International Journal of Advance Research in Science and Engineering
Vol. No.6, Issue No. 09 , September 2017
www.ijarse.com
IJARSE
ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

Characterization of S_2 Space Via Ideals

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ABSTRACT

In this paper we will give some characterizations of S_2 mod $\mathfrak{I}$ spaces. Also Examples are given throughout the paper.

***Keywords and phrases*: S_2 mod $\mathfrak{I}$, θ_I-closed.**

2000 **MSC**: 54A05, 54D10, 54D30.

I. INTRODUCTION

In [2], Csàszàr, introduced S_1 and S_2 spaces and discussed some properties of these spaces and in [3], Janković gave various characterizations of S_1 and S_2 spaces using the θ-closure of a set. On the other hand separation axioms with respect to an ideal and various properties and characterizations were also discussed by many authors. Ideals in topological spaces were introduced by Kuratowski[4] and further studied by Vaidyanathaswamy[5]. Corresponding to an ideal a new topology $\tau^*(\mathfrak{I}, \tau)$ called the *-topology was given which is generally finer than the original topology having the kuratowski closure operator $cl^*(A) = A \cup A^*(\mathfrak{I},\tau)$[6], where $A^*(\mathfrak{I},\tau) = \{x \in X : U \cap A \notin \mathfrak{I}$ for every open subset U of x in X called a local function of A with respect to $\mathfrak{I}$ and τ. We will write τ^* for $\tau^*(\mathfrak{I}, \tau)$.

The following section contains some definitions and results that will be used in our further sections.

Definition 1.1.[4]: Let (X, τ) be a topological space. An ideal $\mathfrak{I}$ on X is a collection of non-empty subsets of X such that (a) $\phi \in \mathfrak{I}$ (b) $A \in \mathfrak{I}$ and $B \in \mathfrak{I}$ implies $A \cup B \in \mathfrak{I}$ (c) $B \in \mathfrak{I}$ and $A \subset B$ implies $A \in \mathfrak{I}$.

Definition 1.2.[2]: A topological space (X, τ) is said to be S_2 space if for every pair of distinct points x and y, whenever one of them has a open set not containing the other then there exist disjoint open subsets containing them.

Definition 1.3.: An ideal space $(X,\tau, \mathfrak{I})$ is said to be **S_2 mod $\mathfrak{I}$** if for every pair of distinct points x and y in X, whenever x has a τ- open subset not containing y, there exist open nhds. U and V such that $x \in U$, $y \in V$ and $U \cap V \in \mathfrak{I}$.

Definition 1.4.[1]: Let $(X,\tau,\mathfrak{I})$ be an ideal space. Then for any subset A of X, a point x is said to be in the θ_I closure of A if for every open subset U of x in X, $cl^*(U)\cap A\neq\phi$. The collection of all such points is denoted by $cl_{\theta_I}(A)$. Also A is said to be θ_I closed if $cl_{\theta_I}(A)$=A.

Definition 1.5.[3]: Let (X,τ) be a topological space and $x \in X$ be any element. Then

a) $Ker\{x\} = \cap\{G : G \in \tau(x)\}$, where τ(x) denotes the collection of all open subsets of x.
b) $<x> = cl\{x\}\cap ker\{x\}$.

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II. RESULTS

Theorem 2.1.: Let $(X,\tau,\mathfrak{I})$ be an ideal space. Then in each of the following cases X is S_2 mod $\mathfrak{I}$.

(i) $\langle x\rangle = cl_{\theta_I}\{x\}$ for every $x\epsilon X$.

(ii) $\langle x\rangle$ is θ_I closed for every $x\epsilon X$.

(iii) $cl_{\theta_I}\{x\}$ =ker{x} for every $x\epsilon X$.

(iv) cl{x} is θ_I closed for every $x\epsilon X$.

(v) Ker{x} is θ_I closed for every $x\epsilon X$.

(vi) If U be any open subset of x in X then $cl_{\theta_I}\{x\}\subset U$.

Proof: Let x,y be two distinct points of X such that y has a open set not containing x then y $\notin$cl{x}and so y $\notin\langle x\rangle$. Therefore by (i) y $\notin cl_{\theta_I}\{x\}$ and so y has a open set H such that x $\notin$cl*(H) which means $x\notin H\cup H^*$. Hence there exist open set G such that x ϵ G and $G\cap H\ \epsilon\ \mathfrak{I}$. Now (ii) implies that $cl_{\theta_I} < x >=\langle x\rangle$ and so y$\notin$ $cl_{\theta_I} < x >$. Further, similarly by (i), X is S_2 mod $\mathfrak{I}$, since $x\epsilon\langle x\rangle$. In (iii)y$\notin$cl{x} implies that y has a open set not containing x and so x$\notin$ker{y}. Therefore, by (iii) x$\notin$ $cl_{\theta_I}\{y\}$. Hence X is S_2 mod $\mathfrak{I}$. Further in (iv) y$\notin$cl{x} implies that y$\notin$ $cl_{\theta_I}(cl\{x\})$ and so y$\notin$ $cl_{\theta_I}\{x\}$, since $x\epsilon$cl{x}. Hence the result follows by (i). In (v) y $\notin$cl{x} implies that y has a open set not containing x and so x$\notin$ker{y}. Therefore, x$\notin$ $cl_{\theta_I}(\ker\{y\})$ and so x$\notin$ $cl_{\theta_I}\{y\}$, since y ϵ ker{y} and in (vi) y $\notin$cl{x} implies that y ϵ X-cl{x} which means that X-cl{x} is the open set containing y and so $cl_{\theta_I}\{y\}\subset$ X-cl{x}. Therefore, x$\notin$ $cl_{\theta_I}\{y\}$ and hence X is S_2 mod $\mathfrak{I}$.

The following Example 2.2 shows that the converse is not true.

Example 2.2. : Let X={a,b,c}, τ={ϕ, {a}, {b}, {a,b}, X} and $\mathfrak{I}$={ ϕ,{a},{b},{a,b}} and so $\tau^*=\wp(X)$. Then it can be seen easily that X is S_2 mod $\mathfrak{I}$. Further,(X, τ^*) is discrete topological space and hence T_2 space. Therefore, $cl_{\theta_I}\{x\}$=cl{x} for every $x\epsilon X$, since cl*(U)=U for every open subset U of X. Now for the subset A={a}, $cl_{\theta_I}\{a\}$ = cl{a}={a,c}, ker{a}={a} and so $\langle a\rangle$={a}. Therefore, (i) $\langle a\rangle\neq cl_{\theta_I}\{a\}$, (ii) $cl_{\theta_I} < a >$ $=cl_{\theta_I}\{a\}$={a,c}$\neq\langle a\rangle$ and so $\langle a\rangle$ is not θ_I closed, (iii) $cl_{\theta_I}\{a\}$={a,c}$\neq${a}=ker{a}, (iv) $cl_{\theta_I}(\ker\{a\})$= $cl_{\theta_I}\{a\}$={a,c}$\neq$ker{a} and hence ker{a} is not θ_I closed and (v) a ϵ{a} is open set but $cl_{\theta_I}\{a\}$={a,c} $\not\subset$ {a}.

Theorem 2.3.: Let $(X,\tau,\mathfrak{I})$ be S_2 mod $\mathfrak{I}$ space and X is S_1. Then $x\epsilon$ $cl_{\theta_I}\{y\}$ if and only if y ϵ $cl_{\theta_I}\{x\}$.

Proof: Let $x\epsilon$ $cl_{\theta_I}\{y\}$ then to prove y ϵ $cl_{\theta_I}\{x\}$. Let if possible, y$\notin$ $cl_{\theta_I}\{x\}$ then by the proof of part (i) of Theorem 2.1 there exist open set G and H such that x ϵ G, y ϵ H and $G\cap H\epsilon\mathfrak{I}$. Also x $\notin$ H implies that y has a open set not containing x and so X is S_1 implies that x has a open set W not containing y. Further U=G$\cap$W is the open set containing x but not y. Also $U\cap V\epsilon\mathfrak{I}$ implies that y $\notin$ cl*(U) and so x $\notin$ $cl_{\theta_I}\{y\}$. Hence $x\epsilon$ $cl_{\theta_I}\{y\}$ if and only if y ϵ $cl_{\theta_I}\{x\}$.

Remark 2.4.: In the above Theorem 2.3 if the space is not S_1 then the result need not be true. In the above Example 2.2 $(X,\tau,\mathfrak{I})$ is S_2 mod $\mathfrak{I}$ but not S_1, since a has a open set {a} not containing c but X is the only open subset containing c. And c ϵ $cl_{\theta_I}\{a\}$ but a $\notin$ $cl_{\theta_I}\{c\}$.

Even though we have seen that in Example 2.2 that if the space is S_2 mod $\mathfrak{I}$ then the converse of Theorem 2.1 need not be true. But the following Theorem 2.5 shows that if we assume S_1 space then the converse of Theorem 2.1 also holds.

Theorem 2.5.: Let $(X,\tau,\mathfrak{I})$ be S_2 mod $\mathfrak{I}$ space and X is S_1. Then each of the following is true.

IJARSE
ISSN (O) 2319 - 8354
ISSN (P) 2319 - 8346

(i) <x> = $cl_{\theta_I}\{x\}$ for every xϵX.

(ii) <x> is θ_I closed for every xϵX .

(iii) $cl_{\theta_I}\{x\}$ =ker{x} for every xϵX .

(iv) cl{x} is θ_I closed for every xϵX .

(v) Ker{x} is θ_I closed for every xϵX.

(vi) If U be any open subset of x in X then $cl_{\theta_I}\{x\} \subset$U.

Proof: (i) Let x ϵ X be any element. Then <x> =cl{x}$\cap$ker{x} $\subset$cl{x} $\subset cl_{\theta_I}\{x\}$. Conversely, let y $\notin$ <x> then either y$\notin$cl{x} or y $\notin$ ker{x}. Now y$\notin$cl{x} and X is S_2 mod $\mathfrak{I}$ implies that there exist open subset W such that y ϵW but x $\notin$ W and U$\cap$W $\epsilon\mathfrak{I}$ where U and W are open sets containing x and y respectively and so U$\cap$G$\epsilon\mathfrak{I}$ where G=V$\cap$W and hence (i) holds. Also in case of y$\notin$ker{x}, X is S_1 implies that x$\notin$ker{y} which means that y has a open set not containing x and so y $\notin$ cl{x}. Hence (i) holds. The proof of (ii) and (iii) follows from(i) and hence is omitted. (iv) Let x ϵ X be any element then we will prove that cl{x} is θ_I closed. As cl{x}$\subset cl_{\theta_I}(cl\{x\})$, so we will prove that $cl_{\theta_I}(cl\{x\})\subset$cl{x}. Let y$\notin$cl{x}, then there exist open sets G and H such that xϵG, yϵH and x $\notin$H such that G$\cap$H ϵ $\mathfrak{I}$. Then for any z ϵ cl{x} and z ϵ H implies that x ϵ H, which is not possible. Also z $\notin$ H*, since z ϵ cl{x} and X is S_1 implies that x ϵ cl{z} and so z ϵ G. But G$\cap$H ϵ $\mathfrak{I}$ implies that z $\notin$ H*. Hence cl*(H)$\cap$cl{x}=ϕ implies that y $\notin$ $cl_{\theta_I}(cl\{x\})$. The proof of (v) and (vi) is also obvious and hence is omitted.

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