Optimizing Inventory Policy for Deteriorating Items with Permissible delay in payment under the Effect of Price Discounting on Lost Sales

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ABSTRACT

In this paper an inventory model is developed with permissible delay in payment including seasonal quadratic demand with time proportional deterioration. Shortages are assumed with partial backlogging. Price discount is offered for reducing the lost sales of back ordered quantity and increasing the profit per unit time. Further a credit period is permitted by the supplier to the retailer to earn interest on the sales revenue till the credit period. The retailer pays interest on the amount not sold till the credit period. The retailer also gets some revenue from sales of deteriorated quantity and it is retained to fulfill daily needs. Numerical illustrations and sensitivity analysis of the optimal solution with respect to various parameters is carried out to describe the model.

Keywords: Deterioration, Permissible delay in payment, Price discounting, Seasonal demand.

I. INTRODUCTION

Inventory System is one of the main streams of the Operations Research which is essential in business enterprises and Industries. Interest in the subject is constantly increasing, and its development in recent years closely parallels the development of operations research in general. Deterioration of things in an inventory control system is a realistic feature in marketing era. Because of deterioration the goods in the inventory system become decay, obsolete, damaged or devalued. In any inventory control problem a major factor is the maintenance of inventory of deteriorating items. Many products decay or deteriorate over time. Product such as fruits, vegetables, food stuffs are subject to direct spoilage during storage. First exponential deteriorating inventory model developed by Ghare and Schrader [1]. Sometime later, Covert and Philip [2] extended Ghare and Schrader’s constant exponential deterioration rate with two-parameter of Weibull distribution. Since then there are so many authors presented their models using deterioration factor in different ways like He Y and Huang H [3], Chakrabarty [4], Teng et al[5], Dave and patel [6], Garg et al.[7] manna and chaudhary [8] ,Dye [9], Sana [10], Sarkar [11], Chang [12], Haidar [13], Guchhait [14] and Pandey and vaish [15].

A valuation access where items are initially mentioned at artificially but are then offered for sale to be minimizing cost to the consumer. This process termed as Price discount. Price discount on unit selling price of goods attracts the customers to buy more and more. Thus price discount is important factors which enhance the demand which in turn increases the total profit per unit time. Ardalan [16] introduced an inventory model in
form of temporary price discount. Since then there are so many authors presented their models using price
discount factor in different ways like. Wee [17], Sana and Chaudhuri [18], Lin [19], Huang [20], Paul [21]
Sarkar [22] and Liying [23].
It has been seen many times that stock ends before the arrival of next replenishment and some customers do not
want wait up to the next replenishment. This is termed as partial backlogging. There are so many authors
presented their models using price discount factor in different ways like Geethal and Udaykumari [24], Chang
and Dye [25] and Pandey et al [26].
Normally, the payment for any consignment is done by retailer to the supplier immediately. But in present days,
due to tough competition in market and for attracting more customers a credit period has to be allotted to retailer
by the suppliers to attain the maximum profit. No interest has to pay during this time credit period but beyond it,
the retailer has to be pay an interest on the amount in stock. Before the end of time period of trade credit limit,
the retailer can accumulate the revenue and earn so much profit and interest. This is termed as permissible delay
in payment. Hence it is more profitable to the retailer to delay his payment till the very last day of the settlement
period. First time inventory model with permissible delay in payment was developed by Goyal [27] using
suitable conditions. After that many authors presented inventory models using permissible delay factor like
Aggarwal and Jaggi [28] Jamal et al. [29], Chang and Dye [25], Ouyang et al. [30], Tsao and Sheen [31] and
Teng et al [32], Jaggi [33] described an inventory model with effects of inspection on retailer's ordering policy
for deteriorating items with time-dependent demand. Jun li [34] introduced an inventory model with credit
period. Khanna [35] developed an inventory policy for deteriorating imperfect quality items including
permissible delay in payment.
The present paper is based on of Pandey and Vaish (2017) papers. In Pandey and Vaish paper an economic
ordered quantity model is developed considering seasonal quadratic type of demand and instantaneous
deterioration with time dependent. In the present model introduce permissible delay in payment factor. In both
cases profit are considered due to credit period. Shortages are allowed. A fraction of demand is backordered
which depends on waiting time up to the next replenishment and a price discount is given on the backordered
quantity. Numerical illustration, tables and graphs are presented to describe the model. In addition sensitivity
analysis of the optimal solution with respect to various parameters involved in inventory problem is carried out.

II. ASSUMPTIONS & NOTATIONS

1. Demand pattern is \( D(t) = at(t-T) \) which is quadratic in nature.

2. Shortages are allowed and are partial backlogged. The backlogged rate is
\[
\frac{1}{1+\delta(T-t)}
\]
where \( \delta > 0 \).

3. \( d (0 \leq d \leq 1) \) is the percentage offer on unit selling price on backordered quantity declared at the start of the
stock out period. \( \alpha = (1-d)^n \). Where \( d \rightarrow 0, \alpha \rightarrow 1 \)

4. The permissible delay in payments is also assumed in the model. There can be two cases regarding the credit
period \( M: 0 \leq M < t, t \leq T \) and \( 0 \leq t, M \leq T \) the two cases are considered separately.

5. Instantaneous deterioration with linearly time dependent which is \( \theta(t) = \theta t \).  

\( p \) Selling price
\( C \) Purchasing cost
Deterioration coefficient, $\theta << 1$

$T$ Cycle length

$t_1$ The time at which inventory level becomes zero

$Q_1$ Initial inventory level at the beginning of each cycle and

$Q_2$ Backordered quantity

$Q$ Ordered Quantity ($Q_1 + Q_2$)

$DQ$ Total Deteriorated quantity

$p_r$ Reduced selling price of each deteriorated unit

$M$ trade credit period

$I_p$ Interest paid

$I_e$ Interest earned

$h$ Holding cost

$s$ Shortage cost

$l$ Lost sale cost

$A$ Ordering cost

$\delta$ Backlogging Rate

$R$ Sales revenue per replenishment cycle

$I(t)$ The inventory level at time $t$

$F(t_1, T)$ Profit per unit time

$t_1^*, F^*(t_1, d), Q^*$ represents the optimal values of $t_1, F(t_1, T), Q$ respectively.

The behavior of the inventory level during cycle $T$ is depicted in figure 1.

The permissible delay in payments is also assumed in the model. There can be two cases regarding the credit period $M$ in Fig (1): $0 \leq M \leq t_1 \leq T$ and $0 \leq t_1 \leq M \leq T$ the two cases are considered separately.
III. MATHEMATICAL MODELING

The differential equations showing the fluctuation of inventory with time $t$ are shown as below:

$$\frac{dI(t)}{dt} = -\theta I(t) - a(T - t) \quad 0 \leq t \leq t_1$$

... (1)

$$\frac{dI(t)}{dt} = -\alpha a(T - t) \quad t_1 \leq t \leq T$$

... (2)

With boundary condition $I(t_1) = 0$ the solutions of these equations are given by:

$$I(t) = a\left\{ \left( \frac{t_1^2}{2} - \frac{t_1^3}{3} + \frac{\theta t_1^4}{8} - \frac{\theta t_1^5}{10} \right) - \left( \frac{T}{2} + \frac{\theta T}{4} - \frac{\theta T}{6} \right) t_1^2 + \frac{t_1^3}{3} + \frac{\theta T}{8} t_1^4 - \frac{\theta}{15} t_1^5 \right\} \quad 0 \leq t \leq t_1$$

... (3)

$$I(t) = a(1 - d)^n \left( \frac{t_1^2 - t^2}{2\delta} + \frac{(T - t_1 - t)}{\delta^2} + \left( \frac{1 + \delta T}{\delta^3} \right) \log \left[ 1 + \delta(T - t_1) \right] - \log \left[ 1 + \delta(T - t) \right] \right)$$

$t_1 \leq t \leq T$ ... (4)

Sales revenue $R = p Q_1 + p (1-d) Q_2 + p_d Q$ where

$$Q_1 = \int_0^{t_1} a(T - t) dt$$

$$Q_1 = \frac{apT t_1^2}{2} - \frac{ata t_1^3}{3}$$

... (5)

$$Q_2 = -I(T)$$

$$Q_2 = a(1 - d)^n(1 - d) \left( \frac{T^2 - t_1^2}{2\delta} + \frac{(T - t_1)}{\delta^2} - \left( \frac{1 + \delta T}{\delta^3} \right) \log \left[ 1 + \delta(T - t_1) \right] \right)$$

... (6)

$$R = ap \left( \frac{t_1^2 T}{2} - \frac{t_1^3}{3} + (1 - d)^n (1 - d) \left( \frac{T^2 - t_1^2}{2\delta} + \frac{(T - t_1)}{\delta^2} - \left( \frac{1 + \delta T}{\delta^3} \right) \log \left[ 1 + \delta(T - t_1) \right] \right) \right)$$

... (7)

Purchasing cost $PC = (Q_1 + Q_2) C = C (I_1(0) + (- I_2(T)))$

$$= aC \left( \frac{t_1^2 T}{2} - \frac{t_1^3}{3} + \frac{\theta t_1^4}{8} - \frac{\theta t_1^5}{10} + (1 - d)^n \left( \frac{T^2 - t_1^2}{2\delta} + \frac{(T - t_1)}{\delta^2} - \left( \frac{1 + \delta T}{\delta^3} \right) \log \left[ 1 + \delta(T - t_1) \right] \right) \right)$$

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Deteriorated Quantity $DQ = ap\{ \frac{\theta Tt_1^4}{8} - \frac{\theta t_1^5}{10} \}$

Holding Cost $HC = \int_0^1 I(t)dt = ah \left( \frac{Tt_1^3}{3} - \frac{t_1^4}{4} + \frac{\theta Tt_1^5}{15} - \frac{\theta t_1^6}{18} \right)$

Shortage cost $SC = \{ \frac{T^3}{6} - \frac{t_1^2T}{2} + \frac{t_1^3}{3} \}$

Lost Sale Cost $LSC = \int_{t_1}^T (t(T-t))dt = \left( \frac{T^2}{2\delta} - \frac{T}{\delta^2} \right) + l \left( \frac{T^3}{6} - \frac{t_1^2T}{2} + \frac{t_1^3}{3} \right)$

Ordering Cost $OC = A$

Now Interest Earned and Interest Paid for two cases of credit period are discussed separately as follows;

**Case-1:** $0 \leq M \leq t_1 \leq T$ in this case the retailer can use sales revenue to earn interest during the period $[0, M]$ and the annual interest will be paid for the amount of inventory not sold till the credit period $M$. The total interest earned $IE_1$ and the total interest paid $IP_1$ for this case will be as follows:

$$IE_1 = pl_1 \left( \int_0^M (at(T-t))dt + (1-d_1)MQ_2 \right)$$

$$IE_1 = apl_1 \left( \frac{TM^3}{6} - \frac{M^4}{12} + (1-d)^n (1-d)M \left( \frac{T^2}{2\delta} - \frac{T-t_1}{\delta^2} \right) \log \left[ 1 + \delta (T-t_1) \right] \right)$$

$$IP_1 = CI \left( \int_M^T I(t)dt \right)$$

$$IP_1 = aCI \left( \frac{t_1^2T}{2} - \frac{t_1^3}{3} + \frac{\theta Tt_1^4}{8} - \frac{\theta t_1^5}{10} \right)(t_1 - M) + \left( \frac{T}{2} - \frac{\theta Tt_1^2}{4} - \frac{\theta t_1^3}{6} \right) \left( \frac{t_1^4 - M^4}{3} + \frac{t_1^4 - M^4}{12} \right) + \frac{\theta T(t_1^5 - M^5)}{40} - \frac{\theta (t_1^6 - M^6)}{90} \right)$$

Now total profit per unit time for case 1 $F(t_1, T)$ can be calculated as:

$$F(t_1, T) = \frac{1}{T} [\text{Sales revenue} + \text{interest earned} - \text{purchasing cost} - \text{shortage cost}]$$
- lost sale cost – holding cost – ordering cost – interest paid

### Total Profit per Unit Time for case 1:

\[
F(t_i) = \frac{a}{T} \left[ \frac{t_i^2}{2} - \frac{t_i^3}{3} + (1-d)^{-n}(1-d) \left( \frac{T^2 - t_i^2}{26} + \frac{T - t_i}{\delta^2} - \frac{1 + \delta T}{\delta^3} \right) \log \left[ 1 + \delta (T - t_i) \right] \right] + \frac{p}{8} \left[ \frac{\theta t_i^4}{10} \right]
\]

\[
+ p \left[ T^2 \cdot \frac{M^3}{6} + (1-d)^{-n}(1-d)M \left( \frac{T^2 - t_i^2}{26} + \frac{T - t_i}{\delta^2} - \frac{1 + \delta T}{\delta^3} \right) \log \left[ 1 + \delta (T - t_i) \right] \right] -
\]

\[
C_l \left[ \frac{t_i^2}{2} - \frac{t_i^3}{3} + \frac{\theta t_i^4}{8} - \frac{\theta t_i^6}{10} \right] (1 - M - M) \left( \frac{T^2 + \delta T}{2} + \frac{\theta t_i^4}{4} - \frac{\theta t_i^6}{6} \right) + \frac{\theta t_i^2 - M^2}{12} + \frac{\theta t_i^4 - M^4}{40} - \frac{\theta t_i^6 - M^6}{90}
\]

\[
- C \left[ \frac{t_i^2}{2} - \frac{t_i^3}{3} + \frac{\theta t_i^4}{8} - \frac{\theta t_i^6}{10} \right] (1 - d)^{-n} \left( \frac{T^2 - t_i^2}{26} + \frac{T - t_i}{\delta^2} - \frac{1 + \delta T}{\delta^3} \right) \log \left[ 1 + \delta (T - t_i) \right] \left[ \left( \frac{T^2 - t_i^2}{26} + \frac{T - t_i}{\delta^2} - \frac{1 + \delta T}{\delta^3} \right) \log \left[ 1 + \delta (T - t_i) \right] \right] -
\]

\[
- h \left[ \frac{T_i^2}{6} - \frac{T_i^3}{2} + \frac{\theta t_i^4}{3} \right] - \left( \frac{T_i^2}{6} - \frac{T_i^3}{2} + \frac{\theta t_i^4}{3} \right) - A \right]
\]

... (14)

**Case-2:** 0 ≤ t_i ≤ M ≤ T in this in case the retailer can use sales revenue to earn interest during the period [0, M] and as there is no inventory on hand beyond the credit period M so the retailer has to pay no interest. The total earned interest for case 2 IE_2 will be calculated as follows:

\[
IE_2 = p \int_0^{\frac{t_i}{2}} at(T-t)dt + (M - t_i) \left( \frac{t_i}{2} \right) dt + a(1-d)^{-n}(1-d)M \left( \frac{T^2 - t_i^2}{26} + \left( \frac{T - t_i}{\delta^2} - \frac{1 + \delta T}{\delta^3} \right) \log \left[ 1 + \delta (T - t_i) \right] \right)
\]

\[
IE_2 = ap \int \left[ \frac{T_i^2}{3} - \frac{T_i^4}{4} + (M - t_i) \left( \frac{T_i^2}{2} - \frac{T_i^3}{3} \right) + (1-d)^{-n}(1-d)M \left( \frac{T^2 - t_i^2}{26} + \left( \frac{T - t_i}{\delta^2} - \frac{1 + \delta T}{\delta^3} \right) \log \left[ 1 + \delta (T - t_i) \right] \right) \right]
\]

Now total profit per unit time for case 2 F(T_i, T) can be calculated as:

\[
F(T_i, T) = \left[ \text{Sales revenue + interest earned} - \text{purchasing cost} - \text{shortage cost} \right]
\]

- lost sale cost – holding cost – ordering cost

### Total Profit per Unit Time for case 2:

\[
F(t_i) = \frac{a}{T} \left[ \frac{t_i^2}{2} - \frac{t_i^3}{3} + (1-d)^{-n}(1-d) \left( \frac{T^2 - t_i^2}{26} + \left( \frac{T - t_i}{\delta^2} - \frac{1 + \delta T}{\delta^3} \right) \log \left[ 1 + \delta (T - t_i) \right] \right) \right] -
\]

\[
+ \frac{p}{8} \left[ \frac{\theta t_i^4}{10} \right] + \frac{p}{8} \left[ T^2 \cdot \frac{M^3}{6} + (1-d)^{-n}(1-d)M \left( \frac{T^2 - t_i^2}{26} + \left( \frac{T - t_i}{\delta^2} - \frac{1 + \delta T}{\delta^3} \right) \log \left[ 1 + \delta (T - t_i) \right] \right) \right] -
\]

\[
- C \left[ \frac{t_i^2}{2} - \frac{t_i^3}{3} + \frac{\theta t_i^4}{8} - \frac{\theta t_i^6}{10} \right] (1 - M - M) \left( \frac{T^2 + \delta T}{2} + \frac{\theta t_i^4}{4} - \frac{\theta t_i^6}{6} \right) + \frac{\theta t_i^2 - M^2}{12} + \frac{\theta t_i^4 - M^4}{40} - \frac{\theta t_i^6 - M^6}{90}
\]

\[
- C \left[ \frac{t_i^2}{2} - \frac{t_i^3}{3} + \frac{\theta t_i^4}{8} - \frac{\theta t_i^6}{10} \right] (1 - d)^{-n} \left( \frac{T^2 - t_i^2}{26} + \left( \frac{T - t_i}{\delta^2} - \frac{1 + \delta T}{\delta^3} \right) \log \left[ 1 + \delta (T - t_i) \right] \right) -
\]

\[
- h \left[ \frac{T_i^2}{6} - \frac{T_i^3}{2} + \frac{\theta t_i^4}{3} \right] - \left( \frac{T_i^2}{6} - \frac{T_i^3}{2} + \frac{\theta t_i^4}{3} \right) - A \right]
\]

... (14)
IV. SOLUTION PROCEDURE

In the model unit time profit is a function of two variables $t_1$ and $T$. To find out the optimal solution

$$\frac{\partial F(t_1, T)}{\partial t_1} = 0 \quad \frac{\partial F(t_1, T)}{\partial T} = 0$$

... (16)

The optimal values of $t_1$ and $d$ are obtained by solving these equations simultaneously provided

$$\frac{\partial^2 F(t_1, T)}{\partial t_1^2} \cdot \frac{\partial^2 F(t_1, T)}{\partial T^2} - \left( \frac{\partial^2 F(t_1, T)}{\partial t_1 \partial T} \right)^2 > 0$$

... (17)

NUMERICAL ILLUSTRATION FOR CASE-1

$M = 0.5$, $\theta = 0.009$, $I_p = 0.1$ units, $I_e = 0.1$ units, $p = 550$, $n = 5$, $p_r = 200$, $s = 15$ rs/unit, $l = 13$ rs/unit, $a = 40000$, $A = 100$ rs/ order

$h = 1.5$ rs/unit, $d = 0.15$, $C = 120$, $\delta = 2.2$

Applying the solution procedure described above the optimal values obtained is as follows:

$t_1^* = 0.648923$, $T^* = 1.13489$, $F^*(t_1, T) = 386156$ rs, $Q^* = 11205.7$ units

Effects of various parameters on Total Profit per Unit Time For Case-1

Effects of parameter $\theta$ on Total Profit per Unit Time

<table>
<thead>
<tr>
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<th>$\theta$</th>
<th>$t_1$</th>
<th>$T$</th>
<th>$F$</th>
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TABLE-1 & FIGURE-2
Effects of parameter "d" on Total Profit per Unit Time

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<th>t1</th>
<th>T</th>
<th>F</th>
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**TABLE-2& FIGURE-3**

Effects of parameter "n" on Total Profit per Unit Time

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**TABLE-3& FIGURE-4**

Effects of parameter "a" on Total Profit per Unit Time

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<td>1.13489</td>
<td>424772</td>
</tr>
<tr>
<td>15%</td>
<td>46000</td>
<td>0.648923</td>
<td>1.13489</td>
<td>444080</td>
</tr>
</tbody>
</table>

**TABLE-4& FIGURE-5**
V. OBSERVATIONS

1. From table (1) it is observed that as the rate of backlogging (θ) decreases, the unit time profit of the system increases.

2. From table (2) it is observed that as discount (d) increases the unit time profit of the system increases.

3. From table (3) it is observed that as (n) increases the unit time profit of the system increases.

4. Table (4) reveals that as the demand factor (a) increases, the unit time profit of the system also.

5. From Above sensitive table it has been observed (a) is insensitive to T, t₁ and moderate sensitive to F(t₁,T). d and θ are negligible sensitive to t₁,T and moderate sensitive to F(t₁,T). ( n) is moderate sensitive to t₁,T and F(t₁,T).

NUMERICAL ILLUSTRATION FOR CASE-2

M= 0.75, θ = 0.009, I_p = 0.12 units, I_e = 0.10 units, p= 550, n = 5, p, = 200, s= 15 rs/unit, l=13 rs/unit, a=600, A = 100rs/ order

h= 1.5 rs/unit, d = 0.15, C = 120, δ = 2.2.

Applying the solution procedure described above the optimal values obtained is as follows:
t₁*= 0.650123, T* = 1.13911, F*(t₁,T) = 589849rs, Q* = 11325.19 units.
Effects of various parameters on Total Profit per Unit Time For Case-II

Effects of parameter "θ" on Total Profit per Unit Time

<table>
<thead>
<tr>
<th>%change in θ</th>
<th>θ</th>
<th>t1</th>
<th>T</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>-15%</td>
<td>0.00765</td>
<td>0.650248</td>
<td>1.13921</td>
<td>590785</td>
</tr>
<tr>
<td>-10%</td>
<td>0.0081</td>
<td>0.650201</td>
<td>1.13917</td>
<td>590414</td>
</tr>
<tr>
<td>-5%</td>
<td>0.00855</td>
<td>0.650162</td>
<td>1.13914</td>
<td>590132</td>
</tr>
<tr>
<td>0</td>
<td>0.009</td>
<td>0.650123</td>
<td>1.13911</td>
<td>589849</td>
</tr>
<tr>
<td>5%</td>
<td>0.0135</td>
<td>0.649773</td>
<td>1.13883</td>
<td>589562</td>
</tr>
<tr>
<td>10%</td>
<td>0.0099</td>
<td>0.650051</td>
<td>1.13906</td>
<td>589336</td>
</tr>
<tr>
<td>15%</td>
<td>0.01035</td>
<td>0.650015</td>
<td>1.13903</td>
<td>589089</td>
</tr>
</tbody>
</table>

TABLE-5 & FIGURE-6

Effects of parameter "d" on Total Profit per Unit Time

<table>
<thead>
<tr>
<th>%change in d</th>
<th>d</th>
<th>t1</th>
<th>T</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>-15%</td>
<td>0.1275</td>
<td>0.638989</td>
<td>1.13142</td>
<td>349821</td>
</tr>
<tr>
<td>-10%</td>
<td>0.135</td>
<td>0.643069</td>
<td>1.13423</td>
<td>430529</td>
</tr>
<tr>
<td>-5%</td>
<td>0.1425</td>
<td>0.646688</td>
<td>1.13677</td>
<td>510206</td>
</tr>
<tr>
<td>0</td>
<td>0.15</td>
<td>0.650123</td>
<td>1.13911</td>
<td>589849</td>
</tr>
<tr>
<td>5%</td>
<td>0.1575</td>
<td>0.653306</td>
<td>1.14124</td>
<td>669426</td>
</tr>
<tr>
<td>10%</td>
<td>0.165</td>
<td>0.656215</td>
<td>1.14314</td>
<td>748852</td>
</tr>
<tr>
<td>15%</td>
<td>0.1725</td>
<td>0.658892</td>
<td>1.14464</td>
<td>828551</td>
</tr>
</tbody>
</table>

TABLE-6 & FIGURE-7

Effects of parameter "n" on Total Profit per Unit Time

<table>
<thead>
<tr>
<th>%change in n</th>
<th>n</th>
<th>t1</th>
<th>T</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>-15%</td>
<td>4.25</td>
<td>0.635379</td>
<td>1.12888</td>
<td>281947</td>
</tr>
<tr>
<td>-10%</td>
<td>4.5</td>
<td>0.640847</td>
<td>1.13272</td>
<td>386612</td>
</tr>
<tr>
<td>-5%</td>
<td>4.75</td>
<td>0.645699</td>
<td>1.13611</td>
<td>488940</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>0.650123</td>
<td>1.13911</td>
<td>589849</td>
</tr>
<tr>
<td>5%</td>
<td>5.25</td>
<td>0.654081</td>
<td>1.14174</td>
<td>689485</td>
</tr>
<tr>
<td>10%</td>
<td>5.5</td>
<td>0.657608</td>
<td>1.14401</td>
<td>788197</td>
</tr>
<tr>
<td>15%</td>
<td>5.75</td>
<td>0.66072</td>
<td>1.14591</td>
<td>885979</td>
</tr>
</tbody>
</table>
### TABLE 7 & FIGURE 8

Effects of parameter ''a'' on Total Profit per Unit Time

<table>
<thead>
<tr>
<th>%change in parameter</th>
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<th>a</th>
<th>t1</th>
<th>T</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>-15%</td>
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<td>1.13911</td>
<td>501371</td>
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</tr>
<tr>
<td>-10%</td>
<td>36000</td>
<td>0.650123</td>
<td>1.13911</td>
<td>530866</td>
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</tr>
<tr>
<td>-5%</td>
<td>38000</td>
<td>0.650123</td>
<td>1.13911</td>
<td>560358</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>40000</td>
<td>0.650123</td>
<td>1.13911</td>
<td>589849</td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>42000</td>
<td>0.650123</td>
<td>1.13911</td>
<td>619344</td>
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</tr>
<tr>
<td>10%</td>
<td>44000</td>
<td>0.650123</td>
<td>1.13911</td>
<td>648836</td>
<td></td>
</tr>
<tr>
<td>15%</td>
<td>46000</td>
<td>0.650123</td>
<td>1.13911</td>
<td>678326</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 8 & FIGURE 9

Sensitivity Analysis

<table>
<thead>
<tr>
<th>parameter</th>
<th>% change</th>
<th>%change t₁</th>
<th>%change T</th>
<th>%change F(t₁)</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ</td>
<td>-15</td>
<td>0.00019227</td>
<td>0.00008778</td>
<td>0.001586847</td>
</tr>
<tr>
<td></td>
<td>-5</td>
<td>0.0000598</td>
<td>0.00002633</td>
<td>0.000479784</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>-0.0005384</td>
<td>-0.000245806</td>
<td>-0.000486565</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>-0.0001661</td>
<td>-0.00007023</td>
<td>-0.001288465</td>
</tr>
<tr>
<td>d</td>
<td>-15</td>
<td>-0.017126</td>
<td>-0.006750884</td>
<td>-0.406931265</td>
</tr>
<tr>
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<td>-0.002054235</td>
<td>-0.135022692</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.004896</td>
<td>0.001869881</td>
<td>0.134910799</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.01348822</td>
<td>0.004854667</td>
<td>0.404683232</td>
</tr>
<tr>
<td>n</td>
<td>-15</td>
<td>-0.0226788</td>
<td>-0.008980695</td>
<td>-0.522001394</td>
</tr>
<tr>
<td></td>
<td>-5</td>
<td>-0.0068049</td>
<td>-0.002633635</td>
<td>-0.171075987</td>
</tr>
<tr>
<td></td>
<td>5</td>
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<td>0.00230882</td>
<td>0.168917808</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.01629999</td>
<td>0.005969573</td>
<td>0.502043743</td>
</tr>
<tr>
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<td>-15</td>
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<td>0</td>
<td>-0.150001102</td>
</tr>
<tr>
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<td>-5</td>
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<td>-0.049997542</td>
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<tr>
<td></td>
<td>5</td>
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<td>0</td>
<td>0.050004323</td>
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<td>15</td>
<td>0</td>
<td>0</td>
<td>0.149999407</td>
</tr>
</tbody>
</table>
VI. OBSERVATIONS

1. From table (5) it is observed that as the rate of backlogging ($\theta$) decreases, the unit time profit of the system increases.

2. From table (6) it is observed that as discount (d) increases the unit time profit of the system increases.

3. From table (7) it is observed that as (n) increases the unit time profit of the system increases.

4. Table (8) reveals that as the demand factor (a) increases, the unit time profit of the system also.

5. From above sensitive table it has been observed (a) is insensitive to T, t, and moderate sensitive to F(t, T), (d) and (\theta) are negligible sensitive to t, T and (d) is moderate sensitive to F(t, T) and (\theta) is negligible sensitive to F(t, T). (n) is negligible sensitive to t, T and moderate sensitive to F(t, T).

VII. CONCLUSION

In the present paper an inventory model is developed with realistic features of seasonal quadratic demand time dependent deterioration, partial backlogging. The impressive feature of the modal is back order price discount and permissible delay in payment. Because of credit period the retailers earn more interest on the accumulated sales revenue and pays interest on the amount not sold till credit period. The present paper is prepared considering the above mentioned realistic features. Two possible cases for credit period are discussed and the numerical illustrations, and sensitivity analysis represents the reality of the model to practical situations of the market. The present study can be further studied for some other situations of demand and partial backlogging useful for inventory problems.

REFERENCES


