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A FORMAL SOLUTION OF CERTAIN QUADRUPLE INTEGRAL EQUATIONS INVOLVING I-FUNCTIONS

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ABSTRACT

The problem discussed is to obtain the solution of quadruple integral equations involving I-functions. The method followed is that of fractional integration. The given quadruple integral equations have been transformed by the application of fractional Erdelyi-Kober operators to four others integral equations with a common Kernel. Here for the sake of generality the I-function is assumed as unsymmetrical Fourier kernel. Here with the help of theorems of Mellin transform, the solution of Quadruple Integral equations is obtained. Some interesting particular cases have been derived.

I. INTRODUCTION

V.P. Saxena[6] defined the I-function, which is more general hypergeometric function than the Fox's H-Function. Saxena's I-function is defined as,

$$I^{m,n}_{p_i;\,q_i;r}[z] = I^{m,n}_{p_i;\,q_i;r}\left[z\Big/\begin{matrix}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right] = \frac{1}{2\pi i}\int_L t(s)z^s ds \quad [1.1]$$

where

$$t(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}s)\right\}} \quad [1.2]$$

Here $p_i\, and q_i$ are positive integers and m, n are integers satisfying

$0 \le n \le p_i$, $0 \le m \le q_i$,(i=1, 2,………………….,r),r is finite.

$\alpha_j, \beta_j, \alpha_{j_i}, \beta_{j_i}$ are real and positive and $a_j, b_j, a_{j_i}, b_{j_i}$ are complex numbers such that

$\alpha_j(b_n+\nu) \neq \beta_h(\alpha_j-1-\nu)$ for ν= 0,1,2,…………… ; h=1,2,……………m ; j=1,2,……………,r.

L is a suitable Contour of Barnes type which runs from σ-i∞ to σ+i∞, (σ is real) in the complex s-plane such that the points

s = $(\alpha_j-1-\nu) / \alpha_j$: j=1,2,……………,n ; ν= 0,1,2,…………… and

s=$(b_j+\nu) / \beta_j$: j=1,2,……………m; ν= 0,1,2,…………… lie to the left hand side and right hand side of the contour L respectively. The conditions under which [1.1] converges are given as follows:

$$A_i = \sum_{j=1}^{n} \alpha_j - \sum_{j=n+1}^{p_i} \alpha_j + \sum_{j=1}^{m} \beta_j - \sum_{j=m+1}^{q_i} \beta_j \quad : (i=1,2,\ldots \ldots,r) \qquad [1.3]$$

$$B_i = \frac{1}{2}(p_i - q_i) + \sum_{j=1}^{q_i} b_j - \sum_{j=1}^{n} a_j \quad : (i=1,2,\ldots\ldots,r) \qquad [1.4]$$

$$A_i > 0 , |\arg z| < \frac{1}{2}\pi A_i \, and \, B_i \geq 0 \quad : (i=1,2, \ldots\ldots,r) \qquad [1.5]$$

Mathur[4] obtained the solution of simultaneous dual integral equations involving I-functions.

Mathur[5] have also considered the formal solution of triple integral equations involving I-functions. The aim of the present section is to obtain the solution of the quadruple integral equations involving I-functions. The method followed is that of fractional integral operators. By the application of fractional integral operators given equations are transformed into a equation with common Kernel.

II. RESULTS USED IN THE PROOF OF THE SEQUEL

MELLIN TRANSFORM:

$$M\{f(x)\} = F(s) = \int_0^\infty f(x)x^{s-1}dx \qquad [2.1]$$

INVERSE MELLIN TRANSFORM C+I∞ :

$$M^{-1}\{F(s)\} = f(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} F(s)x^{-s}ds \qquad [2.2]$$

For s = c+in, x>0

PERSAVAL THEOREM FOR MELLIN TRANSFORMS:

Let M {f (u)} = F(s) and M {a (u)} = A(s) then

M {a (ux)} = $x^{-s}A(s)$ and

$$\int_0^\infty f(ux)a(u)du = \frac{1}{2\pi i}\int_L x^{-s}F(s)A(1-s)ds \qquad [2.3]$$

FRACTIONAL INTEGRAL FORMULAE:

Fractional Integral formulae have been defined by Fox as follows:

$$\int_0^x \left(x^{\frac{1}{c}} - v^{\frac{1}{c}}\right)^{d-e-1} . v^{\frac{e}{c}-s-1}dv = \frac{e\,\Gamma(d-e)\Gamma(e-cs)}{\Gamma(d-cs)} . x^{\frac{d}{c}-\frac{1}{c}-s} \qquad [2.4]$$

$$\text{provided } d > e \; and \frac{e}{c} > \sigma \quad where\ s = \sigma + it \ and\ 0 < x < 1$$

and

$$\int_x^\infty \left(v^{\frac{1}{c}} - x^{\frac{1}{c}}\right)^{d-e-1} . v^{\frac{1}{c}-\frac{d}{c}-s-1}dv = \frac{c\,\Gamma(d-e)\Gamma(e+cs)}{\Gamma(d+cs)} . x^{-\frac{e}{c}-s} \qquad [2.5]$$

IJARSE
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provided $d > e \ and \frac{e}{c} > \sigma \quad where\ s = \sigma + it\ and\ x > 1$

FRACTIONAL ERDELYI – KOBER OPERATORS:

Fox used the following generalized Erdelyi –Kober Operators:

$$T\,[\gamma, \epsilon{:}\, m]\,\{f(x)\} = \frac{m}{\Gamma\gamma} x^{-\gamma m-\epsilon+m-1} \int_0^x (x^m - v^m)^{\gamma-1} . v^{\epsilon}\, f(v) dv, \qquad [2.6]$$

where 0 < x < 1

and

$$R\,[\gamma, \epsilon{:}\, m]\{f(x)\} = \frac{m}{\Gamma\gamma} . x^{\epsilon} \int_x^{\infty} (v^m - x^m)^{\gamma-1} . v^{-\epsilon-\gamma m+m-1}\, f(v) dv, \qquad [2.7]$$

wherex> 1

The operator T exists if f(x)$\in L_p(0,\infty), p > 1, \gamma > 0 and$

$\epsilon > \frac{1-p}{p}$ and if f(x)can be differentiated sufficient number of times then the operator T exists for both negative and positive value of γ.

The operator R exists if f(x)$\in L_p(0,\infty), p \geq 1$ and If $f(x)$can be differentiated sufficient number of times then the operator R exists.

If m > $\epsilon > \frac{-1}{p}$ while γ can take any negative or positive value.

A THEOREM FOR MELLIN TRANSFORMS:

If M {f (u)} = F(s) and M {g (u)} = G (s) then

$$\int_0^{\infty} g(u) f(u) du = \frac{1}{2\pi i} \lim_{\substack{T \to \infty \\ \& \sigma_0 = R_e(s)}} \int_{\sigma_0 - iT}^{\sigma_0 + iT} G(s) F(1-s) ds \qquad [2.8]$$

Thus if f (ux) is considered to be a function of u with x as a parameter, where x > 0 then

$$M\,\{f\,(ux)\} = x^{-s} M\{f(u)\} \qquad [2.9]$$

From [2.8] and [2.9] we have

$$\int_0^{\infty} g(ux) f(u) du = \frac{1}{2\pi i} \lim_{\substack{T \to \infty \\ \& \sigma_0 = R_e(s)}} \int_{\sigma_0 - iT}^{\sigma_0 + iT} x^{-s} G(s) F(1-s) ds \qquad [2.10]$$

Additional conditions for the validity of [2.10] are that

F(s) $\in L_p(\sigma_0 - i\infty, \sigma_0 + i\infty) and$

$x^{1-\sigma_0}\, g(x) \in L_p(0,\infty), p \geq 1$ where L_p denotes the class of functions g(x) such that

IJARSE
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$$\int_0^{\infty} |g(x)|^p . \frac{dx}{x} < \infty$$ [2.11]

III. THE SOLUTION OF FOLLOWING QUADRUPLE INTEGRAL EQUATIONS INVOLVING I-FUNCTIONS

$$\int_0^{\infty} I_{p_i;\ q_i;r}^{m,n}\left[ux/\begin{matrix}(a_j,\alpha_j)_{1,n},(a_{j_i},\alpha_{j_i})_{n+1,p_i}\\(b_j,\beta_j)_{1,m},(b_{j_i},\beta_{j_i})_{m+1,q_i}\end{matrix}\right]f(u)\,du = \phi_1(x)$$ [3.1]

where $x \in (0, a)$

$$\int_0^{\infty} I_{p_i;\ q_i;r}^{m,n}\left[ux/\begin{matrix}(c_j,\alpha_j)_{1,n},(a_{j_i},\alpha_{j_i})_{n+1,p_i}\\(b_j,\beta_j)_{1,m},(b_{j_i},\beta_{j_i})_{m+1,q_i}\end{matrix}\right]f(u)\,du = \phi_2(x)$$ [3.2]

where $x \in (a, b)$

$$\int_0^{\infty} I_{p_i;\ q_i;r}^{m,n}\left[ux/\begin{matrix}(c_j,\alpha_j)_{1,n},(a_{j_i},\alpha_{j_i})_{n+1,p_i}\\(b_j,\beta_j)_{1,m},(b_{j_i},\beta_{j_i})_{m+1,q_i}\end{matrix}\right]f(u)\,du = \phi_3(x)$$ [3.3]

where $x \in (b, c)$

$$\int_0^{\infty} I_{p_i;\ q_i;r}^{m,n}\left[ux/\begin{matrix}(c_j,\alpha_j)_{1,n},(a_{j_i},\alpha_{j_i})_{n+1,p_i}\\(d_j,\beta_j)_{1,m},(b_{j_i},\beta_{j_i})_{m+1,q_i}\end{matrix}\right]f(u)\,du = \phi_4(x)$$ [3.4]

where $x \in (c, \infty)$

where o < a < 1 and 1 < c < ∞. $\phi_1(x), \phi_2(x), \phi_3(x)$ and $\phi_4(x)$ are prescribed functions and f(x) is unknown function which is to be determined. Applying Persaval's theorem in the integral equations [3.1], [3.2] ,[3.3] and [3.4] under the conditions:

$$-Min_{1\le j\le m} R_e(b_j/\beta_j) < R_e(s) < 1/\alpha_j - -Max_{1\le j\le n} R_e(a_j/\alpha_j)$$ [3.5]

$$-Min_{1\le j\le m} R_e(b_j/\beta_j) < R_e(s) < 1/\alpha_j - -Max_{1\le j\le n} R_e(c_j/\alpha_j)$$ [3.6]

$$-Min_{1\le j\le m} R_e(d_j/\beta_j) < R_e(s) < 1/\alpha_j - -Max_{1\le j\le n} R_e(c_j/\alpha_j)$$ [3.7]

$|arg x| < \frac{1}{2}\pi D_i$,   i= 1,2,…………………,r. [3.8]

where

IJARSE
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$$D_i = \sum_{j=1}^{m} \beta_j - \sum_{j=n+1}^{p_i} \alpha_j + \sum_{j=m+1}^{q_i} \beta_j - \sum_{j=1}^{m} \alpha_j \quad [3.9]$$

where $i = 1,2,\ldots,r$

then the integral equations [3.1], [3.2] ,[3.3] and [3.4]are reduced to the following forms:

$$\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j+\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j-\alpha_j s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{j_i}-\beta_{j_i}s)\prod_{j=n+1}^{p_i}\Gamma(a_{j_i}+\alpha_{j_i}s)\right\}} x^{-s}F(1-s)ds = \phi_1(x) \quad [3.10]$$

where $x \in (0,a)$

$$\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j+\beta_j s)\prod_{j=1}^{n}\Gamma(1-c_j-\alpha_j s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{j_i}-\beta_{j_i}s)\prod_{j=n+1}^{p_i}\Gamma(a_{j_i}+\alpha_{j_i}s)\right\}} x^{-s}F(1-s)ds = \phi_2(x) \quad [3.11]$$

where $x \in (a,b)$

$$\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j+\beta_j s)\prod_{j=1}^{n}\Gamma(1-c_j-\alpha_j s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{j_i}-\beta_{j_i}s)\prod_{j=n+1}^{p_i}\Gamma(a_{j_i}+\alpha_{j_i}s)\right\}} x^{-s}F(1-s)ds = \phi_3(x) \quad [3.12]$$

where $x \in (b,c)$

$$\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(d_j+\beta_j s)\prod_{j=1}^{n}\Gamma(1-c_j-\alpha_j s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{j_i}-\beta_{j_i}s)\prod_{j=n+1}^{p_i}\Gamma(a_{j_i}+\alpha_{j_i}s)\right\}} x^{-s}F(1-s)ds = \phi_4(x) \quad [3.13]$$

where $x \in (c,\infty)$

Here $M\{f(u)\} = F(s)$ [3.14]

Now in integral equation [3.10] replacing x by ν and multiplying both sides of the equation [3.10] by $(x^{1/\alpha_n} - \nu^{1/\alpha_n})^{c_n-\alpha_n-1}.\nu^{(1-c_n)/\alpha_n-1}$ and integrating both sides of integral equation [3.10]with respect to ν from 0 to x where x ϵ (0,a) with 0 < a < 1 and applying well known fractional integral formula [2.4] in equation [3.10], we find

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$$\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j+\beta_j s)\prod_{j=1}^{n-1}\Gamma(1-a_j-\alpha_j s)\,\Gamma(1-c_n-\alpha_n s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{j_i}-\beta_{j_i}s)\prod_{j=n+1}^{p_i}\Gamma(a_{j_i}+\alpha_{j_i}s)\right\}}$$

$$x^{-s}F(1-s)ds = \frac{1}{\alpha_n\Gamma(c_n-a_n)}.x^{a_n/\alpha_n}$$

$$\int_0^x (x^{1/\alpha_n}-v^{1/\alpha_n})^{c_n-a_n-1}.v^{(1-c_n)/\alpha_n-1}\,\phi_1(v)dv \qquad [3.15]$$

where $0 < x < 1$

Using the Erdelyi-Kober operator T from [2.6] in equation [3.15],

For brevity we write,

$$T\,[c_j-a_j\ ,\ (1-c_j)/\alpha_j-1 : 1/\alpha_j]\,\{\phi_1(x)\} = T_j\{\phi_1(x)\} \qquad [3.16]$$

where $x \in (0,a)$

then

$$T\,[c_n-a_n,(1-c_n)/\alpha_n-1 : 1/\alpha_n]\,\{\phi_1(x)\} = T_n\{\phi_1(x)\} \qquad [3.17]$$

where $x \in (0,a)$

Hence from [3.17], the integral equation [3.15] can be written as

$$\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j+\beta_j s)\prod_{j=1}^{n-1}\Gamma(1-a_j-\alpha_j s)\,\Gamma(1-c_n-\alpha_n s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{j_i}-\beta_{j_i}s)\prod_{j=n+1}^{p_i}\Gamma(a_{j_i}+\alpha_{j_i}s)\right\}}$$

$$x^{-s}F(1-s)ds = T_n\{\phi_1(x)\} \qquad [3.18]$$

where $x \in (0,a)$

Now repeating the same process in integral equation [3.18]

for j= n-1,n-2,…….,3,2,1

then the integral equation [3.18] takes the from

$$\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j+\beta_j s)\prod_{j=1}^{n}\Gamma(1-c_j-\alpha_j s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{j_i}-\beta_{j_i}s)\prod_{j=n+1}^{p_i}\Gamma(a_{j_i}+\alpha_{j_i}s)\right\}}$$

$$x^{-s}F(1-s)ds = \prod_{j=1}^{n} T_j\{\phi_1(x)\} \qquad [3.19]$$

where $x \in (0,a)$

Now in integral equation [3.13] replacing x by $\mathcal{V}$ and multiplying both sides of the equation [3.13] by

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IJARSE
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$(\nu^{1/\beta_m} - x^{1/\beta_m})^{d_m - b_m - 1} . \nu^{(1-d_m)/\beta_m - 1}$

and integrating both sides of integral equation [3.13] with respect to ν from x to ∞ , where $x \in (c. \infty)$ with c > 1 and applying well known fractional integral formula [2.5], We find,

$$\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m-1}\Gamma(d_j+\beta_j s)\prod_{j=1}^{n}\Gamma(1-c_j-\alpha_j s)\Gamma(b_m+\beta_m s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}-\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}+\alpha_{ji}s)\right\}} x^{-s}F(1-s)ds$$

$$= \frac{1}{\beta_m\Gamma(d_m-b_m)} . x^{b_m/\beta_m}\int_x^\infty (\nu^{1/\beta_m}-x^{1/\beta_m})^{d_m-b_m-1}$$

$$. \nu^{(1-d_m)/\beta_m - 1}\phi_4(\nu)\, d\nu \qquad [3.20]$$

wherex > 1

Using the Erdely-Kober operator R from [2.7] in equation [3.20]

For brevity we write,

$$\text{R}\,[d_j - b_j\ ,\ b_j/\beta_j\ :1/\beta_j]\ \{\phi_4(x)\} = R_j\{\phi_4(x)\} \qquad [3.21]$$

where $x \in (c, \infty)$

then

$$\text{R}[d_m - b_m\ ,\ b_m/\beta_m : 1/\beta_m]\ \{\phi_4(x)\} = R_m\{\phi_4(x)\} \qquad [3.22]$$

wherex $\in (c, \infty)$

Hence from [3.22], the integral equation [3.20] can be written as

$$\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m-1}\Gamma(d_j+\beta_j s)\prod_{j=1}^{n}\Gamma(1-c_j-\alpha_j s)\,\Gamma(b_m+\beta_m s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}-\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}+\alpha_{ji}s)\right\}}$$

$$x^{-s}F(1-s)ds = R_m\{\phi_4(x)\} \qquad [3.23]$$

wherex $\in (c, \infty)$

Now repeating the same process in integral equation [3.23] for

j=m-1,m-2,…………,3,2,1 then the integral equation [3.23] takes the form

$$\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j+\beta_j s)\prod_{j=1}^{n}\Gamma(1-c_j-\alpha_j s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}-\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}+\alpha_{ji}s)\right\}}$$

$$x^{-s}F(1-s)ds = \prod_{j=1}^{m} R_j\{\phi_4(x)\} \qquad [3.24]$$

where $x \in (c, \infty)$

Now if we set

IJARSE
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$$P(x) = \begin{cases} \prod_{j=1}^{n} T_j\{\phi_1(x)\} & ; x \in (0,a) \\ \phi_2(x) & ; x \in (b,c) \\ \phi_3(x) & ; x \in (b,c) \\ \prod_{j=1}^{m} R_j\{\phi_4(x)\} & ; x \in (c.\infty) \end{cases} \qquad [3.25]$$

then integral equations [3.19],[3.11],[3.12] and [3.24] having common kernel can be put into the compact form as

$$\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j+\beta_j s)\prod_{j=1}^{n}\Gamma(1-c_j-\alpha_j s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}-\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}+\alpha_{ji}s)\right\}}$$

$$x^{-s}F(1-s)ds = p(x) \qquad [3.26]$$

where $x \in (0,\infty)$

In order to solve the integral equation [3.26] , the additional condition required is that the I-function is symmetrical or unsymmetrical Fourier kernel and f(u) is continuous at u=x. For the sake of generality we assume that I-function is unsymmetrical Fourier kernel, In this case the parameters have to satisfy the following set of conditions:

(i) $\sum_{j=1}^{m}\beta_j - \sum_{j=n+1}^{p_i}\alpha_j = \sum_{j=m+1}^{q_i}\beta_j - \sum_{j=1}^{n}\alpha_j$: (i=1,2,……..,r)

(ii) $\sum_{j=1}^{m}b_j - \sum_{j=n+1}^{p_i}a_j = \sum_{j=m+1}^{q_i}b_j - \sum_{j=1}^{n}c_j$: (i=1,2, …..,r) [3.27]

(iii) $R_e[(1-c_j)-\alpha_j/2] > \frac{\alpha_j}{2D_i}$; (j=1,2,… ….,n ; i=1,2,…..r)

(iv) $R_e[(a_j+\alpha_j/2)] > \frac{\alpha_j}{2D_i}$; (j=n+1,2,… …., p_i ; i=1,2,…..r)

(v) $R_e[(d_j+\beta_j/2)] > \frac{\beta_j}{2D_i}$; (j=1,2,….,m ; i=1,2,…..r) [3.28]

(vi) $R_e[(1-b_j)-\beta_j/2] > \frac{\beta_j}{2D_i}$; (j=m+1,…., q_i ; i=1,2,…..r)

where

$$D_i = \sum_{j=1}^{m}\beta_j - \sum_{j=n+1}^{p_i}\alpha_j + \sum_{j=m+1}^{q_i}\beta_j - \sum_{j=1}^{n}\alpha_j \qquad [3.29]$$

where i=1,2,…..,r

Now with the help of theorem V. P Saxena[6], we find its respective reciprocal kernel

IJARSE
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$$I_{p_i;\ q_i;r}^{m,n}\left[x/\begin{matrix}(a_{j_i}+\alpha_{j_i},\alpha_{j_i})_{1,n},(c_j-\alpha_j,\alpha_j)_{n+1,p_i}\\(b_{j_i}-\beta_{j_i},\beta_{j_i})_{1,m},(b_j+\beta_j,\beta_j)_{m+1,q_i}\end{matrix}\right] \quad [3.30]$$

Taking Mellin transform and making use of the theorem of V. P. Saxena(1971) in integral equation [3.26], we have the following solution:

$$f(x)=\frac{1}{2\pi i}\lim_{T\to\infty}\int_{\frac{1}{2}-iT}^{\frac{1}{2}+iT}\frac{\prod_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_j-\beta_j+\beta_j s)\prod_{j=n+1}^{p_i}\Gamma(c_j-\alpha_j+\alpha_j s)\right\}}{\prod_{j=1}^{m}\Gamma(b_{j_i}-\beta_{j_i}+\beta_{j_i}s)\prod_{j=1}^{n}\Gamma(1-a_{j_i}-\alpha_{j_i}-\alpha_j s)}$$

$$.x^{-s}P(1-s)ds \quad [3.31]$$

whereM $\{p(x)\} = P(s)$

Now using Parseval's theorem in [3.31], we find the solution finally,

$$f(x)=\sum_{i=1}^{r}\int_0^\infty H_i[ux]p(u)du \quad [3.32]$$

IV. PARTICULAR CASE

If we put r=1 then the integral equations [3.1], [3.2] ,[3.3] and [3.4] reduce to the Quadruple Integral equations involving Fox's H-functions as given below:

$$\int_0^\infty H_{p+n,q+m}^{m,n}\left[ux/\begin{matrix}(a_j,\alpha_j)\\(b_j,\beta_j)\end{matrix}\right]f(u)du=\phi_1(x) \quad [4.1]$$

where $x\in(0,a)$

$$\int_0^\infty H_{p+n,q+m}^{m,n}\left[ux/\begin{matrix}(c_j,\alpha_j)\\(b_j,\beta_j)\end{matrix}\right]f(u)\,du=\phi_2(x) \quad [4.2]$$

where $x\in(a,b)$

$$\int_0^\infty H_{p+n,q+m}^{m,n}\left[ux/\begin{matrix}(c_j,\alpha_j)\\(b_j,\beta_j)\end{matrix}\right]f(u)\,du=\phi_3(x) \quad [4.3]$$

where $x\in(b,c)$

$$\int_0^\infty H_{p+n,q+m}^{m,n}\left[ux/\begin{matrix}(c_j,\alpha_j)\\(d_j,\beta_j)\end{matrix}\right]f(u)\,du=\phi_4(x) \quad [4.4]$$

where $x\in(c,\infty)$

where $0<a<1$ and $1<c<\infty$ and $\phi_1(x)$, $\phi_2(x)$, $\phi_3(x)$ and $\phi_4(x)$ are prescribed Functions.

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The solution of Quadruple Integral equations [4.1],[4.2],, [4.3] and [4.4] involving Fox's H-functions is obtained by putting r =1 in solution [3.32] and so ,far solution obtained is,

$$f(x) = \int_0^\infty H[ux]p(u)du \quad [4.5]$$

V. CONCLUSION

In this paper , we obtain the solutions of quadruple integral equations involving I-function method using by fractional integration.The given quadruple integral equations have been transformed by the application of fractional Erdelyi-Kober operators to four others integral equations with a common Kernel.

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