

FUZZY INVENTORY MODEL WITH SINGLE ITEM UNDER TIME DEPENDENT DEMAND AND HOLDING COST

S. S. Routray¹, S.K. Paikray², S. Mishra³, U.K. Misra⁴

¹Dept. of Mathematics, CET, Bhubaneswar, Odisha (India)

²Dept. of Mathematics, VSSUT, Burla, Odisha (India)

³Dept. of Mathematics, Govt. Science College, Malkangiri, Odisha (India)

⁴Dept. of Mathematics, NIST, Palurhills, Berhampur, Odisha (India)

ABSTRACT

The objective of this model is to discuss the inventory model for time varying demand and time dependent holding cost. Mathematical model has been developed for determining the optimal order quantity, the optimal cycle time and optimal total inventory cost in fuzzy environment. For defuzzification, graded unit preference integration method is used. Numerical examples are given to validate the proposed model. Sensitivity analysis has been carried out to analyze the effect of changes in the optimal solution with respect to the changes in various parameters.

Keywords: Fuzzy Inventory system, time dependent demand, holding cost.

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I. INTRODUCTION

Stock administration is utilized to minimize the stock conveying cost. In conventional EOQ model, the demand rate is thought to be consistent. In real life, it is as often as possible watched that interest for a specific item can be impacted by inner elements, such as price, time and availability. The adjustment in the interest because of stock or advertising choices relies upon interest flexibility. In this manner, when the interest rate is steady, the impact of variability of the holding expense of the total inventory cost functions of such models has likewise been considered. Different models have been proposed for consistent interest rate with steady holding cost. Teng et al. [17] built up an EOQ model on ideal evaluating and requesting arrangement under permissible deferral in installments by expecting that the offering cost is essentially higher than the purchase cost. They built up a suitable model for a retailer to establish its ideal cost and lot size, simultaneously, when the supplier offered an admissible postponement in installment. Muhlemann and Valtis-Spanopoulos [14] examined the consistent rate EOQ model but with variable holding cost expressed as a percentage of the average value of capital investigated in stock. Vander Veen [19] exhibited an EOQ stock framework with the holding cost as a nonlinear function of stock. Weiss [21] examined

conventional EOQ model with the holding cost per unit adjusted as a nonlinear function of the length over time for which an item was held in stock. Goh [6] presented an EOQ model with general request and holding cost capacity and interest rate for an item was considered a function of existing stock level and conveying cost per unit was permitted to change.

Fuzzy set theory has been applied to inventory problems to handle the uncertainties related to the demand or cost coefficients. An extended review of the application of the fuzzy set theory in inventory management can be found in [7]. The advantage of using the fuzzy set theory in modeling the inventory problems is its ability to quantify vagueness and imprecision.

In certain situations, uncertainties are due to fuzziness, primarily introduced by Zadeh[26], is applicable. In 1970, Zadeh et al[27] proposed some strategies for decision making in fuzzy environment. Jain[11] worked on decision making in the presence of fuzzy variables. Kacprzyk et al[12] discussed some long-term inventory policy-making through fuzzy-decision making models. Wide applications of fuzzy set theory can be found in Zimmerman[28], and Park[15]. In basic EOQ model we identify the order size that minimizes the sum of annual costs of inventory holding and fixed setup to place orders. In this model, there are some assumptions:

- The demand is known, fixed and independent,
- Quantity discounts are not allowed
- Inventory replenishment is instantaneous
- Only variable costs are setup cost and inventory holding cost
- No safety stock

Thus, EOQ model serves as a useful approximation to many real life problems. In literature, there are many papers on fuzzified problems of EOQ model. Urgeletti [18] treated EOQ model in fuzzy sense, and used triangular fuzzy number. Chen and Wang[3] used trapezoidal fuzzy number to fuzzify the order cost, inventory cost, and backorder cost in the total cost of inventory model without backorder. Then, they found the estimate of the total cost in the fuzzy sense by functional principle.

Vujosevic et al[20] used trapezoidal fuzzy number to fuzzify the order cost in the total cost of inventory model with backorder. Then, they got fuzzy total cost. They obtained the estimate of the total fuzzy cost through centroid to defuzzify.

Further, in a series of papers, Yao et al.[24,23,25], considered the fuzzified problems for the inventory with or without backorder models. In [24], they applied the extension principle to obtain the fuzzy total cost, and then, they defuzzified the fuzzy total cost by centroid. In [23], they considered the fuzzified problems for the inventory with or without backorder models using trapezoidal fuzzy number. In [25], they considered the fuzzified problems for the inventory without backorder models and they fuzzified the order quantity q as the triangular fuzzy number. For defuzzification, the study shows that the signed distance method is better than centroid method Yao & Lee [25].

Kao and Hsu [13] considered a single-period inventory model with fuzzy demand. Hsieh[10] analyzed some production inventory models in fuzzy sense and he proposed some optimal strategies. Syed & Aziz [16] used

trapezoidal fuzzy number. De and Rawat [5], proposed an EOQ model without shortage cost by using triangular fuzzy number. The total cost has been computed by using signed distance method.

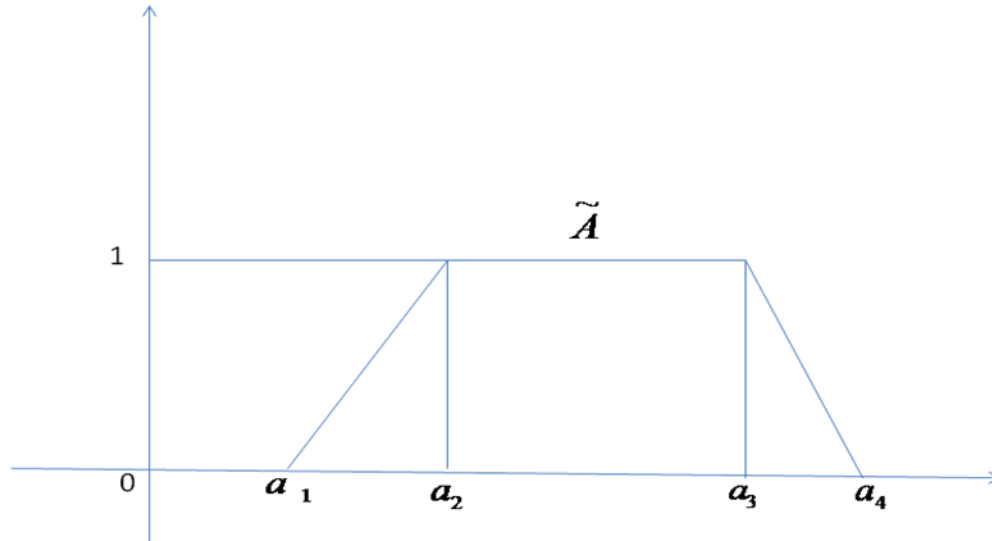


Figure 1 : The Trapezoidal Fuzzy Number \tilde{A}

In the proposed study, a fuzzy inventory model has been developed where, we consider the demand rate is time varying and holding cost is constant .The main objective of this paper is to obtain minimum total inventory cost, order quantity and corresponding order cycle. An algorithm that minimizes the total inventory cost is developed. Numerical examples are discussed to illustrate the procedure of solving the model. The proposed model is developed in both the crisp and fuzzy environments. In fuzzy environment, the related inventory parameters i.e. the inventory holding cost and ordering cost are fuzzified as the trapezoidal fuzzy numbers and then apply the Graded Mean Integration Representation method for defuzzification. The objective is to obtain fuzzy optimal solution to minimize the total cost per time unit of an inventory control system based on the fuzzy arithmetical operations under Function Principle.

1.1. DEFINITION AND PRINCIPLES

Suppose \tilde{A} is a generalized fuzzy number as shown in Figure 1, and is described as any fuzzy subset of the real line R , whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions.

- (1) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval $[0, 1]$,
- (2) $\mu_{\tilde{A}}(x) = 0, -\infty < x \leq a_1$,
- (3) $\mu_{\tilde{A}}(x) = L(x)$, is strictly increasing on $[a_1, a_2]$,
- (4) $\mu_{\tilde{A}}(x) = 1, a_2 \leq x \leq a_3$,
- (5) $\mu_{\tilde{A}}(x) = R(x)$, is strictly increasing on $[a_3, a_4]$,

(6) $\mu_{\tilde{A}}(x) = 0, a_4 \leq x < \infty$,

where a_1, a_2, a_3 and a_4 are real numbers.

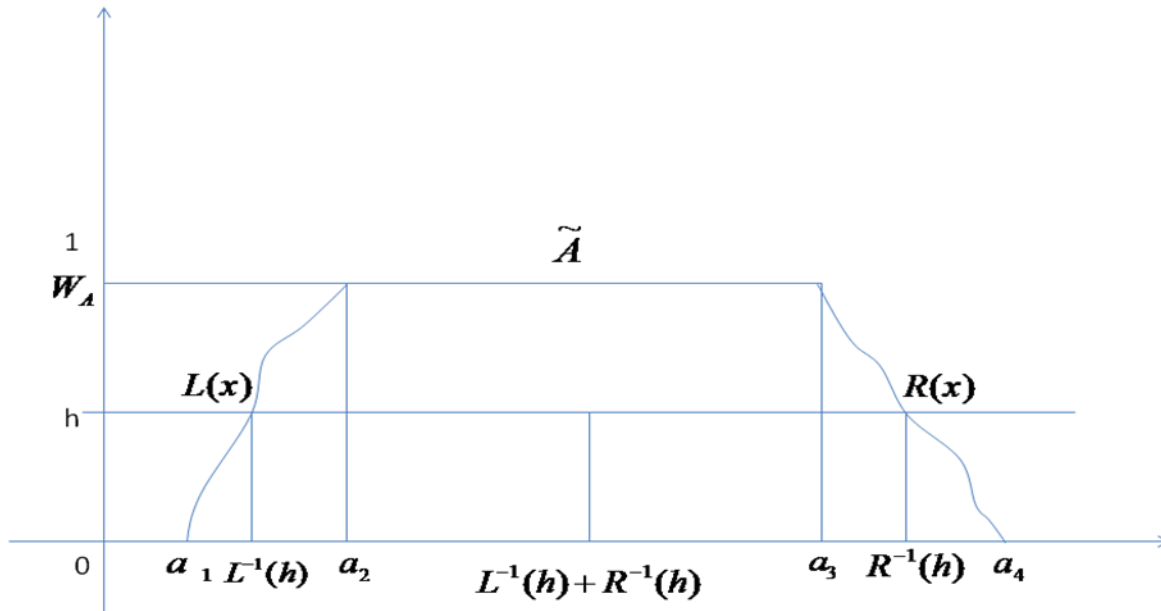


Figure 2 : Generalized Fuzzy Number

In 1998, Chen and Hsieh [1,2,4] propose graded mean integration representation for representing generalized fuzzy number. Now we describe graded mean integration representation (GMIR) as follows.

Suppose L^{-1} and R^{-1} are inverse functions of functions L and R, respectively, and the graded mean h-level value of generalized fuzzy number $A = (c, a, b, d : w)_{LR}$ is $h[L^{-1}(h) + R^{-1}(h)]/2$ as Figure:2. Then the graded mean integration representation of generalized fuzzy number based on the integral value of graded mean h-level is

$$(1.1) \quad P(A) = \int_0^w h(L^{-1}(h) + R^{-1}(h)/2) dh / \int_0^w h dh,$$

where h is between 0 and w and $0 < w \leq 1$. Generalized trapezoidal fuzzy number and generalized triangular fuzzy number are denoted as $(c, a, b, d : w)$ and $(c, a, d : w)$ respectively. Chen and Hsieh [1,2,4] already find the general formulae of the representation of generalized trapezoidal fuzzy number, or generalized triangular fuzzy number as follows.

Suppose $A = (c, a, b, d : w)$ is a trapezoidal fuzzy number. Since,

$$L(x) = w \left(\frac{x - c}{a - c} \right), c \leq x \leq a, \text{ and } R(x) = w \left(\frac{x - d}{b - d} \right), b \leq x \leq d,$$

then $L^{-1}(h) = c + (a - c)h/w, 0 \leq h \leq w, R^{-1}(h) = d - (d - b)h/w, 0 \leq h \leq w,$

and
$$\frac{L^{-1}(h) + R^{-1}(h)}{2} = \frac{c + d + (a - c - d + b)h/w}{2}.$$

By formula (1.1), the graded mean integration representation of A is

$$(1.2) \quad P(A) = \int_0^w h(c + d + (a - c - d + b)h/w/2)dh / \int_0^w h dh = \frac{c + 2a + 2b + d}{6}.$$

The fuzzy arithmetical operations under Function Principle

Here, we describe some fuzzy arithmetical operations under Function Principle as follows.

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers. Then,

1. The addition of \tilde{A} and \tilde{B} is

$$\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

where $a_1, b_1, a_2, b_2, a_3, b_3, a_4$ and b_4 are any real numbers.

2. The multiplication of \tilde{A} and \tilde{B} is

$$\tilde{A} \otimes \tilde{B} = (c_1, c_2, c_3, c_4),$$

Where $T = \{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$, $T_1 = \{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$,

$$c_1 = \min T, c_2 = \min T_1, c_3 = \max T_1, c_4 = \max T.$$

Also, if $a_1, b_1, a_2, b_2, a_3, b_3, a_4$ and b_4 are non zero positive real numbers, then

$$\tilde{A} \otimes \tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4), \text{ Where } \tilde{A} \otimes \tilde{B} \text{ is a trapezoidal fuzzy number.}$$

3. $-\tilde{B} = (-b_4, -b_3, -b_2, -b_1)$, then the subtraction of \tilde{A} and \tilde{B} is

$$\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$$

where $a_1, b_1, a_2, b_2, a_3, b_3, a_4$ and b_4 are any real numbers.

4. $1/\tilde{B} = \tilde{B}^{-1} = (1/b_4, 1/b_3, 1/b_2, 1/b_1)$ where b_1, b_2, b_3 and b_4 , are all positive real numbers.

If $a_1, b_1, a_2, b_2, a_3, b_3, a_4$ and b_4 are all positive real numbers, then the division of \tilde{A} and \tilde{B} is

$$\tilde{A} \oslash \tilde{B} = (a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1).$$

5. Let $\alpha \in R$, then

$$\begin{cases} (i) \alpha \geq 0, \alpha \otimes \tilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4), \\ (ii) \alpha < 0, \alpha \otimes \tilde{A} = (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1). \end{cases}$$

Example:

Suppose $\tilde{A} = (1,2,3,4)$ and $\tilde{B} = (1,3,4,6)$ are two trapezoidal fuzzy numbers and $\alpha = 2.5$. Then,

- i. $\tilde{A} \oplus \tilde{B} = (2,5,7,10)$,
- ii. $\tilde{A} \otimes \tilde{B} = (1,6,12,24)$,
- iii. $\tilde{A} \ominus \tilde{B} = (-5,-2,0,3)$,
- iv. $\tilde{A} \oslash \tilde{B} = (0.2,0.5,1,4)$,
- v. $\alpha \otimes \tilde{B} = (2.5,7.5,10,15)$.

II. ASSUMPTIONS AND NOTATIONS:

Following assumptions are made for the proposed model:

- i. Single inventory will be used.
- ii. Lead time is zero.
- iii. The model is studied when shortages are not allowed.
- iv. The demand rate $R(t)$ is decreasing function of time with increase of β .
- v. The holding cost is constant.

Following notations are made for the given model:

$I(t)$ = On hand inventory level at any time $t, t \geq 0$.

T = The length of cycle time.

A = The ordering cost per unit time

λ = The constant annual demand rate.

$h(t)$ = The time dependent holding cost.

$R(t)$ = The time varying demand rate i.e. $R(t) = \lambda t^{-\beta}$ $\lambda > 0, 0 < \beta < 1$. Here β is the demand parameter of the demand curve.

U = Total inventory cost per cycle

Q = Ordering quantity

III. FORMULATION

Let $I(t)$ be the on-hand inventory level at any time $t \geq 0$. The demand rate is assumed to be positive in its entire domain. The amount of stock depletes in the period $[0, T]$ due to the effect of demand. By this process, the stock reaches zero at time T . Hence, the inventory level at any instant of time t is described as follows.

At time $t + \Delta t$, the on-hand inventory in the interval $[0, T]$ will be

$$I(t + \Delta t) = I(t) - d(t) \Delta t$$

Dividing by Δt and then taking as $\Delta t \rightarrow 0$ we get

$$(3.1) \quad \frac{dI(t)}{dt} = -\lambda t^{-\beta}; \quad 0 \leq t \leq T$$

With the condition

$$(3.2) \quad I(T) = 0.$$

The solution of the differential equation (3.1) is given by,

$$(3.3) \quad I(t) = \frac{\lambda}{1-\beta} (T^{1-\beta} - t^{1-\beta})$$

Now Q is the ordering quantity of stock which is given by

$$(3.4) \quad Q = \frac{\lambda T^{1-\beta}}{1-\beta} \quad \text{where } 0 < \beta < 1.$$

From (3.4), we obtain

$$(3.5) \quad T = \left[\frac{(1-\beta)Q}{\lambda} \right]^{\frac{1}{1-\beta}}.$$

Now the average total cost per cycle is given by

$$(3.6) \quad U(Q) = \frac{1}{T} [\text{Ordering Cost} + \text{Holding Cost}]$$

$$= \frac{1}{T} \left[A + h \int_0^T I(t) dt \right]$$

$$= \frac{A}{T} + \frac{h \lambda}{T(1-\beta)} \int_0^T (T^{1-\beta} - t^{1-\beta}) dt = \frac{A}{T} + \frac{h \lambda T^{1-\beta}}{2-\beta}$$

Using (3.5), we obtain

$$(3.7) \quad U(Q) = \frac{A}{T} + \frac{h \lambda T^{1-\beta}}{2-\beta} = A \left[\frac{\lambda}{(1-\beta)} \right]^{\frac{1}{1-\beta}} Q^{\frac{-1}{1-\beta}} + \frac{h(1-\beta)Q}{(2-\beta)}.$$

The necessary condition for minimization of $U(Q)$ is,

$$(3.8) \quad \frac{\partial U(Q)}{\partial Q} = 0 .$$

The sufficient condition for minimization of $U(Q)$ is for $Q > 0$,

$$(3.9) \quad \frac{\partial^2 U(Q)}{\partial Q^2} > 0 .$$

$$(3.10) \quad \frac{\partial U(Q)}{\partial Q} = \frac{-A}{(1-\beta)} \left[\frac{\lambda}{(1-\beta)} \right]^{\frac{1}{(1-\beta)}} Q^{\frac{-(2-\beta)}{(1-\beta)}} + \frac{h(1-\beta)}{(2-\beta)} .$$

$$(3.11) \quad \frac{\partial^2 U(Q)}{\partial Q^2} = \frac{A(2-\beta)}{(1-\beta)^2} \left[\frac{\lambda}{(1-\beta)} \right]^{\frac{1}{(1-\beta)}} Q^{\frac{-(3-2\beta)}{(1-\beta)}} .$$

Now the function $U(Q)$ will be maximum if

$$(3.12) \quad \frac{\partial^2 U(Q)}{\partial Q^2} > 0 \quad \text{which is obvious from (3.11).}$$

Now from (3.8) we have

$$(3.13) \quad \frac{-A}{(1-\beta)} \left[\frac{\lambda}{(1-\beta)} \right]^{\frac{1}{(1-\beta)}} Q^{\frac{-(2-\beta)}{(1-\beta)}} + \frac{h(1-\beta)}{(2-\beta)} = 0 .$$

Now on solving (3.13), implies minimizing total cost to determine optimal Q^* given by

$$Q^* = \left[\frac{\lambda}{1-\beta} \right]^{\frac{1}{2-\beta}} \cdot \left[\frac{A(2-\beta)}{h(1-\beta)^2} \right]^{\frac{1-\beta}{2-\beta}} \quad \text{and hence the optimal cost } U(Q^*) \text{ can be evaluated.}$$

If $\beta \rightarrow 0$, then the optimal $Q^* = \sqrt{\frac{2A\lambda}{h}}$

$$\text{and } U(Q) = \frac{A\lambda}{Q} + \frac{hQ}{2} .$$

IV. FUZZY MODEL AND SOLUTION PROCEDURE

We consider the model in fuzzy environment. Due to fuzziness, it is not easy to define all the parameters precisely. We use the following variables

\tilde{A} : fuzzy Ordering cost,

\tilde{h}_c : fuzzy carrying cost,

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$, $\tilde{h}_c = (h_1, h_2, h_3, h_4)$, are nonnegative trapezoidal fuzzy numbers.

The total average cost per unit time is given by

$$(4.1) \quad \tilde{C}_{avg}(Q) = (\alpha \otimes \tilde{A}) \oplus (\theta \otimes \tilde{h}_c)$$

$$\text{where } \alpha = \left[\frac{\lambda}{(1-\beta)} \right]^{\frac{1}{1-\beta}} Q^{\frac{-1}{(1-\beta)}}, \theta = \frac{(1-\beta)Q}{(2-\beta)}.$$

$$(4.2) \quad \text{Now } \tilde{C}_{avg}(Q) = (\tilde{C}_{avg_1}(Q), \tilde{C}_{avg_2}(Q), \tilde{C}_{avg_3}(Q), \tilde{C}_{avg_4}(Q),)$$

$$\tilde{C}_{avg_1}(Q) = (\alpha A_1 + \theta h_1)$$

$$\tilde{C}_{avg_2}(Q) = (\alpha A_2 + \theta h_2)$$

$$\tilde{C}_{avg_3}(Q) = (\alpha A_3 + \theta h_3)$$

$$\tilde{C}_{avg_4}(Q) = (\alpha A_4 + \theta h_4)$$

Now on defuzzifying the fuzzy total average cost $\tilde{C}_{avg}(Q)$ we have

$$(4.3) \quad P(\tilde{C}_{avg}(Q)) = \frac{1}{6}(\alpha A_1 + 2\alpha A_2 + 2\alpha A_3 + \alpha A_4) + \frac{1}{6}(\theta h_1 + 2\theta h_2 + 2\theta h_3 + \theta h_4)$$

$$= \frac{\alpha}{6}(A_1 + 2A_2 + 2A_3 + A_4) + \frac{\theta}{6}(h_1 + 2h_2 + 2h_3 + h_4)$$

To minimize the average total cost per unit time, the optimal value of Q can be obtained by solving the following equation

$$(4.4) \quad \frac{d(P(\tilde{C}_{avg}(Q)))}{dQ} = \frac{\alpha'}{6}(A_1 + 2A_2 + 2A_3 + A_4) + \frac{\theta'}{6}(h_1 + 2h_2 + 2h_3 + h_4)$$

$$\text{where } \alpha' = \frac{-1}{(1-\beta)} \left[\frac{\lambda}{(1-\beta)} \right]^{\frac{1}{1-\beta}} Q^{\frac{-2+\beta}{(1-\beta)}}, \theta' = \frac{(1-\beta)}{(2-\beta)}.$$

Thus minimum value of the total cost $C_{avg}(Q)$ denoted by $C_{avg}^*(Q)$

$$(4.5) \quad C_{avg}^*(Q) = \frac{1}{6}[(\alpha A_1 + \theta h_1)] + \frac{1}{3}[\alpha A_2 + \theta h_2]$$

$$+ \frac{1}{3}[\alpha A_3 + \theta h_3] + \frac{1}{6}[(\alpha A_4 + \theta h_4)]$$

V. COMPUTATIONAL ALGORITHM:

Step-1: Start.

Step-2: Initialize the value of the variables A, λ, h, β .

Step-3: Evaluate $U(Q)$.

Step-4: Evaluate $\frac{\partial U(Q)}{\partial Q}$.

Step-5: Solve the equation $\frac{\partial U(Q)}{\partial Q} = 0$.

Step-6: Choose the solution from Step-5.

Step-7: Evaluate $\frac{\partial^2 U(Q)}{\partial Q^2}$.

Step-8: If the value of Step-7 is greater than zero then this solution is optimal (minimum) and go to Step-10.

Step-9: Otherwise go to Step-6.

Step-10: End.

VI. NUMERICAL EXAMPLES

To illustrate the proposed method, let us consider the following input data:

Crisp Model:

The values of the parameters in proper units are considered as follows:

$$\lambda = 500, A = 400, \beta = 0.2, h = 40$$

Optimal $Q^* = 157.5099, C^* = 5040.3162, T^* = 0.1786$.

Fuzzy Model:

We can apply the fuzzy inventory model with fuzzy order quantity to find the optimal fuzzy total average cost. First, we represent the case of vague value as the type of trapezoidal fuzzy number.

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4) = (200, 400, 400, 600)$

$\tilde{h}_c = (h_1, h_2, h_3, h_4) = (20, 40, 40, 60), \lambda = 500, \beta = 0.2$.

Equation (4.5) can be minimized by using, Matlab Software to determine optimal Q^*, C^* & T^* .

The optimal ordering quantity, average cost and time are found to be $Q^* = 165.438, C^* = 5123.568, T^* = 0.1673$.

VII. SENSITIVITY ANALYSIS

TABLE : 01

	Q*	C*	T*
0.2	157.5099	5040.3162	0.1786
0.3	205.5748	5756.0957	0.1688
0.4	277.6152	6662.7654	0.1601
0.5	391.4868	7829.7353	0.1533
0.6	584.7066	9355.3062	0.1496

TABLE : 2

h	Q*	C*	T*
40	157.5099	5040.3162	0.1786
45	149.4766	5381.1593	0.1673
50	142.6385	5705.539	0.1577
55	136.7225	6015.7892	0.1496
60	131.5361	6313.7336	0.1425

TABLE : 3

A	Q*	C*	T*
400	157.5099	5040.3162	0.1786
420	160.9627	5150.8066	0.1835
440	164.3253	5258.411	0.1883
460	167.6041	5363.3310	0.1930
480	170.8046	5465.746	0.1976

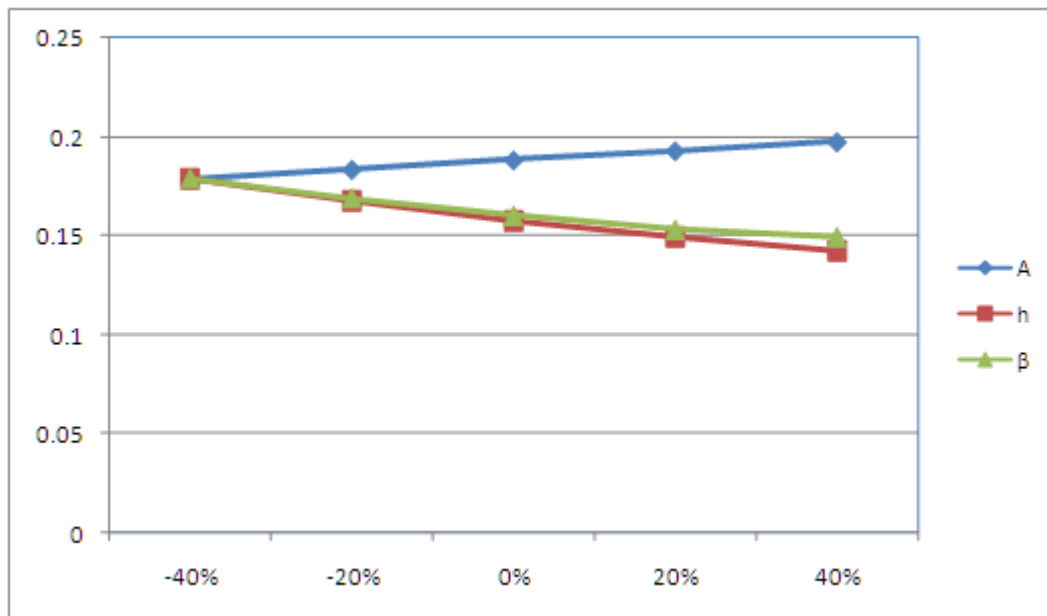
Important points from the table

- ▶ The effect of optimality due to change of values of different parameters associated in this model is discussed below.

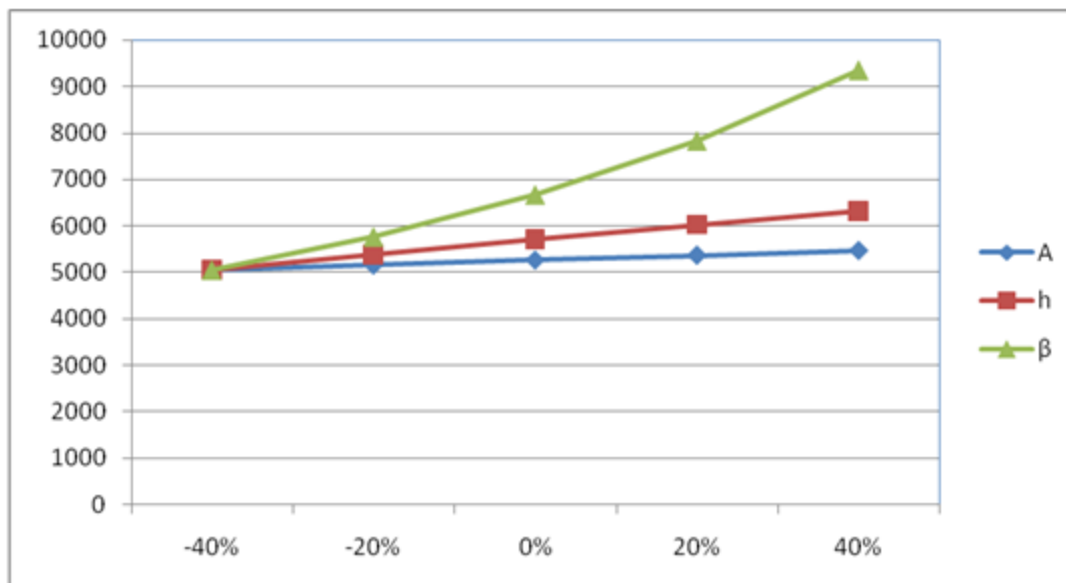
1. Q & C increase while T decreases with increase in value of the parameter β .
2. C increases while Q & T decrease with increase in value of the parameter h .
3. Q, C & T increase with increase in value of the parameter A .

Variation of Time duration, Ordering Quantity and Total cost w.r.t. different parameters.

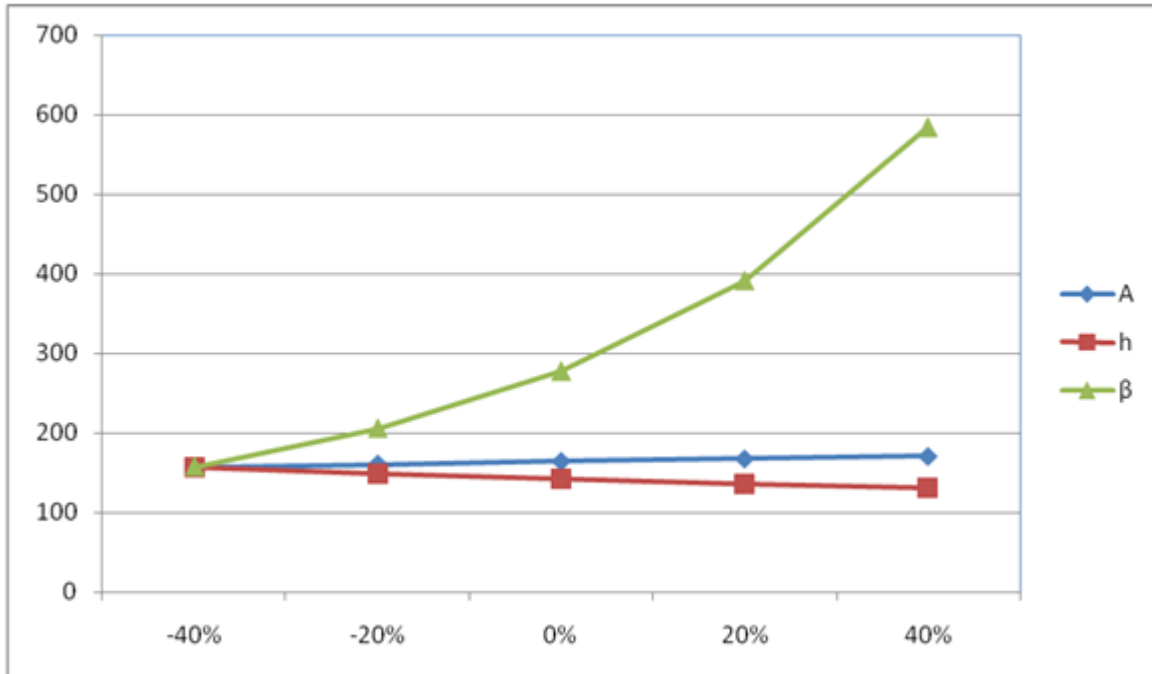
1.Variation of Time duration



2.Variation of Ordering Quantity



3.Variation of Total cost



VIII. CONCLUSION

This paper presented a fuzzy inventory control model for time varying demand and time dependent holding cost respectively. The proposed model is developed in both the crisp and fuzzy environments. In fuzzy environment, all related inventory parameters were assumed to be trapezoidal fuzzy numbers. The optimum results of fuzzy model are defuzzified using graded mean $h=1$ level integration representation method. So, the decision maker, after analyzing the result, can plan for the optimal value for the related parameters.

The model can further be studied for shortage state and for multiple items under identical conditions. This can also be extended for deterioration conditions and also for discounted cash flow approach.

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