

TRANSVERSE VELOCITY OF THE FLUID WITH THE EFFECT OF VOLUME FRACTION IN THE INCOMPRESSIBLE DUSTY FLUID

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ABSTRACT

The presence of contaminating dust particles in fluids can occur naturally. These problems associated with the flow characteristics and their properties are of fundamental interest in the field of fluid mechanics, the effect of finite volume fraction of suspended particulate matter on axially symmetrical jet mixing of incompressible dusty fluid has been considered. The presence of dust particles in a homogeneous fluid makes the dynamical study of flow problems quite complicated. Here we are assuming the velocity and temperature in the jet to differ only slightly from that of surrounding stream, a perturbation method has been employed to linearize the equation those have been solved by using Hankel's Transformation technique. Naturally, the studies of these systems are mathematically interesting and physically useful for various reasons.

key word : *particulate suspension , boundary layer characteristics , volume fraction, incompressible flow.*

NOMENCLATURE :

(x, y, z)	→	Space coordinates
(u, v, w)	→	Velocity components of fluid phase
(u_p, v_p, w_p)	→	Velocity components of particle phase
$(\bar{u}, \bar{v}, \bar{w})$	→	Dimensionless velocity components of fluid phase
$(\bar{u}_p, \bar{v}_p, \bar{w}_p)$	→	Dimensionless velocity components of particle phase
T	→	Temperature of fluid phase

T_p	→	Temperature of particle phase
C_{f_0}, C_{f_1}	→	Skin friction coefficients at the lower and upper plates respectively
C_p, C_s	→	Specific heats of fluid and SPM respectively
K	→	Thermal conductivity
R_e	→	Fluid phase Reynolds number
R_{e_p}	→	Particle phase Reynolds number

INTRODUCTION

In the present chapter, we discussed the effect of volume fraction in axi symmetric jet mixing of incompressible fluid in cylindrical polar coordinates. Assuming the velocity and temperature in the jet to differ only slightly from that of the surrounding stream, a perturbation method has been employed to linearize the governing differential equations. The resulting linearized equations have been solved by using Hankel's transformation technique. Numerical computations have been made to discuss the profiles of transverse perturbation fluid velocity. Consideration of finite volume fraction shows that the magnitude of transverse perturbation fluid and particle velocity reduced significantly.

II. MATHEMATICAL FORMULATION

The equation governing the study two-phase boundary layer flow in axi-symmetric case can be written in cylindrical polar coordinates as

Equation of Continuity in Fluid phase :

$$\frac{\partial}{\partial z}(ru) + \frac{\partial}{\partial r}(rv) = 0 \quad (1)$$

Equation of Motion in fluid phase :

$$(1 - \phi)\rho \left(u \frac{\partial u}{\partial z} + v \frac{\partial u}{\partial r} \right) = \frac{\mu}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{\rho_p}{\tau_m} (u_p - u) \quad (2)$$

Heat equation in fluid phase :

$$(1 - \phi)\rho C_p \left(u \frac{\partial T}{\partial z} + v \frac{\partial T}{\partial r} \right) = K \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + \rho_p C_s \frac{(T_p - T)}{\tau_t} \quad (3)$$

To study the boundary layer flow, we introduce the dimensionless variables are

$$\bar{z} = \frac{z}{\lambda}, \bar{r} = \frac{r}{(\tau_m \nu)^{\frac{1}{2}}}, \bar{u} = \frac{u}{U}, \bar{v} = v \left(\frac{\tau_m}{\nu} \right)^{\frac{1}{2}}, \bar{u}_p = \frac{u_p}{U}, \bar{v}_p = v_p \left(\frac{\tau_m}{\nu} \right)^{\frac{1}{2}}, \alpha = \frac{\rho_{p0}}{\rho} = \text{const}$$

$$\bar{\rho}_p = \frac{\rho_p}{\rho_{p_0}}, \bar{T} = \frac{T}{T_0}, \bar{T}_p = \frac{T_p}{T_0}, \lambda = \tau_m U, \tau_m = \frac{2}{3} \frac{C_p}{C_s} \frac{1}{p_r} \tau_T, p_r = \frac{\mu C_p}{K}.$$

Now considering the flow from the orifice under full expansion we can assume that the pressure in the mixing region to be approximately constant. Hence, the pressure at the exit is equal to that of the surrounding stream. Therefore, both the velocity and the temperature in the jet is only slightly different from that of the surrounding stream. The coefficient of viscosity μ and thermal conductivity K are assumed to be constant. Then it is possible to write $u = u_0 + u_1, v = v_1, u_p = u_{p_0} + u_{p_1}, v_p = v_{p_1}, T = T_0 + T_1, T_p = T_{p_0} + T_{p_1}, \rho_p = \rho_{p_1}$ where the subscripts 1 denotes the perturbed values which are much smaller than the basic values with subscripts '0' of the surrounding stream, i.e. $u_0 \gg u_1, u_{p_0} \gg u_{p_1}, T_0 \gg T_1, T_{p_0} \gg T_{p_1}$. Using the dimensionless variable and the perturbation method the non linear equations (1) to (3) written as

$$\frac{\partial}{\partial z}(ru_1) + \frac{\partial}{\partial r}(rv_1) = 0 \tag{4}$$

$$(1-\phi)u_0 \frac{\partial u_1}{\partial z} = \frac{1}{r} \frac{\partial u_1}{\partial r} + \frac{\partial^2 u_1}{\partial r^2} + \alpha \rho_{p_1} (u_{p_0} - u_0) \tag{5}$$

$$(1-\phi)u_0 \frac{\partial T_1}{\partial z} = \frac{1}{p_r} \left(\frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r} \frac{\partial T_1}{\partial r} \right) + \frac{2\alpha}{3p_r} \rho_{p_1} (T_{p_0} - T_0) \tag{6}$$

The boundary conditions for u_1, v_1, u_{p_1} and v_{p_1} are

$$u_1(0, r) = \begin{cases} u_{10}, & r \leq 1 \\ 0, & r > 1 \end{cases} \tag{7}$$

$$\frac{\partial u_1}{\partial r}(z, 0) = 0, u_1(z, \infty) = 0 \tag{8}$$

$$v_1(0, r) = 0 \tag{9}$$

$$u_{p_1}(0, r) = \begin{cases} u_{p_{10}}, & r \leq 1 \\ 0, & r > 1 \end{cases} \tag{10}$$

$$v_{p_1}(0, r) = 0 \tag{11}$$

III.METHOD OF SOLUTION

The governing linearized equation (4) have been solved by using Hankel's transform technique and using the relevant conditions from (7) to (11) we get



Hankels transform of $\frac{\partial}{\partial z}(u_1) + \frac{1}{r} \frac{\partial}{\partial r}(rv_1) = 0$

i.e $H\{\frac{\partial}{\partial z}(u_1)\} + H\{\frac{1}{r} \frac{\partial}{\partial r}(rv_1)\} = 0$

$$\Rightarrow \frac{du_1^*}{dz} + \int_0^\infty \frac{\partial}{\partial r}(rv_1) J_0(pr) dr = 0$$

where $H\{\frac{\partial}{\partial z}(u_1)\} = \frac{du_1^*}{dz}$

$$\Rightarrow \frac{du_1^*}{dz} + \int_0^\infty rv_1 J_1(pr) dr = 0$$

$$\Rightarrow \frac{du_1^*}{dz} + pv_1^* = 0$$

Where $pv_1^* = \int_0^\infty rv_1 J_1(pr) dr$

$$\Rightarrow pv_1^* = -\frac{du_1^*}{dz}$$

Now $u_1^*(z, p) = (u_{10} - \frac{\alpha E \rho_{p10}}{Ap^2}) \frac{J_1(p)}{p} e^{-Ap^2z} + \frac{\alpha E \rho_{p10}}{Ap^2} \frac{J_1(p)}{p}$

$$\frac{du_1^*}{dz} = -Ap^2 (u_{10} - \frac{\alpha E \rho_{p10}}{Ap^2}) \frac{J_1(p)}{p} e^{-Ap^2z}$$

Therefore $v_1^* = (u_{10} Ap - \frac{\alpha E \rho_{p10}}{p}) \frac{J_1(p)}{p} e^{-Ap^2z}$ (12)

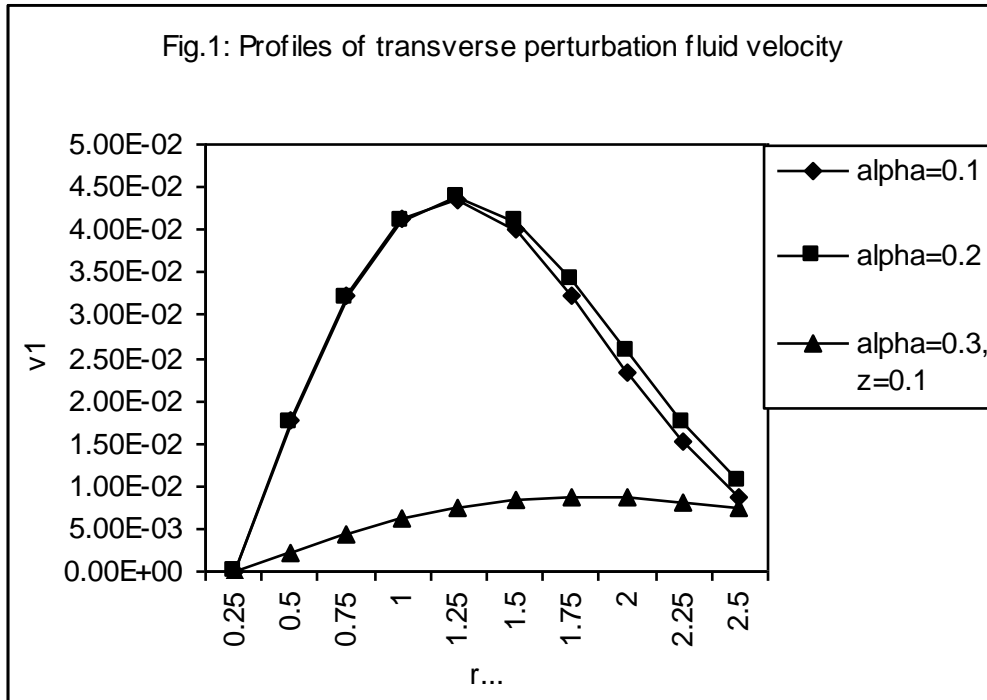
Hankel inversion of (12) gives

$$v_1 = \int_0^\infty \left(Au_{10} p - \frac{\alpha E \rho_{p10}}{p} \right) e^{-Ap^2z} J_1(p) J_1(pr) dp$$
 (13)

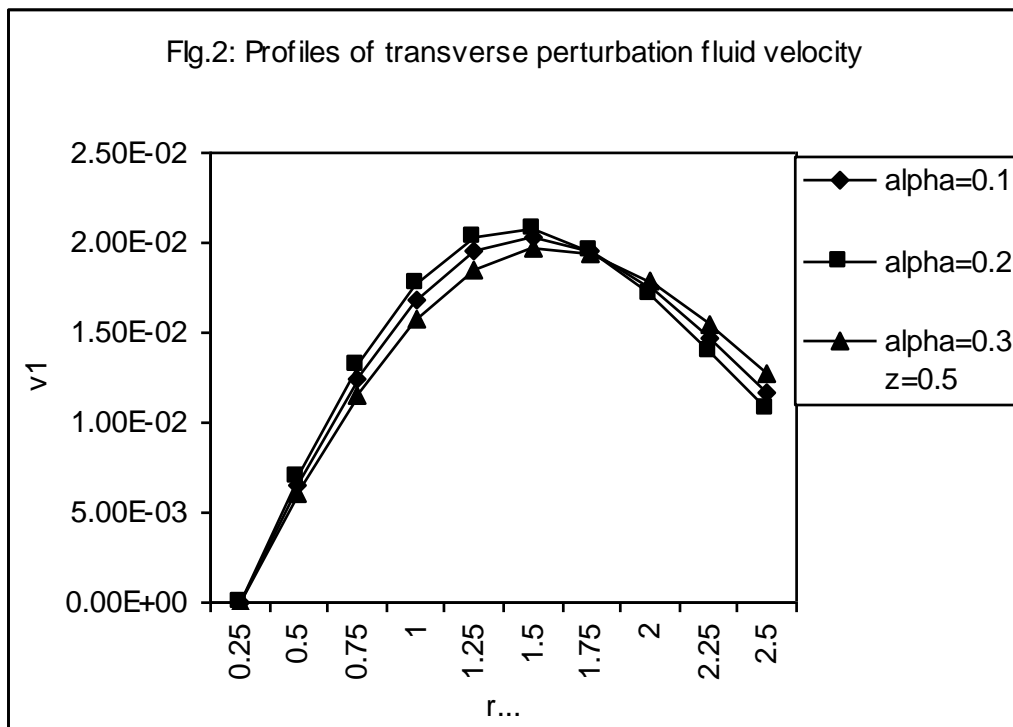
Where $u_1^* = \int_0^\infty r u_1 J_0(pr) dr$ etc

$$A = \frac{1}{(1-\phi)u_0}, E = \frac{u_{p_0} - u_0}{u_0}, C = \frac{2}{3p_r u_{p_0}}, F = \frac{T_{p_0} - T_0}{(1-\phi)u_0},$$

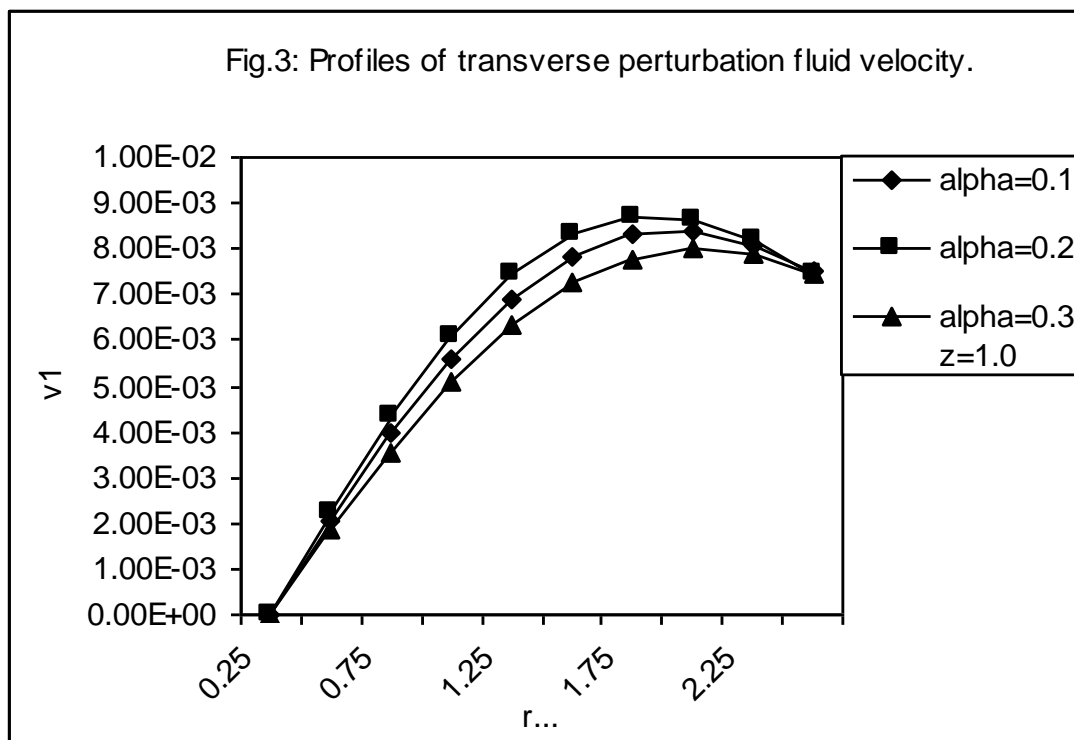
Where J_0 and J_1 are the Bessel function of zero and first order respectively.



(Fig: 1)



(Fig: 2)



(Fig:3)

IV.DISCUSSION OF RESULT AND CONCLUSION

Numerical computation have been made by taking $P_r = 0.72$, $u_{10} = u_{p10} = T_{10} = T_{p10} = \rho_{p10} = 0.1$, $\phi = 0.01$. The velocity and temperature at the exit are taken nearly equal to unity.

Figures 1, 2 and 3 show the profiles of transverse perturbation fluid velocity v_1 for $\alpha = 0.1, 0.2$ and 0.3 and for different values of Z . It is observed that the effect of increase in concentration parameter α of dust particles is to decrease the magnitude of v_1 . It is also observed that the transverse velocity v_1 attain a maximum value at $r = 1.25$. Hence we conclude that consideration of finite volume fraction shows that the magnitude of transverse fluid velocity reduce significantly.

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