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GLOBAL DYNAMICS OF HIGHLY PATHOGENIC AVIAN INFLUENZA EPIDEMIC MODEL WITH VERTICAL TRANSMISSION FUNCTION

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ABSTRACT

In this study, the highly pathogenic avian influenza epidemic model with vertical transmission function in both poultry and human being is investigated. The basic reproduction number R_0 for the model is defined. The equilibriums are established and show that they are global asymptotically stable. Finally a numerical example is also included to illustrate the effectiveness of the proposed model.

***Keywords:* Avian Influenza, Lyapunov Function, Stability, The Basic Reproduction Number, Vertical Transmission function.**

I. INTRODUCTION

The avian influenza (avian flu or bird flu) refers to the influenza that is caused by viruses adapted to birds. Avian influenza A viruses are classified into two categories: low pathogenic avian influenza A (LPAI) and highly pathogenic avian influenza A (HPAI).Avian influenza is a zoonotic disease caused by the transmission of the avian influenza A virus, such as H5N1, H7N1, H7N2, H7N3, H7N7, H9N2 and H7N9, from birds to humans. The avian influenza A H5N1 virus has caused more than 500 human infections worldwide with nearly a 60% death rate since it was first reported in Hong Kong in 1997. The first reports of bird flu in India came from the village of Navapur in the Nandurbar district of Maharashtra on 19 February 2006, Villagers reported a large number of bird deaths in the village. Maharashtra State Animal Husbandry Ministry authorities rushed to the spot. Lab analysis proved that the poultry was indeed affected with the H5N1 virus, nawapur. India had reported outbreaks of Highly Pathogenic Avian influenza at various epicenters in Delhi, Gwalior (MP), Rajpura (Punjab), Hissar (Haryana), Bellary (Karnataka), Allappuzha and Kottayam (Kerala), Ahmedabad (Gujarat), Daman (Daman) and Khordha and Angul (Odisha during October, 2016 to February, 2017. Highly pathogenic avian influenza A H5N1 viruses have spread from Asia to Africa and Europe infecting poultry, humans and wild birds. The avian influenza pathogenic has a high death rate, which is about 100 percent for birds and more than 70 percent for humens [1]. Che [2] proposed the highly pathogenic avian influenza epidemic model with saturated contact rate. Liu et al. [3] studied a nonlinear dynamics of avian influenza epidemic models. Tasmi and Nuraini [4] proposed avian influenza model with optimal vaccination and treatment schedules. Zhao et al. [5] considered stability and persistence of an avian influenza epidemic model with impacts of climate change.

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Avian influenza modeling studies involving humans and birds was carried out in Gumel [6] and Iwami [7]. The number of mathematical modeling studies have been carried out to quantify the potential burden of an influenza pandemic in human being and to assess various control strategies considered by et al. [8-17].

In this study global dynamics of highly pathogenic avian influenza epidemic model with vertical transmission function is proposed. Mathematical model is presented in the second section. The disease free equilibrium and the endemic equilibrium are derived in the third section. In the fourth section, stability analysis of the model is investigated by using stability theory of differential equations. The fifth section contains numerical simulation. In the last sixth section we give conclusion.

II. MATHEMATICAL MODEL

2.1 Basic Model.

Shuqin Che et al. [2] has proposed the following four dimensional system of autonomous differential equation model for the avian influenza

$$\begin{aligned}
\frac{dX}{dt} &= c - \frac{\omega XY}{1+\delta Y} - dX, \\
\frac{dY}{dt} &= \frac{\omega XY}{1+\delta Y} - (d+m)Y, \\
\frac{dS}{dt} &= b - \frac{\beta SY}{1+\delta Y} - \alpha S, \\
\frac{dI}{dt} &= \frac{\beta SY}{1+\delta Y} - (\varepsilon+\alpha+\gamma)I, \\
\frac{dR}{dt} &= \gamma I - \alpha R
\end{aligned} \quad \ldots$$

(2.1)

here $X(t)$ and $Y(t)$ are the numbers of susceptible poultry and infected poultry of birds respectively, $S(t)$, $I(t)$ and $R(t)$ the number of susceptible, infected and recovered of human being respectively. The parameters c and b are respectively the natural birth rate of Avian and human being. d and α are respectively the natural mortality of poultry and human being. m and ε are respectively the poultry and human mortality due to illness. ω stands for infectious rate of susceptible poultry to infected poultry, β stands for infected poultry of the infection rate of susceptible individuals, γ is the recovery rate that infects individuals through treatment. When Y is small, the contact ratio, infected poultry and susceptible poultry, is appromately proportional to the Y : with the increase of Y, the contact rate gradually reaches saturation. When Y is very largy, it is close to a constant ω/δ. The same way to explain $\beta/(1+\delta Y)$, that is to say, δ is a parameter which is effects of infectious diseases, when the contact rate of the disease is saturated.

2.2 Model with Vertical Transmission Function

The model (2.1) with vertical transmission function in both poultry and human being is given by

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$$\frac{dX}{dt} = c - \frac{\omega XY}{1+\delta Y} - dX - pcY,$$
$$\frac{dY}{dt} = \frac{\omega XY}{1+\delta Y} - (d+m)Y + pcY,$$
$$\frac{dS}{dt} = b - \frac{\beta SY}{1+\delta Y} - \alpha S - qbI,$$
$$\frac{dI}{dt} = \frac{\beta SY}{1+\delta Y} - (\varepsilon+\alpha+\gamma)I + qbI,$$
$$\frac{dR}{dt} = \gamma I - \alpha R.$$

(2.2)

where p, q are suitable constant. The rest of the parameters have similar meaning as for as the model (2.1).

III. EQUILIBRIA OF THE SYSTEM

The first four equation of system (2.2) do not contain R, by the method of Vanden Driessche and Watmough Diekmann [10]

$$\begin{aligned}\frac{dX}{dt} &= c - \frac{\omega XY}{1+\delta Y} - dX - pcY,\\ \frac{dY}{dt} &= \frac{\omega XY}{1+\delta Y} - (d+m)Y + pcY,\\ \frac{dS}{dt} &= b - \frac{\beta SY}{1+\delta Y} - \alpha S - qbI,\\ \frac{dI}{dt} &= \frac{\beta SY}{1+\delta Y} - (\varepsilon+\alpha+\gamma)I + qbI.\end{aligned} \qquad (3.1)$$

It can be checked that the system (3.1) has two non-negative equilibrium and one of them disease free equilibrium $E_0\left(X^o, Y^o, S^o, I^o\right) = \left(\frac{c}{d}, 0, \frac{b}{\alpha}, 0\right)$ we can get the basic reproductive numbers of the system (3.1), $R_0 = \frac{c\omega}{d(d+m-pc)}$

Existence of $E_+\left(X^*, Y^*, S^*, I^*\right)$

Here X^*, Y^*, S^*, I^* are the positive solution of the following algebraic equation,

$$c - \frac{\omega XY}{1+\delta Y} - dX - pcY = 0,$$
$$\frac{\omega X}{1+\delta Y} - (d+m) + pc = 0,$$
$$b - \frac{\beta SY}{1+\delta Y} - \alpha S - qbI = 0,$$
$$\frac{\beta SY}{1+\delta Y} - (\varepsilon+\alpha+\gamma)I + qbI = 0.$$

(3.2) Solving (3.2) we get

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$$X^* = \frac{(d+m+c\delta)(d+m-pc)}{(d+m)(d\delta+\omega)-pcd\delta}, \qquad Y^* = \frac{c\omega - d(d+m-pc)}{(d+m)(d\delta+\omega)-pcd\delta}$$

$$S^* = \frac{b(1+\delta Y^*)(\varepsilon+\alpha+\gamma-qb)}{[\beta Y^* + \alpha(1+\delta Y^*)](\varepsilon+\alpha+\gamma-qb)+qb\beta Y^*}, \quad I^* = \frac{b\beta Y^*}{[\beta Y^* + \alpha(1+\delta Y^*)](\varepsilon+\alpha+\gamma-qb)+qb\beta Y^*}.$$

Where $(d+m) > pc, (d+m)(\omega+d\delta) > pcd\delta, (\varepsilon+\alpha+\gamma) > qb.$

Theorem 3.1. If $R_0 \le 1,$ the system (2.2) only exists the disease-free equilibrium $E_0\left(\frac{c}{d}, 0, \frac{b}{\alpha}, 0\right),$ when $(d+m) > pc, (d+m)(\omega+d\delta) > pcd\delta$ and $R_0 > 1,$ there exists only one endemic equilibrium

$$E_+\left(\frac{(d+m+c\delta)(d+m-pc)}{(d+m)(d\delta+\omega)-pcd\delta}, Y^*, \frac{b(1+\delta Y^*)(\varepsilon+\alpha+\gamma-qb)}{[\beta Y^* + \alpha(1+\delta Y^*)](\varepsilon+\alpha+\gamma-qb)+qb\beta Y^*}, \frac{b\beta Y^*}{[\beta Y^* + \alpha(1+\delta Y^*)](\varepsilon+\alpha+\gamma-qb)+qb\beta Y^*}\right)$$

where

$$Y^* = \frac{c\omega - d(d+m-pc)}{(d+m)(d\delta+\omega)-pcd\delta}$$

IV. LINEAR STABILITY ANALYSIS

Theorem 4.1 The disease free equilibrium E_0 is locally asymptotically stable if $R_0 \le 1,$ and disease free equilibrium E_0 is unstable if $R_0 > 1.$

Proof. The Jacobian matrix of system (3.1) is

$$J = \begin{bmatrix} -d - \frac{\omega Y}{1+\delta Y} & -\frac{\omega X(1+\delta Y) - \delta\omega XY}{(1+\delta Y)^2} - pc & 0 & 0 \\ \frac{\omega Y}{1+\delta Y} & \frac{\omega X(1+\delta Y) - \delta\omega XY}{(1+\delta Y)^2} - (d+m-pc) & 0 & 0 \\ 0 & -\frac{\beta S(1+\delta Y) - \delta\beta SY}{(1+\delta Y)^2} & -\frac{\beta Y}{1+\delta Y} - \alpha & -qb \\ 0 & \frac{\beta S(1+\delta Y) - \delta\beta SY}{(1+\delta Y)^2} & \frac{\beta Y}{1+\delta Y} & -(\varepsilon+\alpha+\gamma-qb) \end{bmatrix}$$

Now, Jacobian matrix of system (3.1) at $E_0\left(\frac{c}{d}, 0, \frac{b}{\alpha}, 0\right),$ is

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$$J_{E_0}=\begin{bmatrix} -d & -\frac{\omega c}{d}-pc & 0 & 0 \\ 0 & \frac{\omega c}{d}-(d+m-pc) & 0 & 0 \\ 0 & -\frac{\beta b}{\alpha} & -\alpha & -qb \\ 0 & \frac{\beta b}{\alpha} & 0 & -(\varepsilon+\alpha+\gamma-qb) \end{bmatrix}$$

The characteristic equation is

$$\begin{vmatrix} -d-\lambda & -\frac{\omega c}{d} & 0 & 0 \\ 0 & \frac{\omega c}{d}-(d+m-pc)-\lambda & 0 & 0 \\ 0 & -\frac{\beta b}{\alpha} & -\alpha-\lambda & -qb \\ 0 & \frac{\beta b}{\alpha} & 0 & -(\varepsilon+\alpha+\gamma-qb)-\lambda \end{vmatrix}=0$$

$$(d+\lambda)(\alpha+\lambda)(\varepsilon+\alpha+\gamma-qb+\lambda)\left[\left\{\frac{\omega c}{d}-(d+m-pc)\right\}-\lambda\right]=0 \qquad (4.1)$$

The roots of (4.1) are $-d$, $-\alpha$, $-(\varepsilon+\alpha+\gamma-qb)$, $\frac{\omega c}{d}-(d+m-pc)$

The first three roots having negative real parts and fourth root $\frac{\omega c}{d}-(d+m-pc)$ will have negative real part if $R_0 \le 1$. Thus all roots of (4.1) have negative real parts so E_0 is locally asymptotically stable if $R_0 \le 1$, and the root $\frac{\omega c}{d}-(d+m-pc)$ will have positive real part if $R_0 > 1$, so E_0 is an unstable.

Theorem 4.2 The disease free equilibrium E_0 is globally asymptotically stable if $R_0 \le 1$..

Proof. Consider the Lyapunov function

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$$
\begin{aligned}
L_1 &= X - X^{\circ}\ln X + Y \\
&= X' - \frac{X^{\circ}}{X}X' + Y' \\
&= \left(1-\frac{X^{\circ}}{X}\right)X' + Y' \\
&= \left(1-\frac{X^{\circ}}{X}\right)\left[c - \frac{\omega XY}{1+\delta Y} - dX - pc\right] + \frac{\omega XY}{1+\delta Y} - (d+m-pc)Y \\
&= \left(1-\frac{X^{\circ}}{X}\right)\left[dX^{\circ} - dX - \frac{\omega XY}{1+\delta Y} - pcY\right] + \frac{\omega XY}{1+\delta Y} - (d+m-pc)Y \\
&\le -\frac{d(X-X^{\circ})^2}{X} + (d+m-pc)Y\left(\frac{\omega X^{\circ}}{(d+m-pc)} - 1\right) \\
&= -\frac{d(X-X^{\circ})^2}{X} + (d+m-pc)Y\left(R_0 - 1\right).
\end{aligned}
$$

When $R_0 \le 1,$ we can get $L_1' \le 0$ and $L_1' = 0$ has no other closed trajectory in addition to E_0 is globally asymptotically stable if and only if $R_0 \le 1.$

Theorem 4.3 The endemic equilibrium E_+ is locally asymptotically stable if $R_0 > 1.$

Proof. The Jacobian matrix of system (3.1) at $E_+(X^*, Y^*, S^*, I^*)$ is

$$
J_{E_+} = \begin{bmatrix}
-d - \frac{\omega Y^*}{1+\delta Y^*} & -\frac{\omega X^*(1+\delta Y^*) - \delta\omega X^* Y^*}{(1+\delta Y^*)^2} - pc & 0 & 0 \\
\frac{\omega Y^*}{1+\delta Y^*} & \frac{\omega X^*(1+\delta Y^*) - \delta\omega X^* Y^*}{(1+\delta Y^*)^2} - (d+m-pc) & 0 & 0 \\
0 & -\frac{\beta S^*(1+\delta Y^*) - \delta\beta S^* Y^*}{(1+\delta Y^*)^2} & -\frac{\beta Y^*}{1+\delta Y^*} - \alpha & -qb \\
0 & \frac{\beta S^*(1+\delta Y^*) - \delta\beta S^* Y^*}{(1+\delta Y^*)^2} & \frac{\beta Y^*}{1+\delta Y^*} & -(\varepsilon + \alpha + \gamma - qb)
\end{bmatrix}
$$

$$
J_{E_+} = \begin{pmatrix} A & 0 \\ B & C \end{pmatrix}
$$

where

$$
A = \begin{bmatrix}
-d - \frac{\omega Y^*}{1+\delta Y^*} & -\frac{\omega X^*(1+\delta Y^*) - \delta\omega X^* Y^*}{(1+\delta Y^*)^2} - pc \\
\frac{\omega Y^*}{1+\delta Y^*} & \frac{\omega X^*(1+\delta Y^*) - \delta\omega X^* Y^*}{(1+\delta Y^*)^2} - (d+m-pc)
\end{bmatrix}, B = \begin{bmatrix}
0 & -\frac{\beta S^*(1+\delta Y^*) - \delta\beta S^* Y^*}{(1+\delta Y^*)^2} \\
0 & \frac{\beta S^*(1+\delta Y^*) - \delta\beta S^* Y^*}{(1+\delta Y^*)^2}
\end{bmatrix},
$$

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$$C=\begin{bmatrix} -\frac{\beta Y^*}{1+\delta Y^*}-\alpha & -qb \\ \frac{\beta Y^*}{1+\delta Y^*} & -(\varepsilon+\alpha+\gamma-qb) \end{bmatrix}, O=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Thus J_{E_+} evaluated is stable if and only if so are A and C. The characteristic equations of the matrix A is

$$\lambda^2+h_1\lambda+h_2=0,$$

where

$$h_1=d+(d+m-pc)\frac{\delta Y^*}{1+\delta Y^*}+\frac{\omega Y^*}{1+\delta Y^*}, h_2=[(d+m)(\omega+d\delta)-pcd\delta]\frac{Y^*}{1+\delta Y^*}$$

here $1+\delta Y^*>0$ when $R_0>1$, $(d+m)(\omega+d\delta)>pcd\delta$, so when $(d+m)>pc$, $(d+m)(\omega+d\delta)>pcd\delta$ and $R_0>1$ then $h_1,h_2>0$ by the Hurwitz criterion. The characteristic roots of matrix A have negative real parts.

The characteristic equation of the matrix C is

$$\left(\lambda+\frac{\beta Y^*}{1+\delta Y^*}+\alpha\right)(\lambda+\varepsilon+\alpha+\gamma)=0.$$

The characteristic roots of C are, $-\frac{\beta Y^*}{1+\delta Y^*}-\alpha, -(\varepsilon+\alpha+\gamma).$

When $R_0>1$, The characteristic roots of C have negative real parts. So all characteristic roots of the Jacobian matrix J_{E_+} have negative real parts if and only if $R_0>1$. Thus the endemic equilibrium E_+ is locally asymptotically stable if $R_0>1$.

Theorem 4.4 The endemic equilibrium E_+ is globally asymptotically stable if $R_0>1$.

Proof: Consider the Lyapunov function

$$L_2=X^*\left(\frac{X}{X^*}-1-\ln\frac{X}{X^*}\right)+Y^*\left(\frac{Y}{Y^*}-1-\ln\frac{Y}{Y^*}\right)$$

Then

$$L_2' = X^*\left(\frac{X'}{X^*} - \frac{X^*}{X}\cdot\frac{X'}{X^*}\right) + Y^*\left(\frac{Y'}{Y^*} - \frac{Y^*}{Y}\frac{Y'}{Y^*}\right)$$

$$L_2' = \left(1 - \frac{X^*}{X}\right)X' + \left(1 - \frac{Y^*}{Y}\right)Y' = C\left(2 - \frac{X^*}{X} - \frac{X}{X^*}\right).$$

By the relationship of arithmetic mean and geometric mean.

We know that

$$2 - \frac{X^*}{X} - \frac{X}{X^*} \le 0.$$

i.e. $L_2' \le 0$, if and only if $(X,Y) = (X^*,Y^*), L_2' = 0.$ Thus by LaSalle invariance principle $E_+(X^*,Y^*,S^*,I^*)$ is globally asymptotically stable.

V. NUMERICAL SIMULATION

In this section we present computer simulation of solution of the system. To illustrate the results numerically, choose $c = 3,\ \beta = 0.04,\ d = 0.06, b = 2,$ $m = 2, \varepsilon = 1.2,\ \gamma = 0.502, \delta = 0.08, \alpha = 0.08,\ p = 0.02, q = 0.03,\ \omega = 0.038$ and $\left(X(0), Y(0), S(0), I(0)\right) = (30,10,15,5).$ Then $E_0 = (50,0,25,0),\ R_0 = 0.95 < 1.$ Therefore by theorem (4.2), E_0 is a globally asymptotically stable. Fig. 1 shows that $X(t)$ approaches to its steady-state value while $Y(t)$ approaches to zero as time progresses.

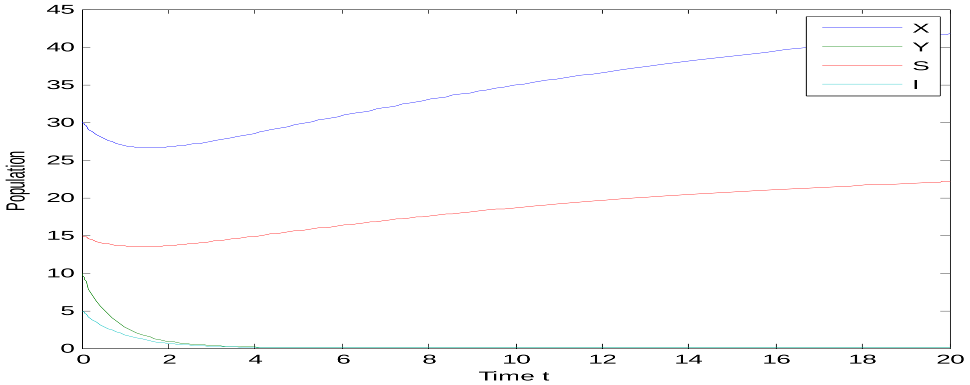

Figure 1. Here $X(0) = 30, Y(0) = 10, S(0) = 15, I(0) = 5, c = 3, \beta = .04, d = .06, b = 2, m = 2, \varepsilon = 1.2,$

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$\omega = .038, \gamma = .502, \delta = .08, \alpha = .08, p = .02, q = .03, R_0 = .95 < 1.$

Similarly Fig. 1 shows that $S(t)$ approaches to its steady-state value while $I(t)$ approaches to zero as time progresses. Finally disease dies out.

Again suppose the parameters as $c = 3,\ \beta = 0.04,\ d = 0.06, b = 2, m = 2, \varepsilon = 1.2,\ \gamma = 0.502, \delta = 0.08,\ \alpha = 0.08,$ $p = 0.02, q = 0.03, \omega = 2,$ and $\left(X(0), Y(0), S(0), I(0)\right) = (30, 10, 15, 5)$. Then

$E_+ = (1.11, 1.42, 15.05, 0.4470),\ R_0 = 50 > 1.$ By theorem (4.4), E_+ is a globally asymptotically stable. Fig. 2 show that $X(t), Y(t), S(t)$ and $I(t)$ approach to their steady-state values as time progresses, the disease will be exist.

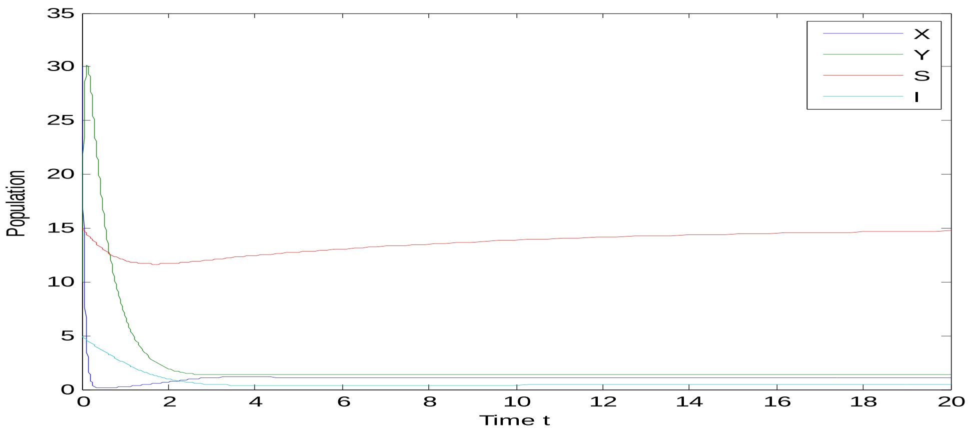

Figure 2. Here $X(0) = 30, Y(0) = 10, S(0) = 15, I(0) = 5, c = 3, \beta = .04, d = .06, b = 2, m = 2, \varepsilon = 1.2,$

$\omega = 2, \gamma = .502, \delta = .08, \alpha = .08, p = .02, q = .03, R_0 = 50 > 1.$

We change the value of p and keeping other parameters fixed, we seen that $I(t)$ increases as p increases. It follows from Fig. 3.

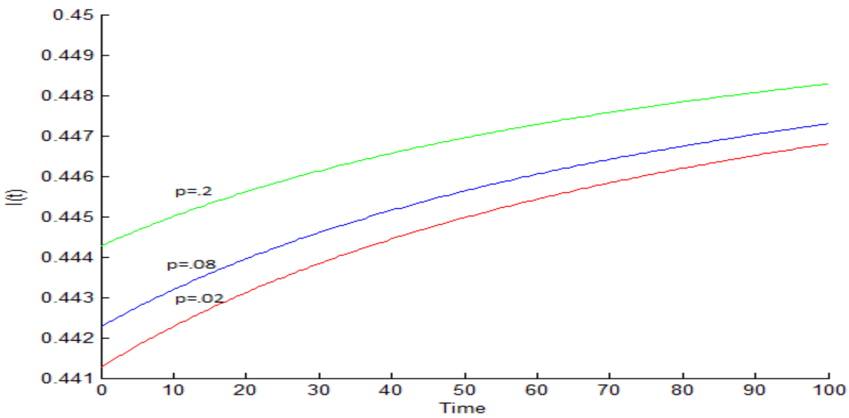

Figure 3. The dependence of $I(t)$ on the parameter p keeping other parameters fixed.

Again we change the value of q and keeping other parameters fixed, we seen that $I(t)$ increases as q increases. It follows from Fig. 4.

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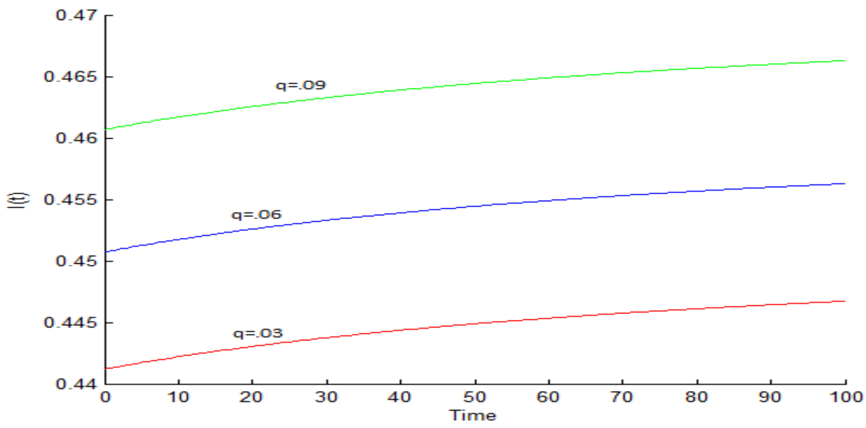

Figure 4. The dependence of $I(t)$ on the parameter q keeping other parameters fixed.

VI. CONCLUSION

In this study we have discussed the global stability of highly pathogenic avian influenza epidemic model with vertical transmission function in poultry and human being. First and second vertical transmission function are taken to represents the interaction between susceptible and infected poultry and human being respectively. Our main aim of mathematical epidemiology is to understand how to control or eradicate diseases. We have proved that if $R_0 \leq 1$ then E_0 is globally asymptotically stable is disease dies out. When $R_0 > 1$ the endemic equilibrium E_+ exists globally stable i.e disease persists. Numerical simulation indicates that when the disease is endemic, the steady state value $I(t)$ of the infectives increases as p, q increases.

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