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Comparative Analysis of Statistical Approach for Face Recognition

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ABSTRACT

The research on face recognition has been actively going on in the recent years because face recognition spans numerous fields and disciplines such as access control, surveillance and security, credit-card verification, criminal identification and digital library. Many methods have been proposed for face recognition but each has its own drawback due to the variety of uncontrolled scenarios such as illumination, pose variations and occlusions. As such in this paper gives the detailed study of different present implementation outline methods along with their comparative measures and result analysis.

Keywords- Principal component statistical approaches eigen feature based methods eigenface fisherface Fisher's Linear Discriminant (FLD) ,singular values,FSV

I. INTRODUCTION

The developing of face recognition system is quite difficult because human faces is quite complex, multidimensional and corresponding on environment changes. For that reason the machine recognition of human faces is a challenching problem due the changes in the face identity and variation between images of the same due to illumination , pose variations and some natural effects.The issues are how the features adopted to represent a face under environmental changes and how we classify a new face image based on the chosen representation.Computers that recognize human faces systems have been applied in many applications such as security system, mug shot matching and model-based video coding. The eigenfaces is well known method for face recognition. Sirovich and Kirby [1] had efficiently representing human faces using principle component analysis. M.A Turk and Alex P. Pentland [2] developed the near real-time eigenfaces systems for face recognition using eigenfaces and Euclidean distance.

Most effort in the literature have been focused mainly on developing feature extraction methods and employing powerful classifiers such as neural networks (NNs) [3,5].The main trend in feature extraction has been representing the data in a lower dimensional space computed through a linear or non-linear transformation satisfying certain properties. Statistical techniques have been widely used for face recognition and in facial analysis to extract the abstract features of the face patterns. Principal component analysis (PCA) [1][7],[8] and linear discriminant analysis (LDA) [3][7] are two main techniques used for data reduction and feature extraction in the appearance-based approaches. Eigen-faces and fisher-faces [6] built based on these two techniques, have been proved to be very successful. LDA algorithm selects features that are most effective for class separability while PCA selects features important for class representation. A study in [10] demonstrated that PCA might

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outperform LDA when the number of samples per class is small and in the case of training set with a large number of samples, the LDA still outperform the PCA. Compared to the PCA method, the computation of the

LDA is much higher [4] and PCA is less sensitive to different training data sets. However, simulations reported in [4] demonstrated an improved performance using the LDA method compared to the PCA approach. When dimensionality of face images is high, LDA is not applicable To resolve this problem we combine the PCA and LDA methods, by applying PCA to preprocessed face images, we get low dimensionality images which are ready for applying LDA. Finally to decrease the error rate in spite of Euclidean distance criteria this was used in [4]. A system is implemented using neural network to classify face images based on its computed LDA features.Kirby and Sirovich [11] showed that any particular face can be (1) economically represented along the eigenpictures coordinate space, and (2) approximately reconstructed using just a small collection of eigenpictures and their corresponding projections ('coefficients').method. A recent major improvement on PCA is to directly manipulate on two-dimensional matrices (not one-dimensional vectors as in traditional PCA), e.g., two-dimensional PCA (2DPCA) [13], generalized low rank approximation of matrices [14], non-iterative generalized low rank approximation of matrices [15] The advantages of manipulating on two-dimensional matrices rather than one-dimensional vectors are [13]: (1) it is simpler and straightforward to use for image feature extraction; it is better in terms of classification performance; and it is computationally more efficient. Based on the viewpoint of minimizing reconstruction error, the above PCA- based methods [12,13–15] are unsupervised methods that do not take the class labels into consideration.Taking the class labels into consideration, LDA aims at projecting face samples to a subspace where the samples belonging same class
The major problem in applying LDA to face recognition is the so-called small sample size (SSS) problem which leads to the singularities of the within-class and between-class scatter matrices. Recently, researchers have exerted great endeavor to deal with this problem. In [6,7], a PCA procedure was applied prior to the LDA procedure, which led to the well known PCA+LDA or Fisher faces method.In [7,8], samples were first projected to the null space of the within-class scatter matrix and then LDA was applied in this null space to yield the optimal (infinite) value of the Fisher's linear discriminant criterion, which led to the so-called discriminant common vectors (DCV) method. In [19, 20], LDA was applied in the range space of the between-class scatter matrix to deal with the SSS problem, which led to the LDA via QR decomposition (LDA/QR) method. In [3] a general and efficient design approach using a radial basis function (RBF) neural classifier to cope with small training sets of high dimension, which is a problem frequently encountered in face recognition, is presented. In order to avoid over-fitting and reduce the computational burden, face features are first extracted by the principal component analysis (PCA) method. Then, the resulting features are further processed by the Fisher's linear discriminant (FLD) technique to acquire lower-dimensional discriminant patterns. These DR methods have been proven to effectively lower the dimensionality of Face Image. Furthermore, in face recognition, PCA and LDA have become de-facto baseline approaches.

These DR methods [1] have been proven to effectively lower the dimensionality of Face Image. Furthermore, in face recognition, PCA and LDA have become de-facto baseline approaches. However, despite of the achieved successes, these FR methods will inevitably lead to poor classification performance in case of great facial

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variations such as expression, lighting, occlusion and so on, due to the fact that the face image A on which they manipulate is very sensitive to these facial variations. It is illustrated that the eigen value of an image are not necessarily be stable hence discrimination of images affected by illumination and other said factors is very difficult.

A paradigm is proposed [2] called Singular value decomposition (SVD) based which uses the singular values(SVD consists of finding the eigenvalues and eigenvectors) for feature extraction which represent algebraic properties of an image and have good stability and good discrimination ability was obtained. But In [3] it is illustrated that singular values of an image are stable and represent the algebraic attributes of an image, being intrinsic but not necessarily visible. Moreover SVs are very sensitive to facial variations such as illumination, occlusions, thus it gives the good discrimination results only when the illumination effect is uniform. Based on the observations a new method is proposed [1] method in which the weights of the facial variation sensitive base images (SVs) are deflated by a parameter α called fractional order singular value decomposition representation (FSVDR) to alleviate facial variations for face recognition and gives the good classification result even in non-uniform effects.

II. GENERAL FACE RECOGNITION SYSTEM

Due to the complexity of the face recognition problem, a modular approach was taken whereby the system was separated into smaller individual stages. Each stage in the designed architecture performs a intermediate task before integrating the modules into a complete system. The face recognition system developed performs three major tasks pre-processing of given face image, extracting the face feature for recognition, and performing classification for the given query sample. The system operates on two phase of operation namely training and testing phase. NN Classifier

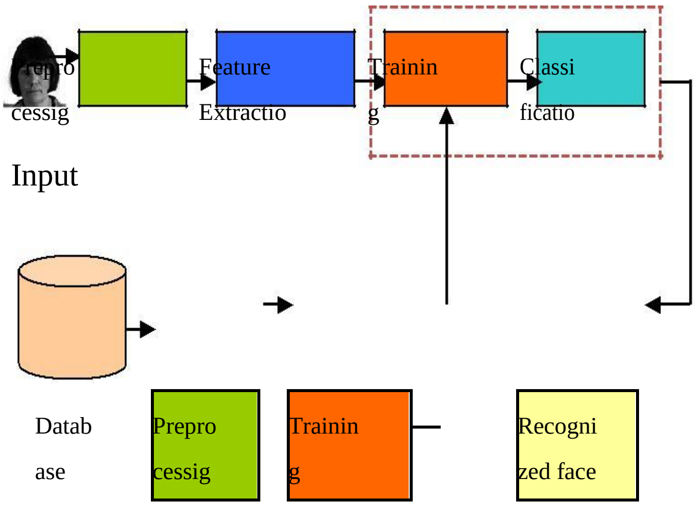

Figure 2.1 Functional Blocks of System

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2.1 Eigen Value Based Approach

2.2.1 Overview

When designing a complex system, it is important to begin with strong foundations and reliable modules before optimizing the design to account for variations. Provided a perfectly aligned standardized database is available, the face recognition module is the most reliable stage in the system. In face recognition still lies in the

normalization and preprocessing of the face images so that they are suitable as input into the recognition module. Hence, the face recognition module was designed and implemented first.

Eigenface approach is one of the earliest appearance-based face recognition methods, which was developed by M. Turk and A Pentland [12] in 1991. This method utilizes the idea of the principal component analysis and decomposes face images into a small set of characteristic feature images called eigenfaces The idea of using eigenfaces was motivated by a technique for efficiently representing pictures of faces using principal component analysis. It is argued that a collection of face images can be approximately reconstructed by storing a small collection of weights for each face and a small set of standard pictures.

The eigenfaces approach for face recognition involves the following initialization operations: [33].

1. Acquire a set of training images.

2. Calculate the eigenfaces from the training set, keeping only the best *M* images with the highest eigen values. These *M* images define the "face space". As new faces are experienced, the eigenfaces can be updated.

3. Calculate the corresponding distribution in *M*-dimensional weight space for each known individual (training image), by projecting their face images onto the face space.

Having initialized the system, the following steps are used to recognize new face images:

1. Given an image to be recognized, calculate the eigen features of the *M* eigenfaces by projecting the it onto each of the eigenfaces.

2. Determined features are further processed using pca so as reduce the dimension of the image so as to have more samples since more eigenfaces will always produce greater classification accuracy, since more information is available.

3. However, the eigenface paradigm, [3] which uses principal component analysis (PCA), yields projection directions that maximize the total scatter across all classes, i.e., across all face images. In choosing the projection which maximizes the total scatter, the PCA retains unwanted variations caused by lighting, facial expression,and other factors [3]. Accordingly, the features produced are not necessarily good for discrimination among classes. In [3], [12],the face features are acquired by using the fisherface or discriminant eigenfeature paradigm. This paradigm aims at overcoming the drawback of the eigenface paradigm by integrating Fisher's linear discriminant (FLD) criteria, while retaining the idea of the eigenface paradigm in projecting faces from a high-dimension image space to a significantly lower-dimensional feature space.

4. These features are classified by using RBF classifier as either a known person or as unknown. The goal of using neural networks is to develop a compact internal representation of faces,which is equivalent to feature extraction. Therefore, the number of hidden neurons is less than that in either input or output layers, which

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results in the network encoding inputs in a smaller dimension that retains most of the important information. smaller dimension that retains most of the important information. Then, the hidden units of the neural network can serve as the input layer of another neural network to classify face images.

2.2.2 Calculating Eigenfaces :

Mathematically, the eigenface approach uses PCA to calculate the principal components and vectors that best account for the distribution of a set of faces within the entire image space. Considering an image as being a point in a very high dimensional space, these principal components are essentially the eigenvectors of the covariance matrix of this set of face images, which Turk and Pentland [12] termed the eigenface. Each individual face can then be represented exactly by a linear combination of eigenfaces, or approximately, by a subset of "best" eigenfaces – characterized by its eigenvalues,

Let the training set of face images be $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_M$. The average face of the set if defined by

$$\Psi = \frac{1}{M}\sum_{n=1}^{M}\Gamma_n . \qquad (2.1)$$

Figure 2.1 Mean Image

Each face differs from the average by the vector $\Phi_n = \Gamma_n - \Psi$. An example training set is shown in Figure 1a, with the average face Ψ shown in Figure 1b. This set of very large vectors is then subject to principal component analysis, which seeks a set of M orthonormal vectors, μ_n , which best describes the distribution of the data. The kth vector, μ_k is chosen such that

$$\lambda_k = \frac{1}{M}\sum_{n=1}^{M}(\mu_k^T \Phi_n)^2 \qquad (2.2)$$

is a maximum, subject to

$$\mu_l^T \mu_k = \begin{cases} 1, l = k \\ 0, otherwise \end{cases} \qquad (2.3)$$

The vectors μ_k and scalars λ_k are the eigenvectors and eigenvalues, respectively, of the covariance matrix

$$C = \frac{1}{M}\sum_{n=1}^{M}\Phi_n \Phi_n^T = AA^T \qquad (2.4)$$

where the matrix $A = [\Phi_1 \Phi_2 ... \Phi_M]$. The matrix C, however, is N^2 by N^2, and determining the N^2 eigenvectors and eigenvalues is an intractable task for typical image sizes. A computationally feasible method is needed to find these eigenvectors.

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If the number of data points in the image space is is less than the dimension of the space (M>N^2),there will be only *M-1* , rather than *N ^2,* meaningful eigenvectors (the remaining eigenvectors will have associated eigen values of zero). Fortunately, we can solve for the 2 *N* - dimensional eigenvectors in this case by first solving for the eigenvectors of and *M* by *M* matrix—e.g., solving a 16 x 16 matrix rather than a 16,384 x 16,384 matrix and then taking appropriate linear

combinations of the face images Φ_n. Consider the eigenvectors v_n of $A^T A$ such that

$$A^T A v_n = \lambda_n v_n \qquad (2.5)$$

Premultiplying both sides by A, we have

$$AA^T A v_n = \lambda_n A v_n \qquad (2.6)$$

from which we see that Av_n are the eigenvectors of $C = AA^T$ .Following this analysis, we construct the M by M matrix $L = A^T A$, where $L_{mn} = \Phi_m^T \Phi_n$, and find the M eigenvectors v_n of L. These vectors determine linear combinations of the M training set face images to form the eigenfaces μ_n :

$$\mu_n = \sum_{k=1}^{M} v_{nk} \Phi_k = A v_n, n = 1, \ldots\ldots M \qquad (2.7)$$

Figure 2.2 Eigen Faces

The associated eigenvalues allow us to rank the eigenvectors according to their usefulness in characterizeing the variation among the images.

2.2.3 FLD for Class Discrimination

The PCA paradigm [3] does not provide any information for class discrimination but dimension reduction. Accordingly, the FLD[3] is applied to the projection of the set of training samples in the eigenface space

$$X = (X_1, X_2, X_3, \ldots\ldots X_n) \subset \Re^{r \times n} .$$

The paradigm finds an optimal subspace for classification in which the ratio of the between-class scatter and the within-class scatter is maximized. Let the between class scatter matrix be defined as

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$$S_B = \sum_{i=1}^{C} n^i (\overline{X_l} - \overline{X})(\overline{X_l} - \overline{X})^T \quad (2.8)$$

and the within-class scatter matrix be defined as

$$S_W = \sum_{i=1}^{C} \sum x_i \in n^i (\overline{X_l} - \overline{X})(\overline{X_l} - \overline{X})^T \quad (2.9)$$

Where $X = (1/n)\sum_{j=1}^{c} X_i$ is the mean image of the ensemble, and $\overline{X}^i = \left(1/n^i \right) \sum_{j=1}^{n^i} X_j^i$ is the mean image of the ith class and c is the number of classes. The optimal subspace by the FLD is determined as follows

$$E_{optimal} = \arg \max_E \frac{|E^T S_B E|}{|E^T S_W R|} = [e_1, e_2, \ldots . e_{C-1}] \quad (2.10)$$

where $(e_1, e_2, e_3, \ldots e_{c-1})$ is the set of generalized eigenvectors of S_B and S_W corresponding to the c-1 largest generalized eigen values $\lambda_i = 1,2,3,\ldots . c-1$ i.e. $S_B E_i = \lambda_i S_W E_i$ (2.11)

Thus the feature vectors P for any query face image Z in the most discriminant sense can be calculated as follows:

$$p = E_{optimal}^T U^T Z \quad (2.12)$$

2.2.4 Classification of Face Image(s) :

The eigenface images calculated from the eigenvectors of L span a basis set with which to describe face images. As mentioned before, the usefulness of eigenvectors varies according their associated eigenvalues. This suggests to pick up only the most meaningful eigenvectors and ignore the rest, in other words, the number of basis function is further reduced from M to M' (M'<M) and the computation is reduced as a consequence. Experiments have shown that the RMS pixel-by-pixel errors in representing cropped versions of face images are about 2% with M=115 and M'=40.In practice, a smaller M' is sufficient for identification, since accurate reconstruction of the image is not a requirement. In this framework, identification becomes a pattern recognition task. The eigenfaces span an M' dimensional subspace of the original *N* 2 image space. The M' most significant eigenvectors of the L matrix are chosen as those with the largest associated eigenvalues. A new face image Γ is transformed into its eigenface components (projected onto "face space") by a simple operation

$$\omega_n = \mu_n(\Gamma - \Psi) \text{ for } n=1,\ldots\ldots,M'.$$

This describes a set of point-by-point image multiplications and summations. The weights form a vector

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$$\Omega^T = [\omega_1, \omega_2, \omega_3 ......\omega_{M'}]$$

that describes the contribution of each eigenface in representing the input face image, treating the eigenfaces as a basis set for face images. The vector may then be used in a standard pattern recognition algorithm to find

which of a number of predefined face classes, if any, best describes the face. The simplest method for determine which face class provides the best description of an input face image is to find the face class k that minimizes

Euclidian distance $\varepsilon_k^2 = \|(\Omega - \Omega_k)^2\|$ where Ω_k is a vector describing k th face class.

The face classes Ω_k are calculated by averaging the results of the eigenface representation over a small number of face images (as few as one) of each individual. A face is classified as "unknown", and optionally used to create a new face class. Because creating vector of weights is equivalent to projecting the original face image onto to low-dimensional face space, many images (most of them looking nothing like a face) will project onto a given pattern vector. This is not a problem for the system; however, since the distance between the image and the face space is simply the squared distance between the mean-adjusted input image

$\Phi_i = \Gamma - \Psi$ and $\Phi_j = \sum_{i=1}^{M} \omega_i \mu_i$, its projection onto face space is :

$$\varepsilon^2 = \|\Phi - \Phi_f\|^2$$

Thus there are four possibilities for an input image and its pattern vector: (1) near face space and near a face class; (2) near face space but not near a known face class; (3) distant from face space and near a face class; (4) distant from face space and not near a known face class. In the first case, an individual is recognized and identified. In the second case, an unknown individual is present. The last two cases indicate that the image is not a face image. Case three typically shows up as a false positive in most recognition systems; in this framework, however, the false recognition may be detected because of the significant distance between the image and the subspace of expected face images. So, the eigenfaces approach for face recognition could be summarized as follows:

Collect a set of characteristic face images of the known individuals. This set should include a number of images for each person, with some variation in expression and in the lighting (say four images of ten people, so M=40). Calculate the (40 x 40) matrix L, find its eigenvectors and eigenvalues, and choose the M' eigenvectors with the highest associated eigenvalues (let M'=10 in this example).Combine the normalized training set of images according to Eq. (6) to produce the (M'=10) eigenfaces k= ,...1..,2…,M. For each known individual, calculate the class vector Ω_k by averaging the eigenface pattern vectors [from Eq.(2.8)] calculated from the original (four) images of the individual. Choose a threshold θ_k that defines the maximum allowable distance from any face class, and a threshold θ that defines the maximum allowable distance from face space [according to Eq. (2.9)].

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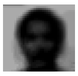
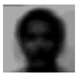
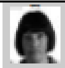
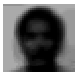
For each new face image to be identified, calculate its pattern vector Ω the distance ε_k to each known class, and the distance ε to face space. If the minimum distance $\varepsilon_k \langle \theta_\varepsilon$ and the distance $\varepsilon \langle \theta$, classify the input face as the individual associated with class vector Ω_k. If the minimum distance $\varepsilon_k \rangle \theta_\varepsilon$ but $\varepsilon \langle \theta$, then the image may be classified as "unknown", and optionally used to begin a new face class. If the new image is classified as a known individual, this image may be added to the original set of familiar face images, and the eigenfaces may be recalculated (steps 1-4). This gives the opportunity to modify the face space as the system encounters more instances of known faces.

2.2.5 Experimental Results

For the implementation of face recognition a well known face database called YALE face database [31] is used. YALE face database contains 165 gray level face images of 15 persons. There are 11 images per subject, and these 11 images are, respectively, under the following different facial expression or configuration: center-light, happy, left-light,glasses, normal, right-light, sad, sleepy, surprised, and wink. In this implementation, all images are sized to a size of 128 x 128.

For classification or recognition purpose for the given training and test samples:

Table 2.1 Result analysis for Eigen based approach

Input Image	Input	Mean Image	MSE	Gradient x1e-010	Epochs	Result
Present in db			0..076125	2.26448	3	 True
right light on			0.475121	3.44942	24	 False
left light on			0.400031	0.092685	14	 False

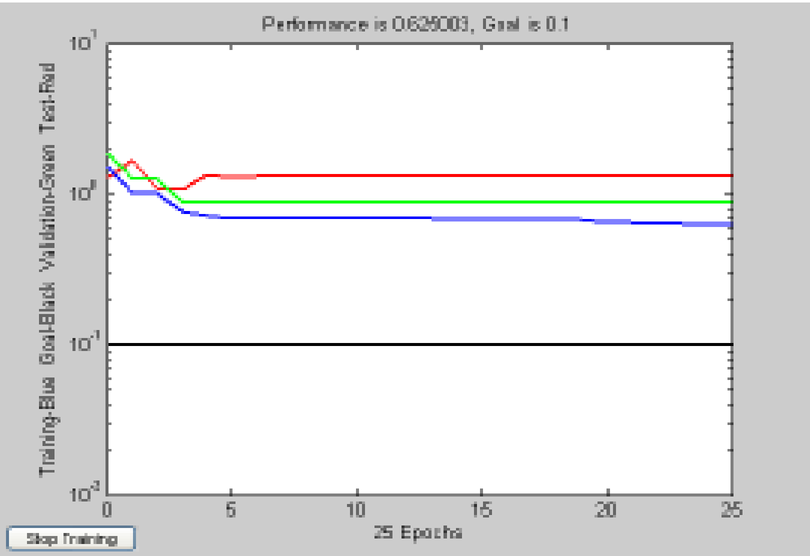


Figure 2.3: Training and Test Performance

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Out of the 12 faces, 8 are correctly classified in the 1st match. Hence the total accuracy for the eigen feature based approach is 66.67% for illuminated affected Yale database. Thus in real time scenarios this method may be inappropriate for the illuminated affected database.

III. SINGULAR VALUE BASED FACE RECOGNITION

3.1 Introduction

The singular value decomposition is a outcome of linear algebra. It plays an interesting fundamental role in many different applications. On such application is in digital image processing. SVD in digital applications provides a robust method of storing large images as smaller, more manageable square ones. This is accomplished by reproducing the original image with each succeeding nonzero singular value. Furthermore, to reduce storage size even further, images may approximated using fewer singular values.

3.2 Singular Value Decomposition

The singular value decomposition of a matrix A of m x n matrix is given in the form,

$$A = U\sum V^T \qquad (3.1)$$

Where U is an m x m orthogonal matrix; V an n x n orthogonal matrix, and Σ is an m x n matrix containing the singular values of A and $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n \geq 0$ along its main diagonal. A similar technique, known as the eigenvalue decomposition (EVD), diagonalizes matrix A, but with this case, A must be a square matrix. The EVD diagonalizes A as

$$A = VDV^{-1} \qquad (3.2)$$

Where D is a diagonal matrix comprised of the eigenvalues, and V is a matrix whose columns contain the corresponding eigenvectors. Where Eigen value decomposition may not be possible for all facial images SVD is the result.

3.2.1 SVD Working Principle

Let A be an m x n matrix. The matrix A^TA is symmetric and can be diagonalized. Working with the symmetric matrix A^TA, two things are true:

1. The eigenvalues of A^TA will be real and nonnegative.
2. The eigenvectors will be orthogonal.

To derive two orthogonal matrices U and V that diagonalizes a m x n matrix A,First, if it is required to factor A as $A = U\sum V^T$ then the following must be true.

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$$A^T A = (U \Sigma V^T)^T (U \Sigma V^T)$$
$$A^T A = V \Sigma^T U^T U \Sigma V^T$$
$$A^T A = V \Sigma^T \Sigma V^T$$
$$A^T A = V \Sigma^2 V^T \qquad (3.3)$$

this implies that Σ^2 contains the eigen values of $A^T A$ and V contains the corresponding eigenvectors. To find the V of the svd , $A = U \sum V^T$ rearrange the eigen values of $A^T A$ in order of decreasing magnitude and $\lambda_1 \ge \lambda_2 \ge ..\lambda_r \ge \lambda_{r+1} = \lambda_{r+2} = ..\lambda_n$ some eigen values are set equal to zero. Define the singular values of A as the square root of the corresponding eigenvalues of the matrix $A^T A$ that is,

$$\bar{\sigma}_j = \sqrt{\lambda_j}, where \quad j = 1,2,.....n \qquad (3.4)$$

re-arranging the eigenvectors of $A^T A$ in the same order as their respective eigenvalues to produce the matrix

$$V = [V_1, V_2.....V_r, V_{r+1}, V_{r+2},......V_n] \qquad (3.5)$$

Let the rank of A be equal to r. Then r is also the rank of $A^T A$, which is also equal to the number of nonzero eigenvalues. Let $\sigma_j = \sqrt{\lambda_j}$ and $V = [V_1, V_2,.......,V_r]$ and $V_2 = [V_{r+1},......V_n]$ be the set of eigenvectors associated with the non-zero eigenvalues and be the set of eigenvectors associated with zero eigen values. It follows that:

$$AV_2 = (Av_{r+1}, Av_{r+2},......Av_n)$$
$$AV_2 = (0,0,...0)$$
$$= 0 \qquad (3.6)$$

where this zero is the zero matrixes.It is defined earlier that the matrix Σ to be the diagonal matrix with the singular values of A along its main diagonal. From above equation, each zero eigenvalue will result in a singular value equal to zero. Let Σ^1 be a square r x r matrix containing the nonzero singular values $\{\sigma_1, \sigma_2,......\sigma_r\}$ of A along its main diagonal. Therefore matrix Σ may be represented by:

$$\Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \qquad (3.7)$$

Where the singular values along the diagonal are arranged in decreasing magnitude, and the zero singular values are placed at the end of the diagonal. This new matrix Σ, with the correct dimension m x n, is padded with m - r rows of zeros and n - r columns of zeros. To find the orthogonal matrix U. Looking at the equation $AV = U \Sigma$ it follows that :

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$$A[v_1,..,v_r,v_{r+1},.v_n]=[u_1,.u_r,.u_{r+!},.u_n]\begin{bmatrix}\sigma_1 & & & & & \\ & \cdots & & & & \\ & & \sigma_r & & & \\ & & & 0 & & \\ & & & & \cdots & \\ & & & & & 0\end{bmatrix}$$

$$[Av_1,...Av_r,..Av_{r+1},..Av_n]=[\sigma_1 u_1,...,\sigma_r u_r,0..0] \qquad (3.8)$$

Therefore, for i= 1,……r

$$Av_j=\sigma_j u_j \qquad (3.9)$$

Examining above equation σ_j is a scalar value and that v_j and u_j is column vectors and a matrix vector multiplication results in another vector. Therefore, the vector resulting from the multiplication of Av_j is equal to the vector u_j multiplied by the scalar Av_j .It could be observed at the vector Av_j as lying in the direction of the unit vector u_j with absolute length σ_j . Av_j can be calculated from previously found matrix V. Therefore, the unit vector u_j is a result of dividing the vector Av_j by its magnitude σ_j .

$$u_j=\frac{Av_j}{\sigma_j} \qquad (3.10)$$

This Equation is restricted to the first r nonzero singular values. This allows to finding the column vectors of so long as there is no division by zero. Therefore this method allows finding only part of the matrix U. To find the other part where the singular values of A are equal to zero. As seen before in the matrix V, the matrix U may be defined as:

$$U=[U_1,U_2] \qquad (3.11)$$

Letting

$$U_1=[u_1,...,u_2]$$

$$U_2=[u_{r+1},...,u_m]$$ Then

$$Av_1=A[v_1,......v_r]$$

$$=[Av_1,......Av_r]$$

$$=[\sigma_1 u_1,......\sigma_r u_r]$$

$$=[u_1,......u_r]\begin{bmatrix}\sigma_1 & & \\ & \cdots & \\ & & \sigma_r\end{bmatrix}$$

$$Av_1=U_1\Sigma_1 \qquad (3.12)$$

referring to the illustration of the four fundamental subspaces the null space ṆẠ of a matrix A,denotes the set of all nontrivial (non-zero) solutions to equation $Ax=0$. Using above equation $Av_2=0$ where zero represents a zero matrix ….**(3.13)**

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It follows that V_2 forms a basis for the $N(A)$ Also because $Av_j = Ou_j$ (3.14)

Where $j = r+1, r+2,..n$, $Av_j = 0$ and $v_j \in N(A)$ The orthogonal complement to the $N(A)$ is $R(A^T$ since the columns in the matrix V are orthogonal, the remaining vectors $[v_1,......v_2]$ must lie in the subspace corresponding to $R(A^T)$ the From above equation, we see that $u_n = \frac{1}{\sigma_n} Av_n$ .This equation holds the valuable information that the column vectors of U, $[u_1,......u_r]$ are in the column space of A. This is because the column vectors of U are linear combinations of the columns of A. or, in matrix notation $u_j \in R(A)$ where. $j = r+1, r+2,....n$ It now follows that $N(A)$ and $R(A^T)$ are orthogonal complement. Since the matrix U is an orthogonal matrix and the first r column vectors of U have been assigned to lie in the $R(A)$, $[u_{r+1},...u_m]$ must lie in the $N(A^T)$ .The vectors that lie in the $N(A^T)$ are the vectors $[u_{r+1},...u_m]$ which form the matrix U_2 .Once matrix V is derived, the matrix Σ ,and the matrix U, the singular value decomposition can be found for any matrix A . where $U\sum V^T$ actually does diagonalize and equal the matrix A.

$$U\sum V^T = [U_1, U_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} [V_1, V_2]^T$$

$$= [U_1, U_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}$$

$$= [U_1, U_2] \begin{bmatrix} \Sigma_1 & V^T \\ 0 & 0 \end{bmatrix}$$

$$= U_1 \Sigma_1 V_1^T + 0$$

$$= U_1 \Sigma_1 V_1^T$$

$$= AV_1 V_1^T$$

$$= AI$$

$$= A$$

. $$U\sum V^T = A \quad (3.15)$$

The rank r is equal to the number of nonzero eigenvalues referring to four fundamental subspaces,it observed that $[v_1,..., v_r]$ are the eigenvectors corresponding to the nonzero eigenvalues of A , and that the remaining column vectors of Vcorrelate with the eigenvalues equal to zero. So there exists an r nonzero singular value. Thus r is equal to the number of nonzero eigenvalues termed as rank of the matrix. These SVD features are used

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for facial feature decomposition to represent an image in dimensionality reduction (DR) factor. An SVD operation breaks down the matrix A into three separate matrices.

$$A = U\sum V^T \quad = [u_n, \ldots u_n] \begin{bmatrix} \sigma_1 & & \\ & \vdots & \\ & & \sigma_n \end{bmatrix} \begin{bmatrix} v_1^T \\ \\ v_n^T \end{bmatrix}$$

$$= [u_1, \ldots u_n] \begin{bmatrix} \sigma_1 v_1^T \\ \vdots \\ \sigma_n v_n^T \end{bmatrix}$$

$$= \sigma_1 u_1 v_1^T + \ldots + \sigma_n u_n v_n^T$$

$$= \sigma_1 u_1 v^T{}_1 + \ldots + \sigma_r u_r v_r^T \text{ because } \sigma_{r+!} \ldots \sigma_n \text{ are equal to zero} \quad (3.16)$$

Singular values at each iteration are obtained as follows.

1. At 1st iteration SV values the facial information provided is given by $A = \sigma_1 u_1 v_1^T$

2. After n=2 iteration the facial approximation is given by, $A = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T$

3. After n=3 iteration the facial approximation is given by, $A = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \sigma_3 u_3 v_3^T$

From the above observations it could be observed that the facial information's are though presented in high leading images such as the eye, mouth and nose regions they are less definitive to facial expressions. So a same face image with facial variation may not be predicted in such SV approach.To overcome this limitation the SVD based face recognition approach is modified to FSVD approach as presented below.

3.3 Face Recognition Using FSV

The main ideas of FSVD approach are that;

(1) The weights of the leading base images $u_i v_i^T$ should be deflated, since they are very sensitive to the great facial variations within the image matrix A itself.

(2) The weights of base images $u_i v_i^T$ corresponding to relatively small σ_i's should be inflated, since they may be less sensitive to the facial variations within A .

(3) The order of the weights of the base images $u_i v_i^T$ in formulating the new representation of SVD should be retained. More specifically, for each face image matrix A which has the SVD, its FSVD 'B' can be defined as,

$$B = U \sum^{\alpha} V^T \quad (3.16)$$

Where U, Σ and V are the SV matrices, and in order to achieve the above underlying ideas, α is a fractional parameter that satisfies $0 \le \alpha \le 1$ .It is seen that the rank of FSVDR 'B' is r, i.e.,identical to the rank of A as the B matrix is fractional raised the values are inflated retaining the rank of the matrix constant. The $u_i v_i^T, i = 1,2, \ldots r$

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form a set of uv^T which are similar to the base images for the SVD approach.It is observed that the intrinsic characteristic of A, the rank, is retained in the FSVD approach. In fact it has the same uv^T like base images as A, and considering the fact that these base images are the components to compose A and B, the information of A is effectively been passed to B. From the observation it could be observed that:

(1) The FSVD is still like human face under lower SV.

(2) The FSVD deflates the lighting condition in vision. Taking the two face images (c) and (d) under consideration, when α is set to 0.4 and 0.1, from the FSVD alone, it is difficult to tell whether the original face image matrix A is of left light on or right light on.

(3) The FSVD reveals some facial details. In the original face images (a) presented, neither the right eyeball of the left face image nor the left eyeball of the right face image is visible, however, when setting α to 0.8 and 0.1 in FSVD, the eyeballs become visible.

In the case of FSVD thus the fractional parameter and it's optimal selection is an important criterion in making the face recognition process more accurate.

3.4 Fractional Parameter 'α'

In FSV based recognition, α is a key parameter that should be adjusted. On a suitable selectivity of α parameter the recognition system can achieve superior performance to existing recognition performance. Further, in images (which are sensitive to facial variations) are deflated but meanwhile the discriminant information contained in the leading base images may be deflated. Some face images have great facial variations and are perhaps in favor of smaller α's, while some face images have slight facial variations and might be in favor of larger α's. The α learned from the training set is a trade off among all the training samples and thus is only applicable to the unknown sample from the similar distribution. Each DR method has its specific application scope, which leads to the difficulty in designing a unique α selection criterion for all the DR methods. As a result, the criterion for automatic choosing α should be dependent on the training samples, the given testing sample and the specific DR method. To optimally choose the α value minimum argument MSV criterion is used. Mean square variance (MSV) criterion state that,

$$\bar{V} = \frac{1}{c}\sum_{i=1}^{C} SV_i \qquad (3.17)$$

where SV_i is the standard variance of the i^{th} class defined as

$$SV_i = \frac{1}{d}\sum_{k=1}^{d}\sqrt{\frac{1}{N_i - 1}\sum_{j=1}^{N_i}\left(x_{jk}^{i} - m_{ik}\right)^2} \qquad (3.18)$$

Where x_{jk}^{i} and m_{ik} respectively, denote the k^{th} element of the d-dimensional samples and class mean m_i C is the number of classes, and N_i is the number of training samples contained in the i^{th} class. For an optimal selection of α value the MSV value must be optimally chosen. The smaller MSV value represents, compact the

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same class samples are, and on the contrary, the bigger MSV is, the looser the same class samples. When the same class samples are very loose, these samples will lead to biased estimation of the class mean, within class and between-class scatter matrices, while on the contrary, when the same class samples are compact, the estimation of the class mean, within-class and between-class variance matrices may be much more reliable. When the same class samples are compact, it is more likely that these samples can nicely depict the Gaussian distribution from which they are generated and considering the fact it is essential for the same class samples to be compact, namely MSV to be small in the recognition methods. The MSV of the training samples is given as ,

$$\alpha_{opt} = \arg\min_{\alpha} MSV(\alpha) \qquad (3.19)$$

3.5 Experimental Results for Singular Value Based Approach:

For this approach,12 recognition faces were randomly picked from the database, then 36 more images were used as training faces; six training faces were picked for each person with different light illumination effects.

Table 3.1 Result analysis for SVD approach

Input Image	Input	Mean Image	MSE	Gradient x1e-010	Epochs	Result
Present in db			0.0979643	0.758578	3	 True
right light on			0.200011	0.0726394	21	 False
left light on			0.475121.	3.44942	7	 False

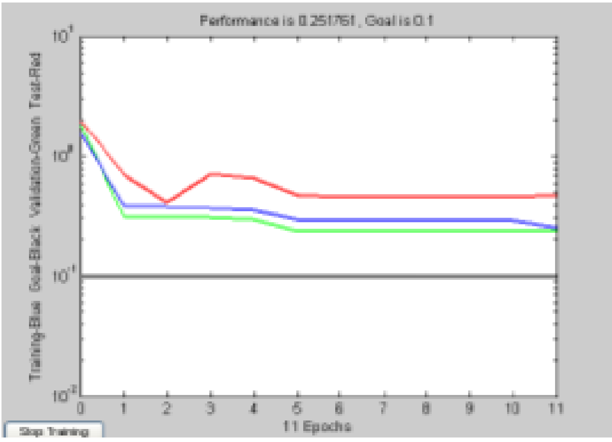

Figure 3.4 : Training and Test Performance

Out of the 12 faces, 9 are correctly classified in the 1st match. Hence the total accuracy for the singular feature based approach is 75%Yale DB.

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3.6 Fractional Singular Value based Approach:

1. Corresponding FSV projection in face-space samples present in database at $\alpha = 1$

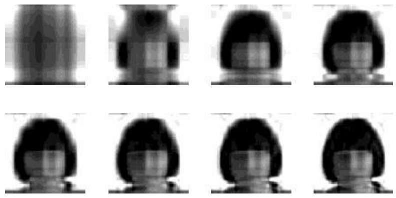


Figure 3.5 FSVD Projections for 8 iterations

2. Corresponding FSV projection in face-space samples having right side light on at $\alpha = 1$

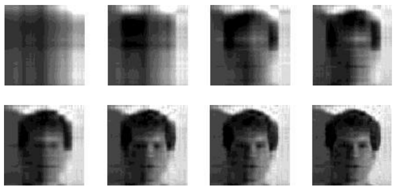


Figure 3.6 FSVD Projections for 8 iterations

3. Corresponding FSV projection in face-space samples having left side light on at $\alpha = 1$

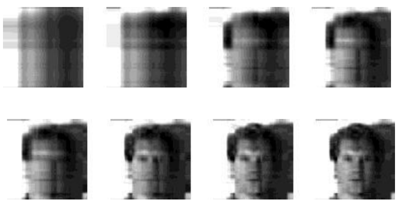
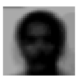
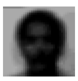
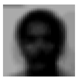


Figure 3.7 FSVD Projections for 8 iterations a

Table 3.2 Result analysis for FSVD approach

Input Image	Input	Mean Image	MSE	Gradient x1e-010	Epochs	Result
Present in db			0.072206.	2.35874	3	 True
right light on			0.0979643	0.758578	6	 True
left light on			0.072206	2.35874	3	 True

From above observation it is found that at $\alpha = 1$ the projected in the face spaces looks like an original images. Hence the recognition accuracy is more as compared to previous two methods.

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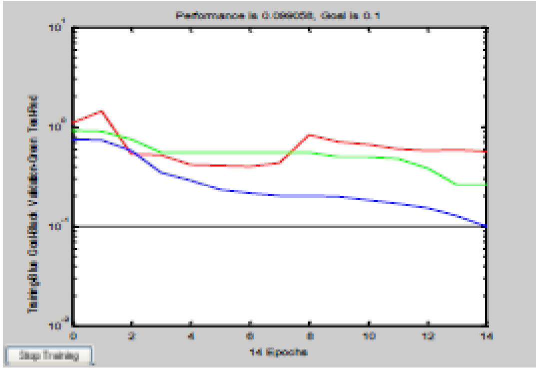
Figure 3.8 Performance Evaluations



Out of the 12 faces, 11are correctly classified in the 1st match at $\alpha = 1$.Total accuracy for the fractional singular feature based approach is 91.96% for illuminated affected Yale database.

IV. CONCLUSION

The objective of this paper is to analyze developed face recognition systems.Here,the effect of environmental and surrounding effect due to illumination are been analyzed using Eigen feature , Singular Feature and Fractional Singular Feature values and the variation of these factors are observed to be effective in face recognition approach. To evaluated performance of the developed approach(s), an evaluation is carried out for recognition accuracy over various face images with lighting effect and expression variation and found that for more accurate results these systems are to be analyzed thoroughly and tested different databases with different effects.

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