse vector is X which contains less number of non-zero valued samples than the zero valued samples. The measurement matrix converts the signal as $\mathbb{R}^N \rightarrow \mathbb{R}^M$. But measurement matrix should satisfy Restricted Isometry Property (RIP) to provide recoverable compressed version of the original signal [9]. For each integer $s = 1, 2, \dots$, RIP property define the isometry constant δ_s of a matrix ϕ as the smallest number such that, $(1 - \delta_s) \|X\|_2 \leq \|\phi X\|_2 \leq (1 + \delta_s) \|X\|_2 \dots (2)$ holds for all s -sparse vectors X [9]. A vector is said to be s -sparse if it has at most s non-zero entries. This property is satisfied by the random matrices like Gaussian, Bernoulli and also structured matrix like Fourier [10]. The most challenging task in CS theory is to recover the original signal from incomplete samples. For reconstruction process, X represents the unknown vector and the problem is to find X from Y given ϕ [3]. This problem is popularly written with l_2 -norm as, $P_2: \operatorname{argmin}_X \|X\|_2 \text{ such that } Y = \phi X \dots (3)$ The minimum norm solution can be obtained from l_2 -norm instead of handling individual element, it measures total energy of the vector X . It cannot reconstruct the original signal properly from compressed version. The number of nonzero elements from X can be counted by replacing squared l_2 -norm with an l_0 -norm [3] as, $P_0: \operatorname{argmin}_X \|X\|_0 \text{ such that } Y = \phi X \dots (4)$ The l_0 -norm solution provides sparse solution but not unique, unlike l_2 -norm solution. But, the l_1 -norm solution provides compromise between l_1 -norm and l_2 -norm solution. In terms of sparsity, it is closer to l_0 -norm where as in terms of uniqueness or being convex, it is closer to l_2 -norm. It can be written as, $P_1: \operatorname{argmin}_X \|X\|_1 \text{ such that } Y = \phi X \dots (5)$ P_1 is a convex optimization problem and can be easily solved by a linear programming (LP). P_1 is also known as basis pursuit (BP) [22]. The compressed signal can be recovered exactly under two conditions first is original signal should be sparse and second is the measurement matrix should satisfy RIP property. There are mainly three types of reconstruction algorithms [23].

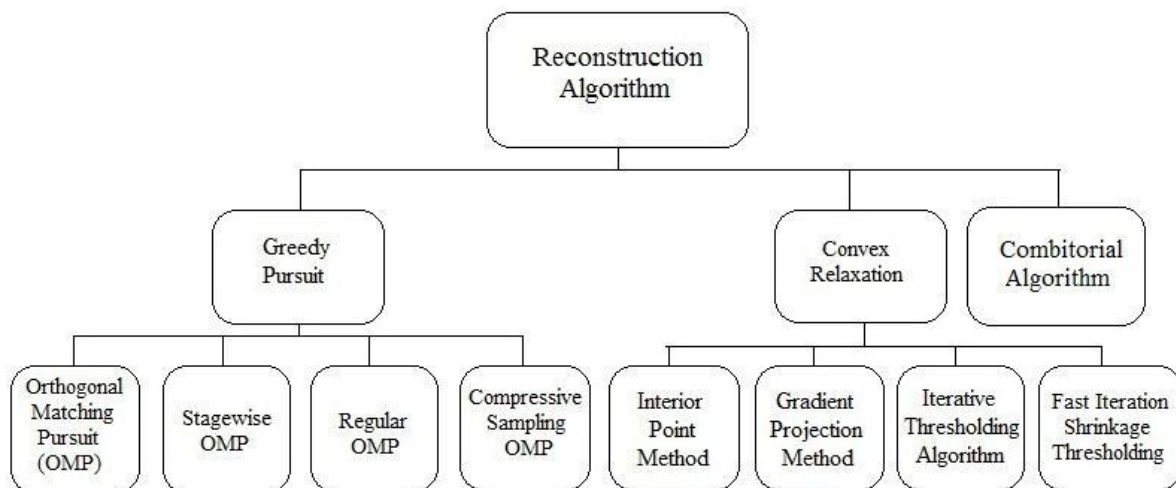


Fig. 2 Classifications of reconstruction algorithms in CS theory

First is the greedy pursuit, such as the orthogonal matching pursuit (OMP) method [24], the stage wise OMP (StOMP) method [25] and the regularized OMP (ROMP) method [26], the compressive sampling matching pursuit (CoSaMP) method [23], where these methods build up an approximation one step at a time. Second is the convex relaxation algorithm, such as the interior-point method [27], the gradient projection method [28] and the iterative thresholding algorithm [29] and the last is the combinatorial algorithms that acquire structured samples of the signal that support rapid reconstruction by group testing [23]. Each algorithm has its own pros and cons in a particular reconstruction problem. So the reconstruction algorithm should be chosen according to the requirement in specific application.

System model

The system described in Fig.3 transmits the *j*th information symbol with *m*-ary pulse position modulation (PPM). In PPM, the delay is added in the signal for modulation which is easy to implement. For transmitting the *j*th information symbol, consider a signal $U_j(t)$ containing *NF* frames of length *TF*, so that the signal length becomes $T = NF \times TF$ and delayed by $Tm = TFm$ for PPM modulation. The transmitted *j*th symbol can be represented as $U_j(t) = \sum_{i=0}^{NF-1} b(t - NF - 1i = 0(i + jNF)TF - c_j Tm)$, where $c_j \in (0, 1, \dots, m-1)$ and $b(t)$ is second derivative of Gaussian pulse with unit energy of duration $Tb \ll Tm$. If $h(t)$ is represented as impulse response of Gaussian communication channel, then the received signal is,

$$r(t) = U_j(t) * h(t) + w_j + I_j(t) \tag{6}$$

where $w_j(t)$ and $I_j(t)$ is the additive noise and interference symbol of bandwidth is *IB*, corresponding to *j*th information symbol respectively and $U_j(t) * h(t) = g_j(t)$ is the received pulse waveform of bandwidth *UB* with duration *Tg*.

For Nyquist-rate sampling of the symbol, we take *N* samples per frame period *TF*, whereas *N/m* samples for each slot. Then the *i*th sampled frame corresponding to *j*th symbol is given by,

$$r_{j,ki} = r(iTF + kTFN) = g_{j,ki} + w_{j,ki} + I_{j,ki} \tag{7}$$

for $k=0,1,\dots,N-1$. We assume that the NBI zero means, unit power elements whereas, w_j, k_i has independent identically distributed (i.i.d.) zero mean Gaussian with variance σ^2 .

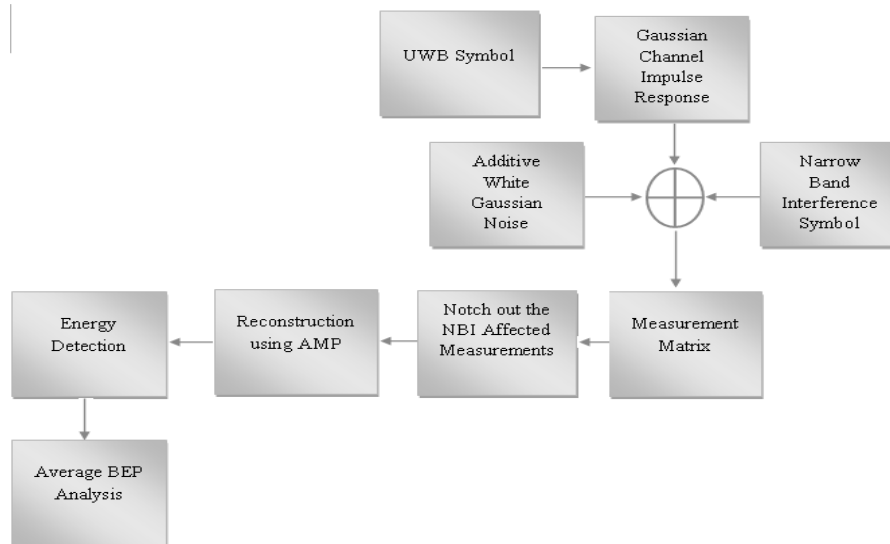


Fig.3 Block diagram of NBI robust UWB energy detector.

The sampling of received signal using Nyquist-rate consumes large amount of energy due to sparse nature of the signal. The zero or near to zero valued samples produced after applying equation (7) are eliminated. By replacing Nyquist sampling with compressed sampling this wastage of energy can be removed.

For compressing the signal, measurement matrix Φ is so chosen that it contains rows which are approximately orthogonal to each other [10]. Now, the received signal is applied to $M \times N$ matrix Φ to get compressed version of signal. For i th frame, applying CS to equation (6), we get,

$$D_{ji} = \Phi r_{ji} = G_{ji} + V_{ji} + Z_j \tag{8}$$

where, D_{ji} is the $M \times 1$ compressed measurement vector. Similarly, Z_{ji} and V_{ji} are the compressed versions of NBI symbol and noise respectively. Along with huge advantages, the UWB symbol has sensitivity towards NBI due to its wide bandwidth feature. The UWB measurements are deteriorated due to addition of NBI symbol and UWB symbol. In this paper, we have applied the ‘notch out’ method [11] to suppress the NBI affected measurements from compressed vector. In this method, first step is to choose Fourier ensemble of magnitude $1/\sqrt{N}$ and its frequency is selected from $(F_c - \Omega_2, F_c + \Omega_2)$, which are decoherent with UWB signal and coherent with NBI, to ensure that only few measurements are affected by NBI. Then, we can implement notch to mitigate NBI affected measurements. For at most 1 NBI we find,

$$s = \text{argmax}_{s \in \{0, 1, \dots, M-1\}} |D_s| \tag{9}$$

Now, take $A \sim \alpha B \gamma$, where $\gamma = UBM$ is test function spacing and α is nothing but the safety factor lies between 4 to 8. The NBI mitigated measurements can be obtained by notching out $A+1$ measurements around the indexes. If $N_I > 1$ NBI is expected in the signal then this notching procedure is performed for

N_I largest values from compressed measurement vector D . To reduce the time required for the frames individually, the NBI eliminated measurements of N_F frames are averaged reconstructing and applied to approximated message passing (AMP) algorithm to reconstruct the original signal [10]. The iterative thresholding (ITH) algorithm has better simplicity and speed than other reconstruction algorithms, but its performance is not good in sparsity-undersampling (SU). However, the AMP performs well in SU along with better speed and simplicity. The AMP can be explained briefly for n th iteration as follows,

$$y_{j[n+1]} = S(y_{j[n]} + \Phi T x_{j[n]}, \tau[n]) \tag{10}$$

where, $x_{j[n]} = D_j - \Phi y_{j[n]} + 1 \mu x_{j[n-1]} \langle S'(y_{j[n-1]} + \Phi T x_{j[n-1]}, \tau[n-1]) \rangle$ (11)

Here, τ is iteratively updating threshold and $\langle S(\cdot) \rangle$ is the average of derivative all samples of soft-thresholding over N samples. The value provided by y_{jn} is the reconstructed vector of signal. Now, these samples are provided to reconstruction based energy detector, which is provided in [10]. For N_m non-zero samples, detection is done as follows,

$$\hat{c}_{j(R-ED)} = \max_{c_j} \sum [1 N_F \sum [\check{b}_k]_{i=N+ck N_m+p N_F-1 i=0}^{2 N_m-1 p=0}] \tag{12}$$

The Nyquist rate energy detection is obtained by replacing reconstructed samples with Nyquist rate samples in (12). For analysis of energy detection, the bit error probability (BEP) of both Nyquist-rate energy detector and reconstruction based energy detector is given in same [10].

$$P_{R-BEP} = 1 - 2 \Gamma(N_2) N_2 [\Gamma(N_4)]^2 [\sigma_r \sigma_\omega \sigma_{r^2} + \sigma_{\omega^2}]_{N_2} \times 2 F_1(1, N_2; N_4 + 1; \sigma_r^2 \sigma_{r^2} + \sigma_{\omega^2}) \tag{13}$$

Where $2F_1(\cdot, \cdot; \cdot; \cdot)$ is the Gaussian hyper geometric function.

$$P_{N-BEP} = 1 - 2 \Gamma(N_2) N_2 [\Gamma(N_4)]^2 [\sigma_{\beta 0} \sigma_{F 0} \sigma_{\beta 0^2} + \sigma_{F 0^2}]_{N_2} \times 2 F_1(1, N_2; +1; \sigma_{\beta 0^2} \sigma_{\beta 0^2} + \sigma_{F 0^2}) \tag{14}$$

Where $\sigma_{\beta 0^2} \triangleq (1 + \sigma_{2 N_F})$ and $\sigma_{F 0^2} \triangleq (\sigma_{2 N_F})$

IV. EXPERIMENTAL RESULT

This section presents the simulation results occurred from the detector developed in previous section. We consider, the UWB signal is transmitted along Gaussian distributed physical channel with the elements having zero mean and unit variance. The received signal is compressed by applying it to the random measurement matrix. Let the number of interference, $N_I=1$ for experiment purpose. But, we can simulate this detector for multiple numbers of interferences. In AMP algorithm, the threshold policy is in the form of $\tau[n] = \delta \sigma \omega[n]$, which is infeasible in practice. So, threshold can be updated as suggested in [9],

$$\tau[n] = \tau + 1 \mu \tau[n-1] \langle S'(x_{k[n-1]} + \Phi T y_{k[n-1]}, \tau[n-1]) \rangle \tag{15}$$

where, τ is a constant.

We have considered four detectors for analysis purpose. The first detector is based on Nyquist rate sampling, the second is based on compressive sampling i.e. nothing but reconstruction based, the third detector is having NBI effect and in forth detector the NBI mitigation method is implemented. All the four detectors are compared with each other with respect to signal to noise ratio (SNR in dB) in Fig.4 - Fig.7 and with respect to

compression ratio (μ) in Fig.8 – Fig.11. All the detectors have same transmission parameters. The transmitted second derivative of Gaussian pulse has duration of 1nsec. For experimentation purpose, we take frame length as 100nsec and number of frames in one symbol is 30, so the duration of symbol is multiplication of N_F and T_F . For every frame, the number of samples is considered to be 200. All the detectors are analyzed with equation (12) and (13) for energy detection and bit error probability respectively. Due to implementation of orthogonal random matrix at compression, the plot of ABEP is following the plot of ED in all the simulated results.

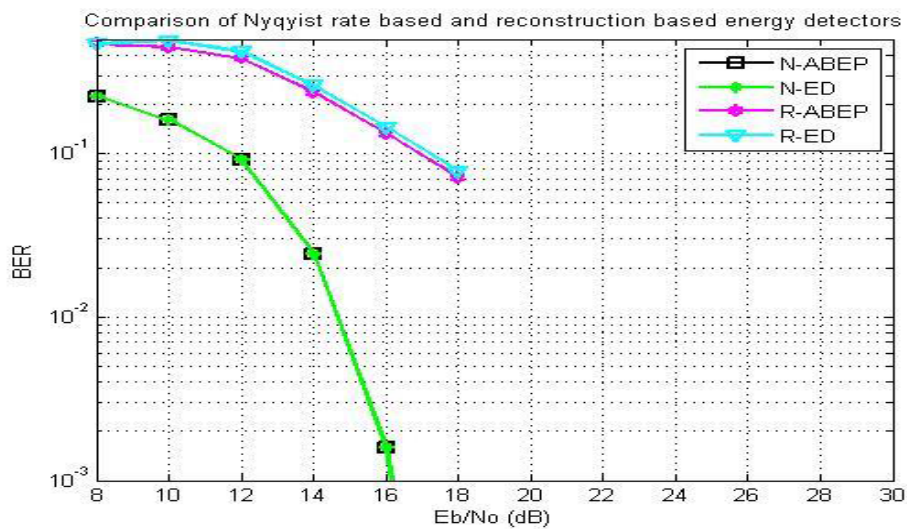


Fig.4 Comparison of Nyquist Rate and reconstruction based energy detectors w.r.t. E_b/N_0 .

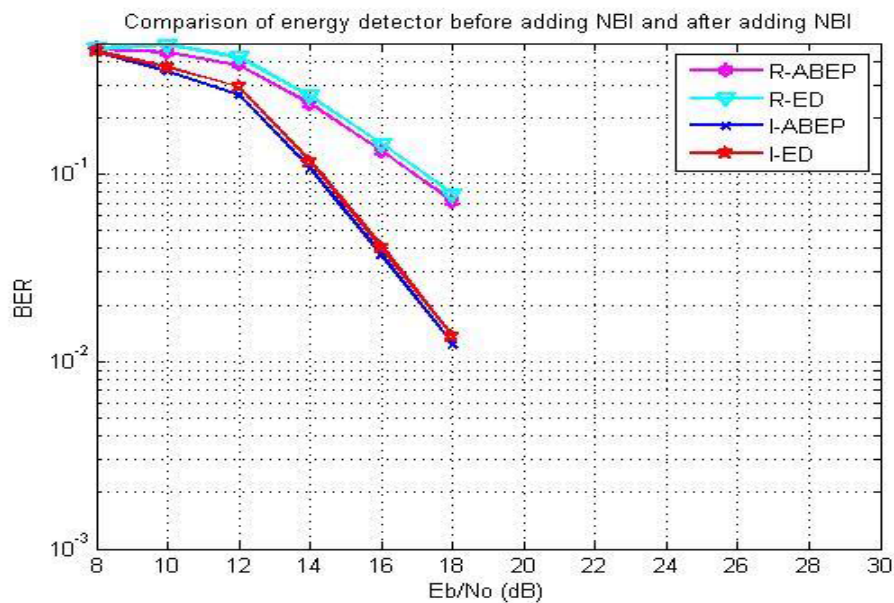


Fig.5 Effect of NBI on reconstruction based energy detector w.r.t. E_b/N_0

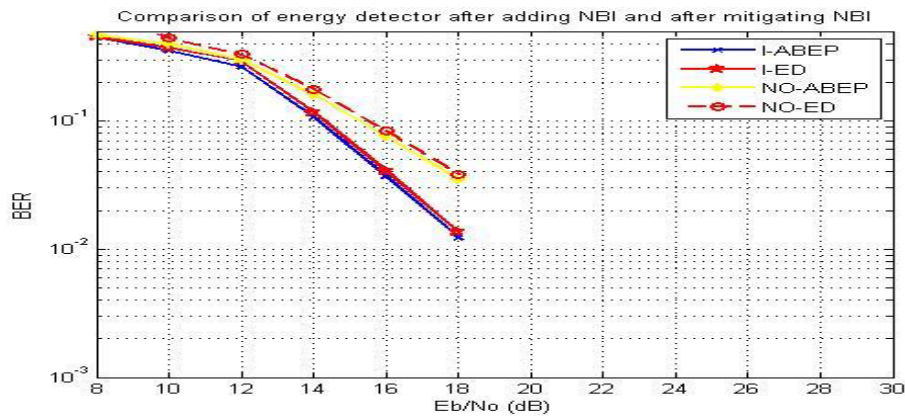


Fig. 6 Comparison of detectors in the presence of NBI and after mitigating NBI w.r.t. Eb/No

In Fig.4 and Fig.8 we can observe that, there is large difference between energy detection of Nyquist rate based and compressive sampling based energy detectors. The CS based energy detectors gives better energy detection than the traditional sampling based energy detectors. After adding the NBI symbol with UWB symbol, the effect can be observed in Fig.5 and Fig.9. Then the notch out method is implemented to eliminate the NBI affected measurements from compressed measurements. After eliminating the NBI effect, it is compared with the detector having NBI effect in Fig.6 and Fig.10. The Fig.7 and Fig.11 shows the effectiveness of the notch out method to remove the NBI from the compressed version of the signal. The energy detection of the detector in the absence of NBI and after mitigating NBI is similar. It is the evident that the notch out method will remove the NBI successfully.

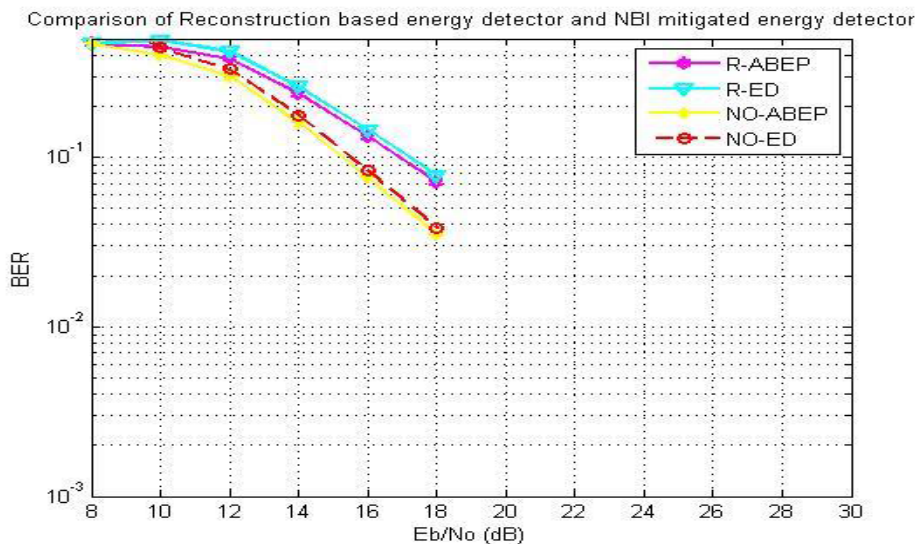


Fig. 7 Comparison of detectors in the absence of NBI and after notching out NBI w.r.t. Eb/No.

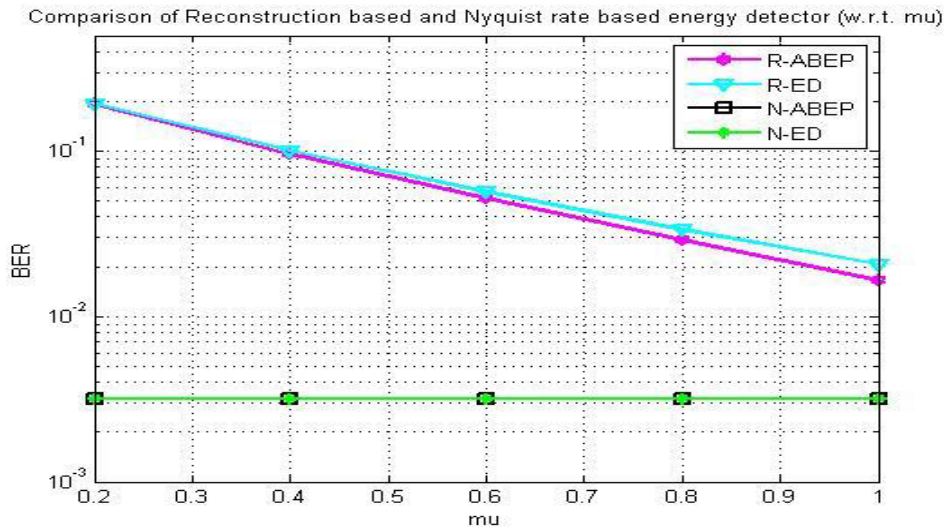


Fig.8 Comparison of Nyquist rate energy detector and compressed sampling based energy detector w.r.t. mu

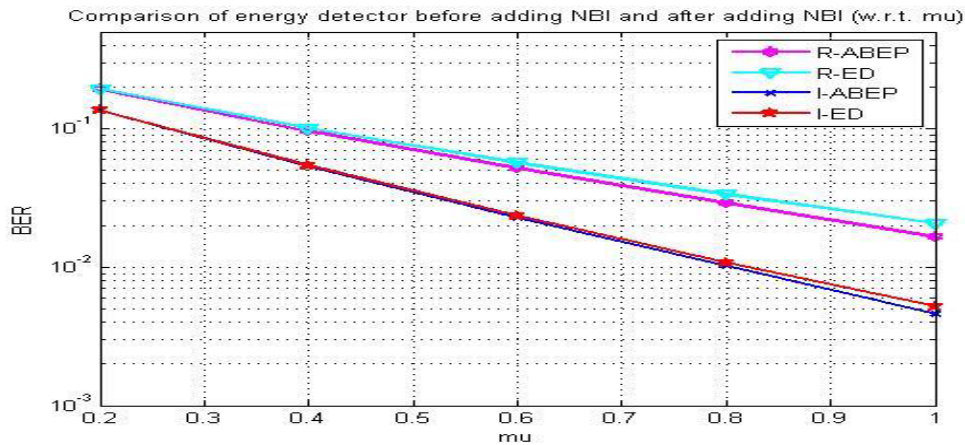


Fig.9 Comparison of energy detector in the absence of and in the presence of NBI

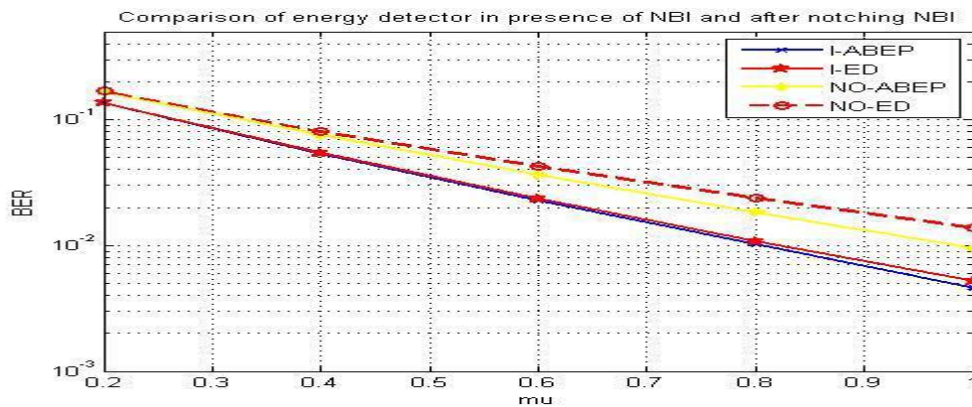


Fig.10 Comparison of energy detector in the presence of NBI and after mitigating NBI.

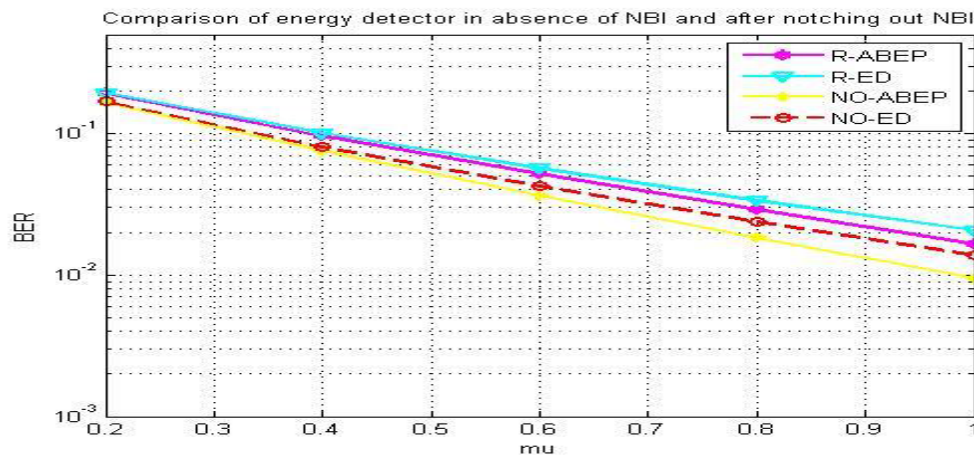


Fig.11 Comparison of energy detector in the absence of NBI and after mitigating NBI w.r.t μ

V. CONCLUSION

In this paper, we have developed compressed sensing-based UWB energy detector for narrowband interference it is robust. Due to reduction in sampling rate than the traditional sampling rate this detector consumes less power. The wideband communication is sensitive to NBI; so that we have implemented the method which mitigates the NBI-affected measurements from compressed version of the received signal without harming much of the other measurements. From the simulated results, thus we have observed that CS-based energy detection is better than the Nyquist rate energy detection

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