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Dynamic System Simplification using Differentiation and Cauer Second Form

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ABSTRACT

The authors proposed a simplification method for reducing the order of linear dynamic systems using differentiation and Cauer second form. In this method, the denominator of the simplified / reduced model is obtained by using differentiation method and numerator is determined via Cauer second form. The proposed method is capable to retain stability, linearity and transient behaviour of the original system. The viability of the method is shown with the help of numerical example taken from literature.

Keywords: Cauer Second Form, Differentiation, Order Reduction, Simplification, Stability.

I. INTRODUCTION

The modelling of large-scale dynamic system plays an important role in diverse field of science and engineering such as complex chemical processes, electrical engineering, pneumatic , thermal, mechanical, etc. and in specially in design and analysis, where control engineers quite often is faced with controlling of a physical system for which an analytic model is represented as a high order system. A mathematical model of a high order system may pose difficulties in its analysis, synthesis and identification. Therefore, it is desirable to approximate it by low order model which retains the main qualitative properties of the original high order system. Many simplification methods have been proposed to reduce the complexity of a high order system from last four decades.

A good number of research papers are available for simplification in frequency [1-4] and time domain [5-6]. Several methods [7-10], which are suggested by mixing two reduction techniques, are available in the literature. In this paper, authors proposed a mixed simplification method in which denominatorpolynomial is obtained by differentiation technique [11] while the coefficients of numerator are obtained by using Cauer second form [12]. The advantages of the Cauer second form are to retain steady-state value and transient nature of the original system.

II. PROBLEM STATEMENT

Consider an n^{th} -order time –invariant single-input single-output (SISO) system described in frequency domain as

$$G(s)=\frac{N(s)}{D(s)}=\frac{a_{21}+a_{22}s+a_{23}s^2+...+a_{2,n}s^{n-1}}{a_{11}+a_{12}s+a_{13}s^2+...+a_{1,n+1}s^n} \quad (1)$$

where $a_{2j};1 \le j \le n$ and $a_{1j};1 \le j \le n+1$ are known scalar constants.

The corresponding $k^{th}(k<n)$ order simplified model is of the form

$$R_K(s)=\frac{N_k(s)}{D_k(s)}=\frac{b_{21}+b_{22}s+b_{23}s^2+\ldots+b_{2,k}s^{k-1}}{b_{11}+b_{12}s+b_{13}s^2+\ldots+b_{1,k+1}s^k} \quad (2)$$

Where $b_{2j};1 \le j \le k$ and $b_{1j}=1 \le j \le k+1$ are known scalar constants.

The aim of this paper is to find the simplified model of the k^{th}-order in the form of equation (2) from the known high order system (1) such that it retains the important characteristics of the original system (1).

III. PROPOSED SIMPLIFICATION METHOD

The proposed method consists of the following two steps:

Step-1: Differentiation method [11] for getting reduced denominator polynomial $D_k(s)$.

The original system consists of a denominator polynomial $D(s)$ as $D(s)=a_{11}+a_{12}s+a_{13}s^2+...+a_{1,n+1}s^n$ (3)

The reciprocal of the $D(s)$ can be written as

$$\dot{D}(s)=s^n D\left(\frac{1}{s}\right)=a_{11}s^n+a_{12}s^{n-1}+...+a_{1,n+1} \quad (4)$$

The equation (4) is successively differentiated $(n-k)$ times, which yield k^{th}-order polynomial as

$$\dot{D}_k(s)=a_{11}s^k+a_{12}s^{k-1}+a_{13}s^{k-2}+...+a_{1,k+1} \quad (5)$$

Now, $\dot{D}_k(s)$ is reciprocated back to get k^{th}-order reduced denominator polynomial as

$$D_k(s)=s^k \dot{D}_k\left(\frac{1}{s}\right)=b_{11}+b_{12}s+b_{13}s^2+\ldots+b_{1,k+1}s^k \quad (6)$$

The purpose of reciprocating the polynomial is to retain the transient behaviour the system.

Step-2: Determination of the numerator $N_k(s)$ of the reduced model using Cauer second form [12].

The coefficients of Cauer second form can be evaluated by using Routh arrays as:

$$h_1=\frac{a_{11}}{a_{21}}\left\langle\begin{matrix} a_{11} & a_{12} & a_{13} & a_{1,n} & a_{1,n+1} \\ a_{21} & a_{22} & a_{23} & a_{2,n} & \end{matrix}\right.$$

$$h_2=\frac{a_{21}}{a_{31}}\left\langle\begin{matrix} a_{21} & a_{22} & a_{23} & a_{2,n} \\ a_{31} & a_{32} & a_{33} & \end{matrix}\right. \quad (7)$$

$$h_3=\frac{a_{31}}{a_{41}}\left\langle\begin{matrix} a_{31} & a_{32} \\ a_{41} & a_{42} \end{matrix}\right.$$

$\vdots$

$\vdots$

The first two rows coefficients are known and taken from the original high order system and rest of the elements are determined by the well known Routh algorithm.

$$a_{i,j} = a_{i-2,j+1} - h_{i-2}a_{i-1,j+1} \quad (8)$$

$$i = 3, 4, \ldots$$
$$j = 1, 2, \ldots$$

$$h_i = \frac{a_{i,1}}{a_{i+1,1}} \qquad i = 1, 2, 3, \ldots, k \quad (9)$$

By matching the coefficients $b_{1,j}\left(j = 1, 2, \ldots (k+1)\right)$ of reduced denominator and Cauer quotations $h_p\left(p = 1, 2, \ldots k\right)$ of equation (8), the coefficients of the reduced numerator $R_k\left(s\right)$ can be obtained. For this purpose, following inverse Routh array is used.

$$b_{i+1,1} = \frac{b_{i,1}}{h_i}; i = 1, 2, \ldots k \quad (10)$$

$$b_{i+1,j+1} = \frac{b_{i,j+1} - b_{i+2,j}}{h_i} \text{ for } i = 1, 2, \ldots (k-j) \text{ and } j = 1, 2, \ldots (k-1) \qquad (11)$$

by applying above equations, the numerator of the reduced order model can be obtained as

$$N_k(s) = b_{21} + b_{22}s + \ldots + b_{2,k}s^{k-1} \quad (12)$$

IV. COMPARISON OF METHOD

In order to check the accuracy of the proposed method, the step responses of the original and reduced systems are compared graphically. But for quantitative comparison, the Relative Integral Square Error (RISE) [13] between the original and reduced model is determined by using Matlab/Simulink. The lower value of RISE indicates the better response and hence validates the effectiveness of the proposed algorithm.

$$\text{RISE} = \int_0^\infty [y(t) - y_k(t)]^2 dt \Big/ \int_0^\infty [y(t) - y(\infty)]^2 dt \qquad (13)$$

Where $y(t)$ and $y_k(t)$ are unit step responses of the original and simplified model respectively and $y(\infty)$ is the steady-state value of the original system.

V. NUMERICAL EXAMPLE

Consider a fourth order system [14] described by its transfer function as

$$G(s) = \frac{24 + 24s + 7s^2 + s^3}{24 + 50s + 35s^2 + 10s^3 + s^4}$$

Procedure is explained for getting 2nd –order simplified model.

using Step-1 of the algorithm, denominator polynomial of second order model is obtained as

$D_2(s) = 288 + 300s + 70s^2$

Routh array for getting the h coefficients as:

$$h_1 = 1\left\langle \begin{matrix} 24 & 50 & 35 & 10 & 1 \\ 24 & 24 & 7 & 1 & \end{matrix} \right.$$

$$h_2 = 0.92308\left\langle \begin{matrix} 24 & 24 & 7 & 1 \\ 26 & 28 & 9 & \end{matrix} \right.$$

$\vdots$

Inverse Routh array can be constructed through h_1 and h_2 as

$$h_1 = 1\left\langle \begin{matrix} 288 & 300 & 70 \\ 288 & -11.99 & \end{matrix} \right.$$

$$h_2 = 0.92308\left\langle \begin{matrix} 288 & -11.99 \\ 311.998 & \end{matrix} \right.$$

$\vdots$

From inverse Routh array, the numerator of the simplified 2nd –order model is determined as

$N_2(s) = 288 - 11.99s$

Hence, a 2nd –order simplified model is obtained and written as

$$R_2(s) = \frac{288 - 11.99s}{288 + 300s + 70s^2}$$

The graphical comparison of the simplified model with the original system is shown in the fig 1. From the step responses comparison, it is clear that the 2nd -order is closely matching the original system response.

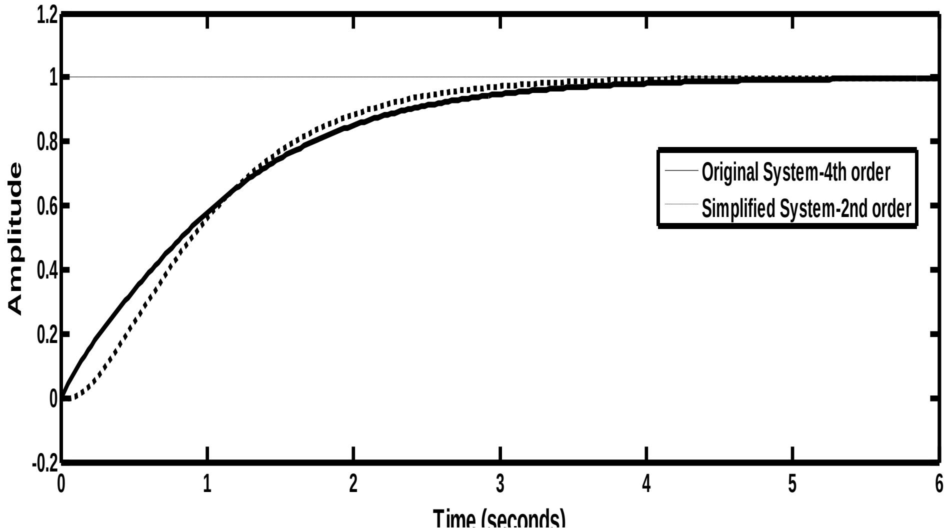

Fig.1 Step response comparison

The RISE is calculated through simulink model between the original 4th –order system and the simplified models of 2nd –order and tabulated in Table 1.

Table-1: Comparison of simplification methods

Method of Simplification	Order of simplified model	RISE
Proposed Method	$\dfrac{288-11.99s}{288+300s+70s^2}$	1.0127
Prasad and Pal [9]	$\dfrac{34.2465+s}{34.2465+239.8082s+s^2}$	1.2050
Parthasarathy et. al [15]	$\dfrac{0.6997+s}{0.6997+1.45771s+s^2}$	0.0217
Shieh and Wei [16]	$\dfrac{2.3014}{2.3014+5.7946s+s^2}$	0.1934

V. CONCLUSIONS

The authors suggested a mixed simplification method for order reduction of linear dynamic systems. in this method, denominator polynomial of the reduced order model is obtained by differentiation method while the coefficients of numerator are determined by using Cauer second form of continued fraction expansion. The method is simple in concept and capable to retain the transient and as well as steady-state characteristics of the original high order system. The viability of the simplification method is illustrated with the help of one example taken from the literature. The simplified second order model is compared with the original system and it is found that reduced model is nearly matching the step response of the original system. The proposed method is also compared with the well known methods already suggested by the authors and it may be concluded that the proposed method is comparable with the available methods in the literature. The proposed method may be extended to multi-inputs multi-outputs systems as well.

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