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Optimal inventory model with single item under various demand conditions

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ABSTRACT

The goal of this model is to talk about the stock model for time fluctuating demand and constant demand with time dependent holding expense and consistent holding cost for case 1 and case 2 separately. Scientific model has been created for deciding the optimal request amount, the optimal process duration and optimal aggregate stock expense for both the cases. Numerical illustrations are given for both cases to approve the proposed model. Sensitivity analysis has been done to analyze the effect of changes in the optimal solution with respect to change in various parameters.

Keywords: holding cost, Inventory system, time dependent demand.

I. INTRODUCTION

Inventory management aims to minimize the inventory carrying cost. Previously EOQ models are constructed assuming demand rate as constant. It is watched that demand for a specific item can be affected by internal factors, for example, value, time and accessibility. Demand elasticity means alteration in demand to the inventory. Thus, when the demand rate is constant, marketing decisions plays an important role in Inventory management.

Different models have been discussed for constant demand rate with constant holding cost. An EOQ model was talked about by Teng et al. [16] on ideal valuing and requesting strategy under allowable postponement in installments was viewed as that the offering cost is unavoidably higher than the buying cost. A suitable model was given by them in which retailer discovered its ideal cost and parcel size, at the same time, when the supplier offered an allowable postponement in installment. A consistent rate EOQ model was given by Muhlemann and Valtis-Spanopoulos [11] with variable holding cost where rate of the normal estimation of capital investigated in stock. An EOQ inventory model was developed by Vander Veen [18] where holding cost is taken as a nonlinear function of inventory. Weiss [19] established an EOQ model where the holding cost is a nonlinear function of length of time an item was held in stock. Goh [6] found an EOQ model with general request and holding cost capacity, demand rate for a thing was considered as an element of existing stock level and conveying cost per unit was allowed to change.

Alfares [1] displayed the stride arrangement of the holding cost in perspective of the stock approach where request rate relies upon stock level and holding cost relies on capacity time. The holding cost per unit thing per

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unit time is taken as expanding capacity of time spent in storage. In a portion of the stock model, it is expected that the holding cost changes with time. An EOQ model was given by Giri et al. [5] for crumbling (deteriorating) things with deficiencies, where both the interest rate and the holding expense are considered as continuous function of time. Datta and Pal [3] talked about deterministic stock model without deficiency where time horizon is boundless and has a level-dependent demand rate up to a specific stock-level and a consistent interest for whatever is left of the cycle. Pal et al. [12] develop a deterministic stock model in which the interest rate was stock-dependent and that the things crumble at a steady rate.

Discounted cash flow approach in inventory was introduced by Dye et al. [4]. They found the ideal stock and evaluating methodologies amplifying the net present estimation of the aggregate benefit over the infinite horizon. Utilizing DCF approach Chung and Liao [2] built up an ideal requesting arrangement of EOQ model under exchange credit depending upon the ordering amount. They examined the ideal request amount of the EOQ model which is reliant on the stock strategy and in addition firm credit arrangement utilizing marked down income (DCF) approach and exchange credit contingent upon the amount requested.

Researchers like Jaggi et al [9], Hsu [8], and Roy et al. [13] created stock lot–size models under exchange credit financing in which the interest rate is consistent. An EOQ model with exchange credit financing was given by Teng et al. [17] for non–decreasing request and ideal arrangement. An EOQ model was built up by Sarkar [14] where delay in installments and time shifting weakening rate was considered and where the retailers were permitted an exchange credit offer by suppliers to purchase more things with various rebate rates on the acquiring cost. Hung [7] found a stock model with summed up interest, deterioration and backorder rates. Hung [7] augmented the model from incline sort request rate and Weibull deterioration rate to arbitrary demand rate and subjective weakening rate in considering partial backorder. Khanra et al. [10] set up an EOQ model for crumbling thing with time–dependent quadratic interest under allowable deferral in installment where an exertion has been made to analyze an EOQ model for deteriorating down thing. Sana [15] set up an EOQ model where time horizon is boundless, ideal offering cost and parcel size with time changing decay and partial backlogging.

In the present section, the interest rate is time changing and holding expense is steady for case 1; and in case 2 the condition is vice-versa. The target of this part is to acquire least aggregate stock cost, request amount and relating request cycle for both cases. An algorithm that minimizes the aggregate stock expense is produced. The Model is tried with Numerical examples.

II. ASSUMPTIONS AND NOTATIONS

The model needs the accompanying suppositions:

i. Inventory utilized are of one sort.
ii. Lead time is zero.
iii. Shortages are not permitted.
iv. The request rate $R(t)$ is decreasing function of time with increase of β for case 1.
v. The annual demand rate λ is constant for case 2.

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vi. The holding expense is time dependent and holding cost parameter h is given by $h(t) = h.t$ for case 2.

vii. The holding expense is steady for case 1.

We utilize the accompanying notations:

$I(t)$ = On hand inventory level at any time t, where $t \geq 0$.

T = The length of cycle time.

A = The requesting cost per unit time

λ = The consistent yearly request rate.

h = The stock holding expense of the thing of case 2

$h(t)$ = The time dependent holding cost for case 1.

$R(t)$ = The time changing interest rate given by $R(t) = \lambda\, t^{-\beta} \quad where\ \lambda > 0,\ 0 < \beta < 1$. Here β is the demand parameter.

U = Total inventory cost per cycle

Q = Ordering quantity

III. FORMULATION

Case: 1

At time t ≥ 0 the inventory level on-hand is $I(t)$. The demand rate is always assumed to be positive. Due to effect of demand the amount of stock reduces in the interval $[0,T]$, and at time T the stock achieves zero. Consequently, the stock level at any moment of time is given as follows.

The inventory on-hand at time $t + \Delta t$ in $[0,T]$ will be:

$I(t + \Delta t) = I(t) - d(t).\Delta t\ ,\ t + \Delta t \in [0,T]$

Dividing by Δt and then taking limit $\Delta t \to 0$ we get

(3.1) $$\frac{dI(t)}{dt} = -\lambda . t^{-\beta}\ ; \quad 0 \leq t \leq T$$

under the boundary condition

(3.2) $$I(T) = 0.$$

We have, solution of the differential equation (3.1) is found to be:

(3.3) $$I(t) = \frac{\lambda}{1-\beta}.\left(T^{1-\beta} - t^{1-\beta}\right)$$

The ordering quantity of stock Q is given by

(3.4) $$Q = \frac{\lambda\, T^{1-\beta}}{1-\beta} \quad where \quad 0 < \beta < 1.$$

From (3.4), we obtain

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(3.5) $$T=\left[\frac{(1-\beta)Q}{\lambda}\right]^{1/(1-\beta)} .$$

Now the average total cost per cycle is given by

(3.6) $$U(Q)=\frac{1}{T}\text{[Ordering Cost + Holding Cost]}$$

$$=\frac{1}{T}\left[A+h\int_0^T I(t)\,dt\right]$$

$$=\frac{A}{T}+\frac{h\,\lambda}{T(1-\beta)}\int_0^T\left(T^{1-\beta}-t^{1-\beta}\right)dt=\frac{A}{T}+\frac{h\,\lambda\,T^{1-\beta}}{2-\beta}$$

Using (3.5), we obtain

(3.7) $$U(Q)\;=\frac{A}{T}+\frac{h\,\lambda\,T^{1-\beta}}{2-\beta}=A\left[\frac{\lambda}{(1-\beta)}\right]^{1/(1-\beta)}Q^{\frac{-1}{(1-\beta)}}+\frac{h(1-\beta)Q}{(2-\beta)} .$$

The necessary condition for minimization of $U(Q)$ is,

(3.8) $$\frac{\partial U(Q)}{\partial Q}=0 .$$

The sufficient condition for minimization of $U(Q)$ is

(3.9) $$\frac{\partial^2 U(Q)}{\partial Q^2}>0 .$$

(3.10) $$\frac{\partial U(Q)}{\partial Q}=\frac{-A}{(1-\beta)}\left[\frac{\lambda}{(1-\beta)}\right]^{1/(1-\beta)}Q^{\frac{-(2-\beta)}{(1-\beta)}}+\frac{h(1-\beta)}{(2-\beta)} .$$

(3.11) $$\frac{\partial^2 U(Q)}{\partial Q^2}=\frac{A(2-\beta)}{(1-\beta)^2}\left[\frac{\lambda}{(1-\beta)}\right]^{1/(1-\beta)}Q^{\frac{-(3-2\beta)}{(1-\beta)}} .$$

Now the function $U(Q)$ will be maximum if

(3.12) $$\frac{\partial^2 U(Q)}{\partial Q^2}>0$$ which is obvious from (3.11).

Now from (3.8) we have

(3.13) $$\frac{-A}{(1-\beta)}\left[\frac{\lambda}{(1-\beta)}\right]^{1/(1-\beta)}Q^{\frac{-(2-\beta)}{(1-\beta)}}+\frac{h(1-\beta)}{(2-\beta)}=0 .$$

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Now on solving (3.13), and using in the above equation we gate optimal Q^* which is given by

$$Q^* = \left[\frac{\lambda}{1-\beta}\right]^{\frac{1}{2-\beta}} . \left[\frac{A(2-\beta)}{h(1-\beta)^2}\right]^{\frac{1-\beta}{2-\beta}}$$ and hence the optimal cost $U(Q^*)$ can be evaluated.

If $\beta \to 0,$ then the optimal $Q^* = \sqrt{\frac{2A\lambda}{h}}$

and $U(Q) = \frac{A\lambda}{Q} + \frac{hQ}{2}$.

Case: 2

At time t ≥0 the inventory level on-hand be $I(t)$. The demand rate is always assumed to be constant. The amount of stock reduces in the period $[0,T]$ because of the impact of demand and the stock achieves zero at time T. subsequently, the stock level at any moment of time is depicted as follows.

The inventory on-hand at time $t+\Delta t$, will be:

$$I(t+\Delta t) = I(t) - d(t).\Delta t \ , \ t+\Delta t \in [0,T]$$

Dividing by Δt and taking limit $\Delta t \to 0,$ we get

(3.14) $$\frac{dI(t)}{dt} = -\lambda \ ; \quad t \in [0,T]$$

With the boundary condition

(3.15) $$I(T) = 0.$$

The solution of the differential equation (3.14) is given by,

(3.16) $$I(t) = \lambda(T-t)$$

The stock ordering quantity Q is given by

(3.17) $$Q = I(0) = \lambda T \quad .$$

From (6.3.17), we obtain

(3.18) $$T = \frac{Q}{\lambda}.$$

Now the average total cost per cycle is given by

(3.19) $$U(Q) = \frac{1}{T}\text{[Ordering Cost + Holding Cost]}$$

$$= \frac{1}{T}\left[A + h\int_0^T t\,.\,I(t)\,dt\right]$$

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$$=\frac{A}{T}+\frac{h.\lambda}{T}.\int_0^T t(T-t)\,dt=\frac{A}{T}+\frac{h\lambda T^2}{6}$$

Using (3.18), we obtain

(3.20) $$U(Q)=\frac{A}{T}+\frac{h\lambda T^2}{6}=\frac{A\lambda}{Q}+\frac{hQ^2}{6\lambda}\ .$$

The necessary condition for minimization of $U(Q)$ is,

(3.21) $$\frac{\partial U(Q)}{\partial Q}=0\ .$$

The sufficient condition for minimization of $U(Q)$ is

(3.22) $$\frac{\partial^2 U(Q)}{\partial Q^2}>0\ .$$

(3.23) $$\frac{\partial U(Q)}{\partial Q}=\frac{-A\lambda}{Q^2}+\frac{hQ}{3\lambda}\ .$$

(3.24) $$\frac{\partial^2 U(Q)}{\partial Q^2}=\frac{2A\lambda}{Q^3}+\frac{h}{3\lambda}\ .$$

Now the function $U(Q)$ will be minimum if

(3.25) $$\frac{\partial^2 U(Q)}{\partial Q^2}>0$$ which is obvious from (3.11).

Now from (3.21) we have

(3.26) $$\frac{-A\lambda}{Q^2}+\frac{hQ}{3\lambda}=0\ .$$

Now on solving (3.26), implies minimizing total cost to determine optimal Q^* given by

$Q^*=\left[\frac{3A\lambda^2}{h}\right]^{1/3}$ and hence the optimal cost $U(Q^*)$ can be evaluated.

IV. COMPUTATIONAL ALGORITHM

Step-1: Start.

Step-2: Initialize the value of the variables A, λ, h, β.

Step-3: Evaluate $U(Q)$.

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Step-4: Evaluate $\frac{\partial U(Q)}{\partial Q}$.

Step-5: Solve the equation $\frac{\partial U(Q)}{\partial Q}=0.$

Step-6: Choose the solution from Step-5.

Step-7: Evaluate $\frac{\partial^2 U(Q)}{\partial Q^2}$.

Step-8: If the value of Step-7 is greater than zero then this solution is Optimal (minimum) and move to Step-10.

Step-9: Otherwise move to Step-6.

Step-10: End.

V. NUMERICAL EXAMPLE

The estimations of the parameters in legitimate units are considered as below:

$$\lambda = 500,\ A = 400,\ \beta = 0.2, h = 40$$

For case-1

Optimal $Q^* = 157.5099, U^* = 5040.3162\ , T^* = 0.1786$.

For case-2

Optimal $Q^* = 195.7434, U^* = 1532.6189\ , T^* = 0.3915$.

VI. SENSITIVITY ANALYSIS

TABLE : 01(CASE 1)

β	Q^*	U^*	T^*
0.2	157.5099	5040.3162	0.1786
0.3	205.5748	5756.0957	0.1688
0.4	277.6152	6662.7654	0.1601
0.5	391.4868	7829.7353	0.1533
0.6	584.7066	9355.3062	0.1496

TABLE : 2

	CASE 1			CASE 2		
h	Q^*	U^*	T^*	Q^*	U^*	T^*
40	157.5099	5040.3162	0.1786	195.7434	1532.6189	0.3915
45	149.4766	5381.1593	0.1673	188.2072	1593.9879	0.3764
50	142.6385	5705.539	0.1577	181.7121	1650.9636	0.3634

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55	136.7225	6015.7892	0.1496	176.0298	1704.2569	0.3521
60	131.5361	6313.7336	0.1425	170.9976	1754.4106	0.342

TABLE : 3

	CASE 1			CASE 2		
A	Q^*	U^*	T^*	Q^*	U^*	T^*
400	157.5099	5040.3162	0.1786	195.7434	1532.6189	0.3915
420	160.9627	5150.8066	0.1835	198.9529	1583.2896	0.3979
440	164.3253	5258.411	0.1883	202.062	1633.1621	0.4041
460	167.6041	5363.3310	0.1930	205.0783	1682.2843	0.4102
480	170.8046	5465.746	0.1976	208.0084	1730.6995	0.416

Important points from the table

- The effect of optimality due to change of values of different parameters associated in this model is discussed below.

  For Case: 1

1. Q & U increase as T decreases for increase in value of the parameter β .
2. U increases while Q & T decrease with increase in value of the parameter h .
3. Q, U & T increase with increase in value of the parameter A .

For Case: 2

1. U increases while Q & T decrease with increase in value of the parameter h .
2. Q, U & T increase with increase in value of the parameter A .

Variation of Time duration, Ordering Quantity and Total cost w.r.t. different parameters.

CASE:1

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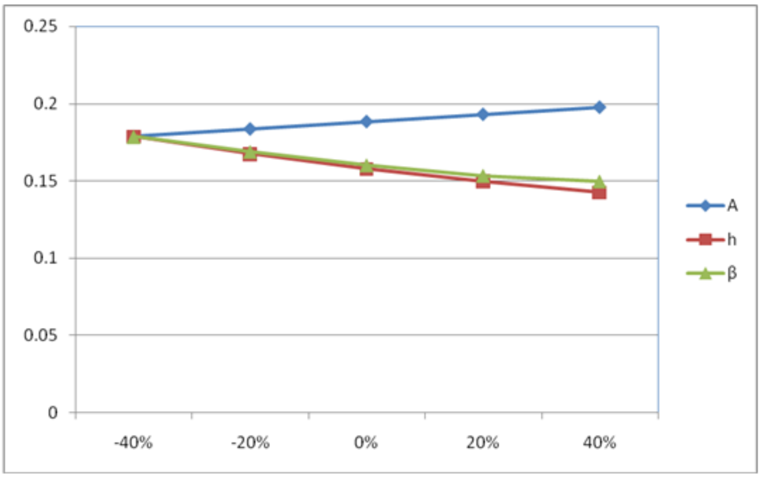
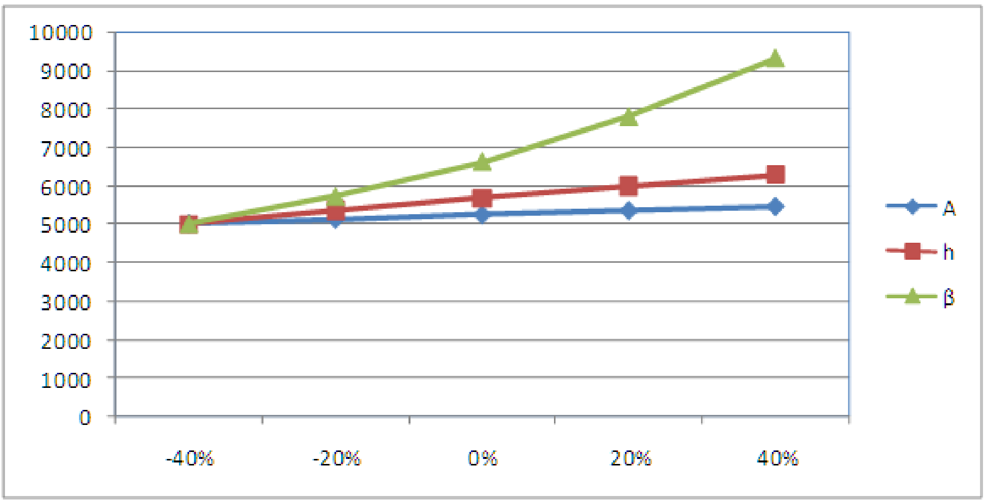
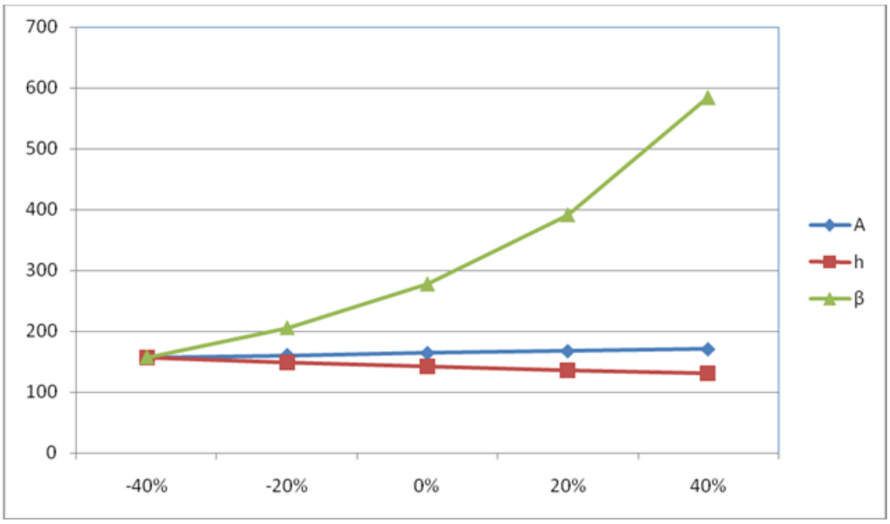
1.Variation of Time duration



2.Variation of Ordering Quantity



3.Variation of Total cost



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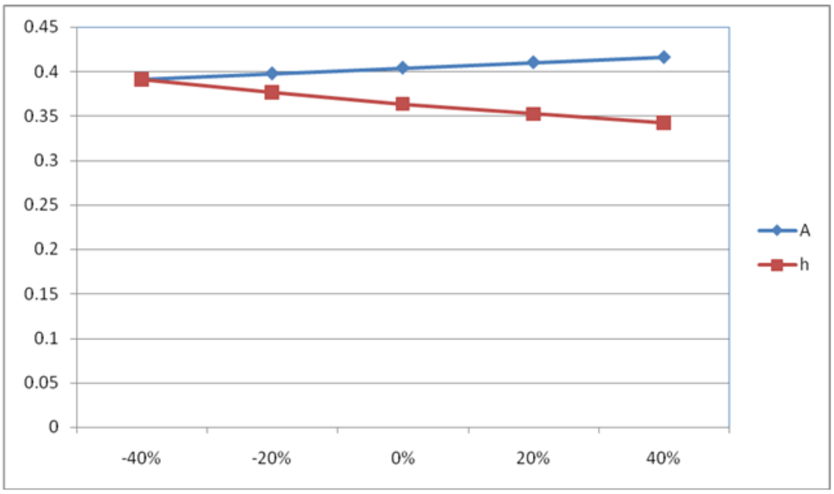
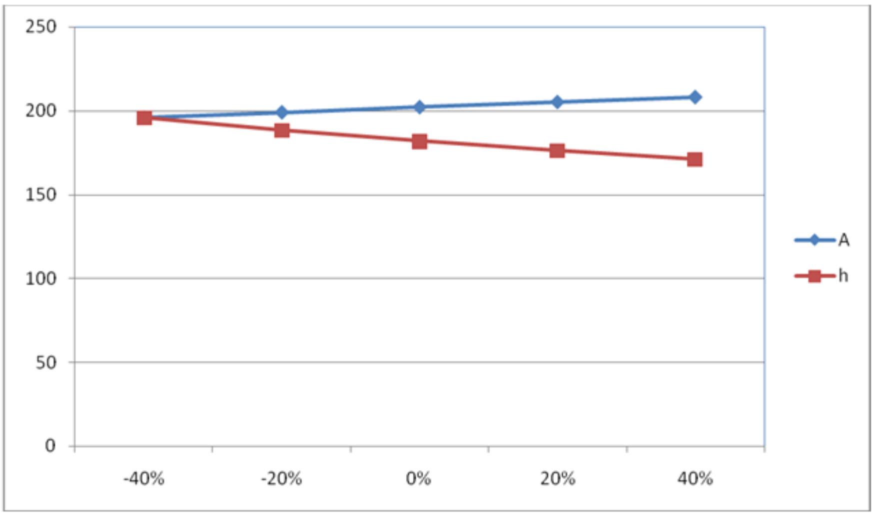
CASE: 2

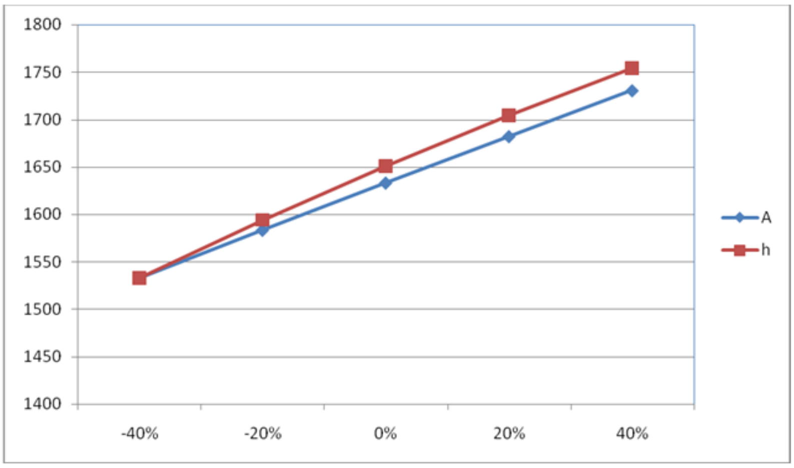
1.**Variation of Time duration**



2.**Variation of Ordering Quantity**



3.**Variation of Total cost**



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VII. CONCLUSION

A deterministic stock model for time fluctuating interest and steady request; and time subordinate holding expense and consistent holding cost for case 1 and case 2 individually are inferred. Mathematical model has been produced for deciding the ideal (optimal) request amount, the ideal process duration and ideal aggregate stock expense for both the cases. Numerical examples are given for both cases to accept the proposed model along with sensitivity investigation.

The model can further be enriched with shortage state and for multiple items under similar conditions. This can also be extended for deterioration conditions and also for discounted cash flow approach.

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