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Third Grade fluid flow with natural Heat Convection through Vertical flat Plates

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ABSTRACT

In this paper, natural heat convection of a third grade fluid flow between two vertical parallel infinite flat plates is considered. The governing flow problem of the conservation laws are reduced to a set of coupled differential equations by similarity transformations. Homotopy analysis Method (HAM) is used to solve the present flow problem. The results comprise good agreement between HAM and earlier literature work. HAM gives rapid convergent series solution which shows that this method is efficient, accurate and has advantages over other methods. Further, the effects of different physical parameters such as Prandlt number, Eckert number and viscoelastic parameter on the flow are discussed in detail.

***Keywords*: Natural heat convection, non-Newtonian fluids, HAM, flat plates.**

I. INTRODUCTION

The study of flows of non-Newtonian fluids attracts much attention in recent years, due to an increasing application in science and industries. The fundamental analysis of non-Newtonian fluids in a boundary layer continuously stretching sheet or an extended surface is of great importance, and it is an essential part in the study of emerging field of fluid dynamics**.** For examples melts, muds, pastes, printing ink, emulsions, cleansers, sugar solution, paints, tomato glue demonstrate the non-Newtonian properties of fluids. Few models of non-Newtonian fluids have been investigated in the literature through the differential, integral, and rate type categories. The majority of non-Newtonian fluid models are concerned with simple models like the power law and grade two or three.

The fluids involved are not simply Newtonian in most of realistic models, and a single model cannot capture the complex rheological properties of non-Newtonian fluids. The different types of non-Newtonian fluids have been studied in different mathematical approaches. For the flow of a particular class of such fluids between vertically standing flat plates natural heat convection has been examined by Bruce and Na [1]. Rajagopal and Na discussed a comprehensive thermodynamic investigation of constitutive relations for few classes of non-Newtonian fluids [2]; one of them is known as third grade fluid and many specialists have utilized this constitutive relation to demonstrate the flow of non-Newtonian fluids [3 - 5].

Many scientists and engineers have to deal with natural heat convection which plays a major role in overall behavior of the flow. In many circumstances it plays a significant role that disregarding it can leads us to extremely poor and unrealistic flow model. In modern days industrial advancement has really strengthened the ease level of human life and a way better expectation for everyday comforts has been accomplished; however further advancement is constantly invited and extent of further change is dependably there. In comparative way regardless of the immense achievement in demonstrating flow models with heat convection, still there is a lot of requirement for new ideas and strategies.

In most of the practical situations the equations describing the flow are highly nonlinear and are not amenable for obtaining analytical solutions. In such situations, the attempts have to develop a semi-numerical / semi-analytical method for the solutions of these problems. The perturbation methods are widely used by the scientists and engineers to obtain the solutions for nonlinear problems. For this purpose many semi-analytical and semi-numerical schemes have been developed to approximate the solution in better way [6 - 8], one of such method is Homotopy Analysis Method (HAM) which is very effective and easy to apply. The HAM was developed by Liao [9] and further modified it in [10] to introduce a non-zero auxiliary parameter which is known as convergence-control parameter $\hbar$, which allows us to adjust the convergence region and rate of approximations of required solution.

In this paper natural heat convection for the flow of a third grade fluid through vertical parallel plates is considered. HAM solution is obtained for the governing problem and compared with the earlier literature work. The results are presented and compared with the help of tables and graphs. The Behaviour of flow for physical parameters is discussed in detail.

II. MATHEMATICAL FORMULATION

In this problem we consider the natural convection of a non-Newtonian fluid, namely the Rivilin-Ericksen of third grade fluid between two infinite parallel vertical flat plates. The stress in such a fluid is related to the motion is given by [11]

$$\tau = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2 [A_1 A_2 + A_2 A_1] + \beta_3 (tr A_1^2) A_1 \tag{2.1}$$

where p is pressure, μ is the co-efficient of viscosity, $\alpha_1, \alpha_2, \beta_1, \beta_2$ and β_3 are material constants and A_1, A_2 and A_3 are kinematical tensors and are given by [12]

$$A_1 = grad\, v + (grad\, v)^T \tag{2.2}$$

and

$$A_n = \frac{d}{dt} A_{n-1} + A_{n-1} L + A_{n-1} L^T, \qquad n = 1,2,..... \tag{2.3}$$

where $\frac{d}{dt}$ denotes material derivatives and $L = grad\, v$.

Consider an incompressible steady flow of a third grade fluid between two vertical flat plates at a distance '2h' apart. The plates are kept at $x = -h$ and $x = +h$ at constant temperature T_0 and T_1 respectively, where $T_0 > T_1$. This difference in temperature causes the fluid near the left wall at $x = -h$ to raise and the fluid near the right wall at $x = +h$ to fall. We seek velocity and temperature fields of the form

$$V = v(x), \qquad T = T(x) \tag{2.4}$$

In the absence of body forces, the physical problem of governing momentum and energy equations becomes [12]

$$\mu\frac{d^2v}{dx^2}+6\beta_3\left(\frac{dv}{dx}\right)^2\frac{d^2v}{dx^2}-\rho_0\gamma(T-T_0)g=0 \tag{2.5}$$

$$K\frac{d^2T}{dx^2}+\mu\left(\frac{dv}{dx}\right)^2+2\beta_3\left(\frac{dv}{dx}\right)^4=0 \tag{2.6}$$

subjected to boundary conditions

$$\begin{aligned} &v(-h)=0, \qquad v(+h)=0,\\ &T(-h)=T_0, \qquad T(+h)=T_1 \end{aligned} \tag{2.7}$$

Define the non-dimensional parameters as [12]

$$v^*=\frac{v}{V_0}, \qquad x^*=\frac{x}{h}, \qquad T^*=\frac{T-T_m}{T_0-T_1}$$

Use the above dimensionless parameters and drop the asterisks, then eqns. (2.5) to (2.7) becomes

$$\mu\frac{d^2v}{dx^2}+6\beta\left(\frac{dv}{dx}\right)^2\frac{d^2v}{dx^2}+T=0 \tag{2.8}$$

$$\frac{d^2T}{dx^2}+Ec\cdot\Pr\left[\left(\frac{dv}{dx}\right)^2+2\beta\left(\frac{dv}{dx}\right)^4\right]=0 \tag{2.9}$$

also boundary conditions becomes

$$\begin{aligned} &v(-1)=0, \qquad v(+1)=0,\\ &T(-1)=\frac{1}{2}, \qquad T(+1)=-\frac{1}{2} \end{aligned} \tag{2.10}$$

where $Ec=\dfrac{V_0^2}{c_p(T_1-T_0)}$ is Eckert number, $\Pr=\dfrac{\mu c_p}{K}$ is Prandlt number, $\beta=\dfrac{(\beta_3V_0^2)}{\mu h^2}$ is viscoelastic parameter and c_p is the specific heat of fluid.

III. HOMOTOPY ANALYSIS METHOD

We seek HAM solution for the Eqn. (2.8-2.9) subjected to the boundary conditions (2.10). We choose the initial guesses which satisfies the boundary conditions automatically and auxiliary linear operators for the functions v and T as

$$v_0(x)=0, \qquad T_0(x)=-\frac{x}{2}, \tag{3.11}$$

and

$$L[v]=v'', \qquad L[T]=T''. \tag{3.12}$$

Here the above linear operator satisfies

$$L_v[C_1x+C_2]=0, \qquad L_T[C_3x+C_4]=0, \tag{3.13}$$

where $C_i : i = 1, 2, 3, 4$ are arbitrary constants to be determined later.

3.1 Zeroth-order Deformation Problem

If $q \in [0,1]$ then the zeroth order deformation problem can be constructed as

$$\left.\begin{aligned}(1-q)L_v[v(x,q)-v_0(x)] &= q\hbar_v \aleph_v[v(x,q),T(x,q)],\\ (1-q)L_T[T(x,q)-T_0(x)] &= q\hbar_T \aleph_T[v(x,q),T(x,q)].\end{aligned}\right\} \tag{3.14}$$

The boundary conditions becomes

$$\left.\begin{aligned}&v(-1,q)=0, \quad v(+1,q)=0,\\ &T(-1,q)=-\frac{1}{2}, \quad T(+1,q)=\frac{1}{2}.\end{aligned}\right\} \tag{3.15}$$

where $q \in [0,1]$ is an embedding parameter. Here $\hbar_v$ and $\hbar_T$ are the non-zero auxiliary parameters. Further $\aleph$ is the non-linear differential operator and is given by

$$\left.\begin{aligned}\aleph_v[v(x,q),T(x,q)] &= \frac{\partial^2 v(x,q)}{\partial x^2}+6\beta\left(\frac{\partial v(x,q)}{\partial x}\right)^2\frac{\partial^2 v(x,q)}{\partial x^2}+T,\\ \aleph_T[v(x,q),T(x,q)] &= \frac{\partial^2 T(x,q)}{\partial x^2}+Ec\cdot\Pr\left[\left(\frac{\partial v(x,q)}{\partial x}\right)^2+2\beta\left(\frac{\partial v(x,q)}{\partial x}\right)^4\right].\end{aligned}\right\} \tag{3.16}$$

For $q=0$ and $q=1$, Eqn. (3.14) have the solutions

$$\left.\begin{aligned}v(x,0)&=v_0(x) \qquad & v(x,1)&=v(x),\\ T(x,0)&=T_0(x) \qquad & T(x,1)&=T(x).\end{aligned}\right\} \tag{3.17}$$

As q vary from 0 to 1, $v(x,q), T(x,q)$ also vary from the initial guesses $v_0(x), T_0(x)$ to the final solutions $v(x), T(x)$. With the help of Taylor's theorem, Eqn. (3.17) can be written as

$$\left.\begin{aligned}v(x,q) &= v_0(x)+\sum_{m=1}^{\infty} v_m(x)q^m,\\ T(x,q) &= T_0(x)+\sum_{m=1}^{\infty} T_m(x)q^m.\end{aligned}\right\} \tag{3.18}$$

where $v_m(x)=\frac{1}{m!}\frac{\partial^m v(x,q)}{\partial q^m}\Big|_{q=0}, \quad T_m(x)=\frac{1}{m!}\frac{\partial^m T(x,q)}{\partial q^m}\Big|_{q=0}$. The convergence of the above two series (3.18) depends on the auxiliary parameters $\hbar_v$ and $\hbar_T$. In order to select the values of $\hbar_v$ and $\hbar_T$ in such a way that the series (3.18) is convergent at $q=1$, we have

$$\left.\begin{aligned} v(x,q) &= v_0(x)+\sum_{m=1}^{\infty} v_m(x), \\ T(x,q) &= T_0(x)+\sum_{m=1}^{\infty} T_m(x). \end{aligned}\right\} \tag{3.19}$$

3.2 mth-order Deformation Problems

Differentiating the zeroth order deformation problem (3.14) $'m'$ times with respect to the embedding parameter q and then dividing by $m!$, finally setting $q=0$. The resulting mth-order deformation problem becomes

$$\left.\begin{aligned} L_v[v_m(x)-\chi_m v_{m-1}(x)] &= h_v \Re_m^v(x), \\ L_T[T_m(x)-\chi_m T_{m-1}(x)] &= h_T \Re_m^T(x). \end{aligned}\right\} \tag{3.20}$$

The homogeneous boundary conditions are

$$\left.\begin{aligned} v_m(-1,q)=0, \quad v_m(+1,q)=0, \\ T_m(-1,q)=0, \quad T_m(+1,q)=0, \end{aligned}\right\} \tag{3.21}$$

where

$$\left.\begin{aligned} \Re_m^v(x) &= v''_{m-1}+6\beta\sum_{n=0}^{m-1}[(v'_n)^2(v''_{m-1-n})]+T_{m-1}, \\ \Re_m^T(x) &= T''_{m-1}+Ec\cdot \Pr\sum_{n=0}^{m-1}[v'_n v'_{m-1-n}+2\beta(v'_n)^4]. \end{aligned}\right\} \tag{3.22}$$

and

$$\chi_m = \begin{cases} 0, & m\le 1; \\ 1, & m>1. \end{cases} \tag{3.23}$$

We use Mathematica to solve the linear system of equations (3.20) with the appropriate homogeneous boundary conditions (3.21) up to first few orders of approximations for the series v and T

$$v_1 = \left(\frac{1}{6}\hbar_v+\frac{1}{12}\hbar_v^2\right)x-\left(\frac{1}{6}\hbar_v+\frac{1}{12}\hbar_v^2\right)x^3,$$

$$T_1 = Ec\cdot \Pr\left[-\frac{1}{48}\hbar_v^2\hbar_T-\frac{x}{2}+\frac{1}{48}\hbar_v^2\hbar_T x^4\right]$$

3.3 Convergence of HAM

The analytic expressions of v and T in terms of series are given in Eqn. (3.19) contains the auxiliary parameters and the convergence of the series strictly depends upon the parameters $\hbar_v$ and $\hbar_T$ which are called as convergence control parameters. These parameters play a major role in predicting the convergence region and rate of approximations. For this purpose, we have drawn the line segment of the $\hbar$ curves parallel to x - axis.

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Fig. 1 and 2 shows the $\hbar$ curves for the series $v(x)$ and $T(x)$ for the 10th order of approximations. It is clearly indicates that the admissible ranges of $\hbar_v$ and $\hbar_T$ are $-1.5 \leq \hbar_v \leq -0.25$ and $-1.7 \leq \hbar_T \leq -0.2$ respectively for $\beta = 0.5, \Pr = 1, Ec = 1$. Our calculation shows that the two series converges in the whole region of x when $\hbar_v = \hbar_T = -1$, other than this $\hbar$ values the results may diverge or converge slowly.

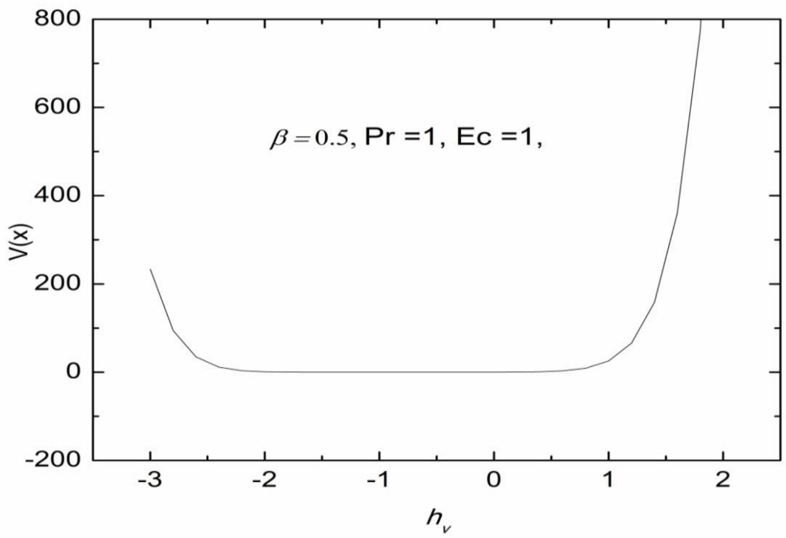

Fig. 1: $\hbar$ curves for the series $v(0.5)$ for $\beta = 0.5, \Pr = 1, Ec = 1$.

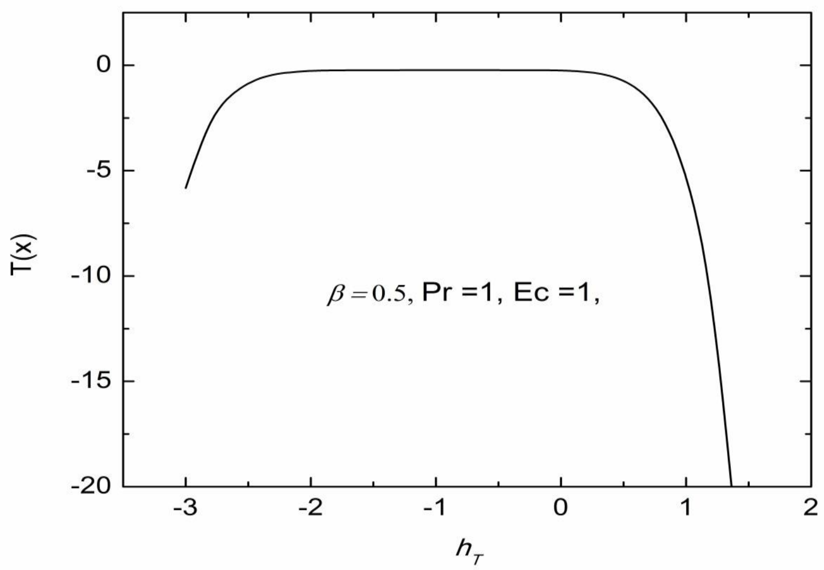

Fig. 2: $\hbar$ **curve for the series** $T(0.5)$ **for** $\beta = 0.5, \Pr = 1, Ec = 1$.

IV. RESULTS AND DISCUSSION

Natural heat convection flow of a third grade fluid between two vertically placed parallel infinite plates is considered. The coupled differential eqns. (2.9-2.10) along with boundary conditions (2.10) of the governing problem is solved by HAM. The effects of Eckert number *Ec*, Prandtl number Pr and viscoelastic parameter β on the flow are shown graphically in Figs. 3-7.

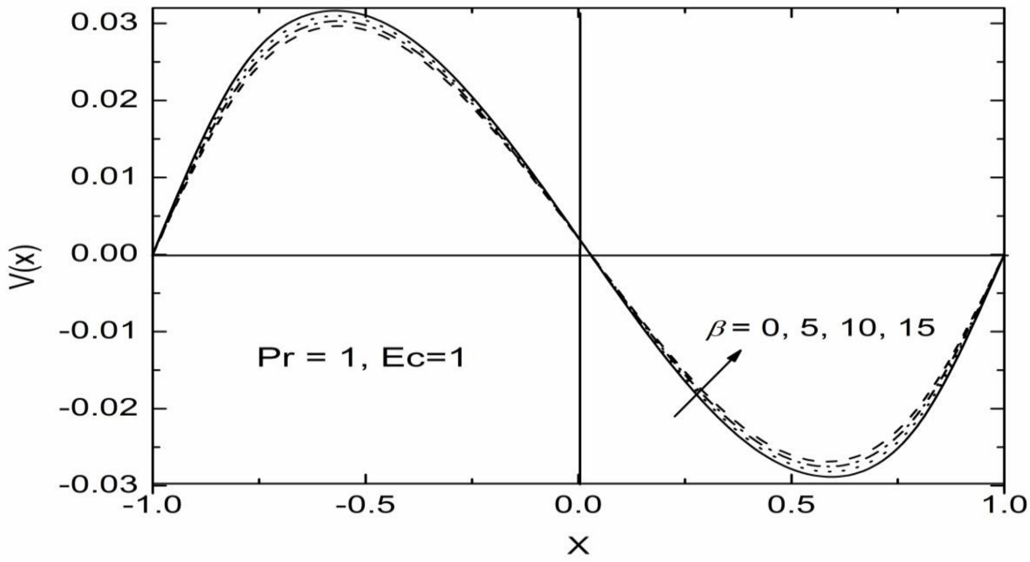

Fig. 3: Effects of β on non-dimensional velocity.

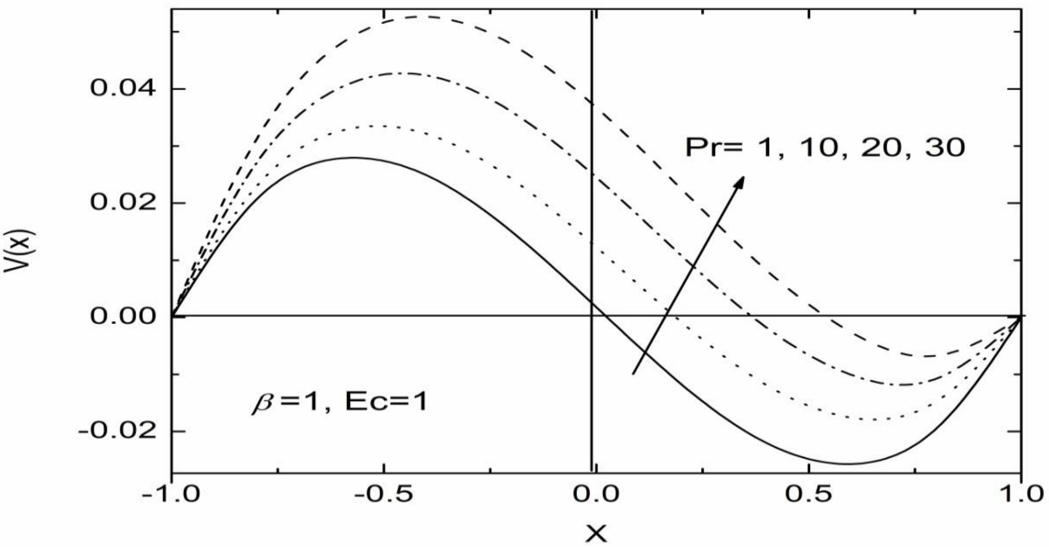

Fig. 4: Effects of Pr on non-dimensional velocity.

Fig. 3 demonstrates the impacts of viscoelastic parameter β on the velocity v of the fluid. The velocity decreases with the increase in viscoelastic parameter. The behavior of velocity and temperature profiles are shown in Figs. 4 and 5 respectively for increasing values of Pr. Both velocity and temperature are increasing functions in this case. For increasing values of *Ec* the similar effects can be observed in Figs. 6 and 7.

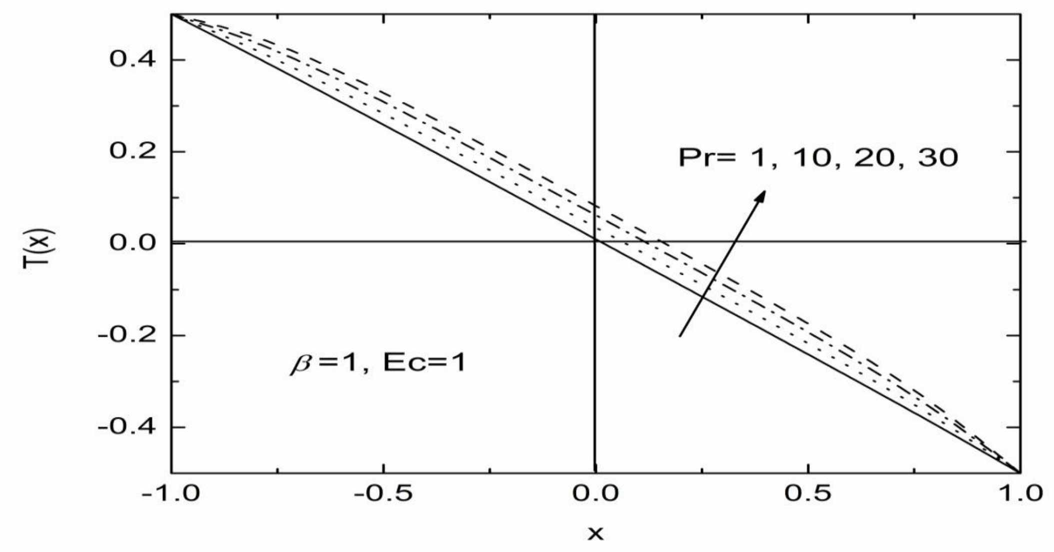

Fig. 5: Effects of Pr on non-dimensional temperature.

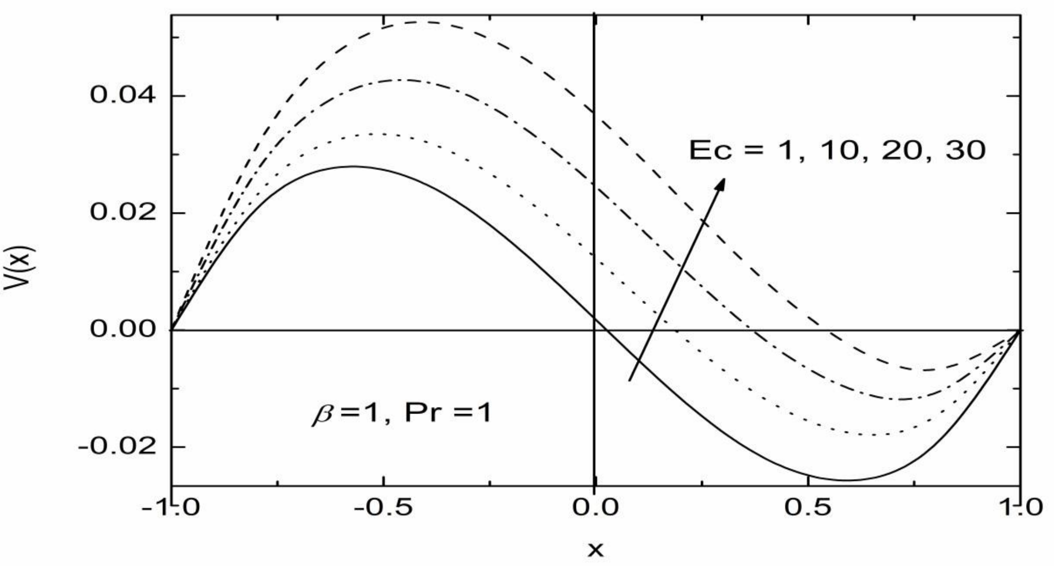

Fig. 6: Effects of *Ec* on non-dimensional velocity.

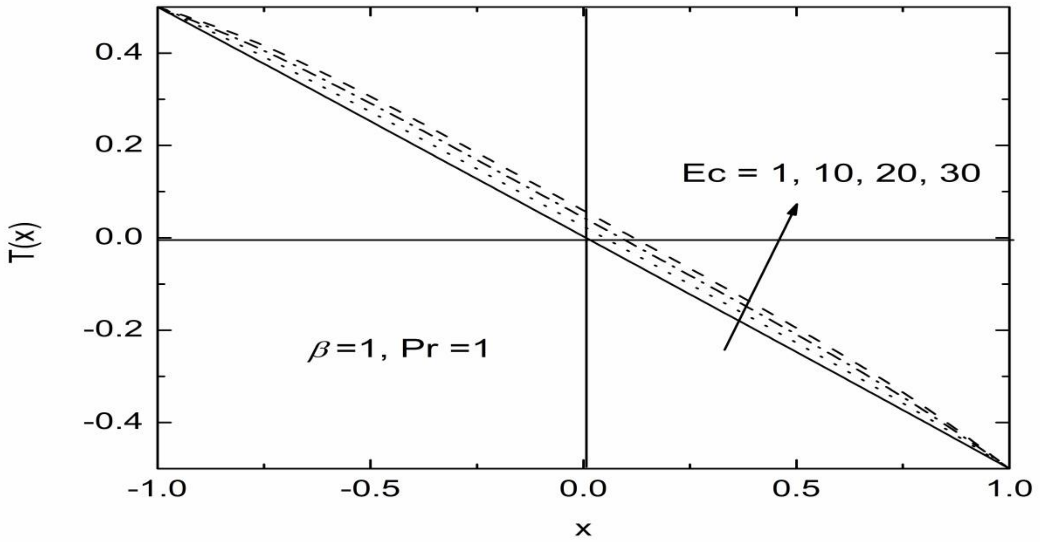

Fig. 7: Effects of *Ec* on non-dimensional temperature.

Table.1 describes the numerical and semi-analytical solutions to the velocity profile for $\beta = 0.5$, Pr =1, Ec =1 and Table.2 shows the temperature distribution results for $\beta = 0.5$, Pr =1, Ec =1. From the Tables. 1 and 2, the Comparison is made with the earlier literature work [13] and an excellent agreement is seen between both the solutions which approve the effectiveness of HAM.

Table 1: Comparison of HAM and numerical solution for $\beta = 0.5$, Pr =1, *Ec* =1.

x	V(x)		
	VPM	HAM	Numerical
-1	0	0	0
-0.9	0.01412053	0.0137045	0.0141168
-0.8	0.02392391	0.0231506	0.0239193
-0.7	0.02979535	0.0287785	0.0297907
-0.6	0.03217724	0.0310484	0.0321727
-0.5	0.03154945	0.0304321	0.031545
-0.4	0.02841114	0.0274075	0.0284069
-0.3	0.02326744	0.0224546	0.0232633
-0.2	0.01662161	0.0160529	0.0166176
-0.1	0.00897182	0.0086809	0.008968
0	0.00081131	0.000815424	0.0008076
0.1	-0.0073694	-0.00706752	-0.007373

0.2	-0.0150791	-0.0144919	-0.015082
0.3	-0.021823	-0.0209809	-0.021826
0.4	-0.0271006	-0.0260558	-0.027104
0.5	-0.0304052	-0.0292364	-0.030408
0.6	-0.0312274	-0.0300413	-0.03123
0.7	-0.0290632	-0.0279904	-0.029066
0.8	-0.023427	-0.0226077	-0.023429
0.9	-0.0138699	-0.0134273	-0.013871
1	0	0	0

Table 2: Comparison of HAM and numerical solution for $\beta = 0.5$, Pr =1, Ec =1.

x	T(x)		
	VPM	HAM	Numerical
-1	0.5	0.5	0.5
-0.9	0.45044049	0.450701	0.4504418
-0.8	0.4007343	0.401203	0.4007359
-0.7	0.35096454	0.351548	0.350966
-0.6	0.30117607	0.301773	0.3011774
-0.5	0.2513854	0.25191	0.2513865
-0.4	0.20158997	0.201985	0.2015909
-0.3	0.15177631	0.152021	0.1517771
-0.2	0.1019269	0.102034	0.1019275
-0.1	0.05202535	0.0520369	0.0520258
0	0.00206022	0.00203697	0.0020605
0.1	-0.0479728	-0.0479633	-0.047973
0.2	-0.09807	-0.0979665	-0.09807
0.3	-0.1482206	-0.14798	-0.148221
0.4	-0.1984082	-0.198016	-0.198409
0.5	-0.2486154	-0.248091	-0.248616
0.6	-0.2988279	-0.298227	-0.298829
0.7	-0.3490424	-0.348453	-0.349043
0.8	-0.399274	-0.398798	-0.399275
0.9	-0.4495659	-0.4493	-0.449566
1	-0.5	-0.5	-0.5

V. CONCLUSION

The non-Newtonian fluid flow between two vertically placed infinite parallel plates is considered. Natural heat convection for the flow is investigated and HAM is used to solve the coupled equations. The HAM results are compared with previous results which show good agreement. The convergence of the HAM is given. The effects of physical parameters on the flow are presented in table and graphs, and are discussed in detail.

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