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Band Matrix Patterns and Data Structures of Interconnection Networks

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ABSTRACT

In this paper we have analyzed the Band matrix patterns of different interconnection networks. We have compared the adjacency matrices of hypercube and Perfect Difference Network (PDN) we have found that the adjacency matrix of hypercube can be partitioned into square matrices where as adjacency matrix of Perfect Difference Network (PDN) cannot be partitioned into square matrices. We have also seen in the analysis of matrix patterns that in Perfect Difference Network (PDN) adjacency matrix there are more palindrome than in a hypercube adjacency matrix. In this paper we have presented the formulas that give us the edges, nodes, squares, cube, and hypercubes in a hypercube. Many interesting properties are presented in the form of lemmas which may be beneficial for the further research in matrix patterns of different data structures. An approach is made to derive symmetric facts within the matrix patterns for interconnection networks.

Keywords: Perfect Difference Set (PDS), Perfect Difference Network (PDN), Complete Graph, Hypercube.

I.INTRODUCTION

In this paper we have analyzed the Band matrix pattern of different interconnection networks. The calculation of total number of edges, nodes, squares, cubes, and hypercubes in a hypercube is done on the basis of mathematical procedures. During the analysis of Perfect Difference Network (PDN) adjacency matrix we have found that the transpose of Perfect Difference Network (PDN) adjacency matrix is same as its adjacency matrix. The palindrome property has been shown with in band matrix of different interconnection networks.. The factor (k) is derived which gives us the connectivity of different architectures. The results are presented in the form of lemmas.

1.1. Hypercube

A hypercube is a multidimensional mesh of nodes with exactly two nodes in each dimension. A d-dimensional hypercube consists of k nodes, where $k=2^n$.[12]

1. A hypercube has n dimensions; where n can be any positive integer (including zero).
2. A hypercube has 2^n vertices.
3. There are n connections (line) that meet at each vertex of a hypercube.
4. All connections at a hypercube vertex meet at right angles with respect to each other.[13][12]
5. The hypercube can be constructed recursively from lower dimensional cubes.
6. An architecture where the degree and diameter of a graph is the same then they will achieve a good balance between, the communication speed and complexity of the topology network. Hypercube achieve this equality, which explains why they are one the today's most popular design.[8].

1.2. Perfect Difference Set

The Perfect Difference Sets were first discussed by J. Singer in 1938 in terms of points and lines in a projective plane of a Galois Field (GF) [14], [15].

Definition 1: Perfect Difference Set - If the set S of δ+1 distinct integers S_0, S_1 ..., S_δ has the property that the $\delta^2 + \delta$ differences $S_i - S_j$ ($0\leq i, j \leq \delta$, $i\neq j$) are distinct modulo $\delta^2 + \delta + 1$, S is called a perfect difference set mod $\delta^2 + \delta + 1$.

The existence of perfect difference sets seems intuitively improbable, at any rate for large δ, but in 1938 J. Singer proved that, whenever δ is a prime or power of prime, say $\delta = p^n$, a perfect difference set mod $p^{2n} + p^n + 1$ exists. [2], [4], [6]

From now we on, let δ denote p^n and we write that $n = \delta^2 + \delta + 1 = p^{2n} + p^n + 1$. S = {s : $|s_i - s_j|$ mod n, where $0\leq i, j\leq \delta$, $i \neq j$, δ is a prime or power of prime and $n = \delta^2 + \delta + 1$} [5].

1.3. Perfect Difference Network

Perfect Difference Network architecture, based on a PDS is designed where each ith node is connected via direct links to nodes $i \pm 1$ and $i \pm s_j$ (mod n), for $2 \leq j \leq \delta$. Each link is bidirectional and the preceding connectivity leads to a chordal ring of δ in-degree and δ out-degree (total degree of a node $d(v) = 2\delta$) and diameter D = 2 [3], [4]. PDN has already been studied for, high performance communication and parallel processing network [4] and some topological properties of PDNs and parallel algorithms [6], [7], [9], [12] were suggested. It was shown that an n-node PDN can emulate the complete network with optimal slow down and balanced message traffic.

Although other interconnection architectures with topological and performance characteristics similar to PDNs exist, we view PDNs as worthy additions to the repertoire of computer system designers.

Alternative network topologies offer additional design points that can be exploited to accommodate the needs of new and emerging technologies. Further study is needed to resolve some open questions and to cost/performance comparisons for PDNs.

IJARSE
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1.4. Perfect Difference Graph

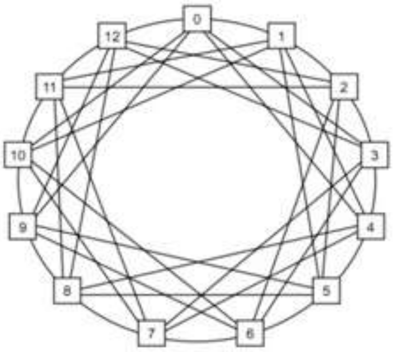
PDGs [3], based on the mathematical notion of perfect difference sets (PDSs), are undirected graphs of degree d = 2δ (where δ is the number of elements in the PDS) and diameter D=2.

Definition: A PDG is an undirected inter-connection graph with n= $\delta^2+\delta+1$vertices, numbered 0 to n-1. In the PDG, each vertex i is connected via undirected edges to vertices $(i \pm s_j)(\text{mod } n)$ for $1\leq j \leq \delta$, where s_j is an element of PDS $\{s_1,s_2,..., s_j\}$ of order δ [10].

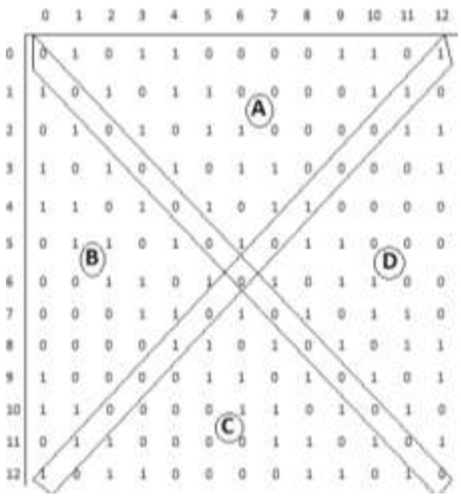
II.ANALYSIS OF MATRIX PATTERNS OF PDN, HYPERCUBE, AND TORUS ARCHITECTURES

The analysis of matrix patterens of differeent architectures is shown in below cases.

Case 1: Adjacency matrix representation of PDN.



(a)



(b)

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Fig.1: (a) PDN with n = 13, δ = 3 and (b) Adjacency matrix representation.

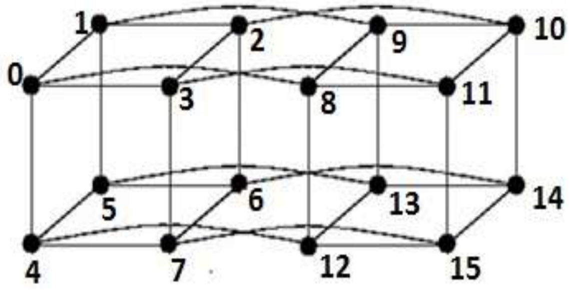
From the Fig.1 (b) we have observed following properties of PDN.

I. In Perfect Difference Network the intersection of diagonals always represent an element.

II. The diagonals of the above matrix partitions it into four equal parts excluding the diagonals i.e. the upper right triangular matrix (A & D) is mirror image of lower left triangular matrix (A & C) and the upper left triangular matrix (A & B) is the mirror image of lower right triangular matrix (C & D)but the involvement of nodes in links is different.

III. Diagonal sub-matrix shown the mirror image of the architecture.

Case 2: Adjacency matrix representation of Hypercube.



(a)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	1	0	1	0	0	0	0	1	1	0	0	0	0	0	0
1	1	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0
2	0	1	0	1	0	0	1	0	0	0	1	0	0	0	0	0
3	1	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0
4	1	0	0	0	0	1	0	1	0	0	0	0	1	0	0	0
5	0	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0
6	0	0	1	0	0	1	0	1	0	0	0	0	0	0	1	0
7	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	1
8	1	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0
9	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0	0
10	0	0	1	0	0	0	0	0	0	1	0	1	0	0	1	0
11	0	0	0	1	0	0	0	0	1	0	1	0	0	0	0	1
12	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0	1
13	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	0
14	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1
15	0	0	0	0	0	0	0	1	0	0	0	1	1	0	1	0

(b)

Fig.2. (a) Hypercube with n = 4 and (b) Adjacency matrix representation.

The Fig.2. (b) shows the matrix representation of hypercube where n =4, In this matrix the intersection of diagonals is left blank or in other words we can say that the intersection of diagonals does not have an element

IJARSE
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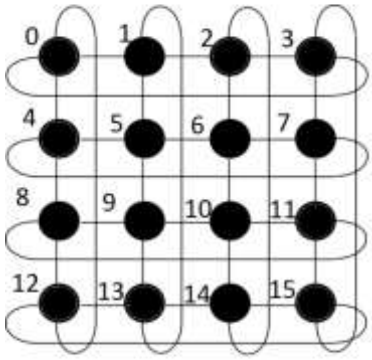
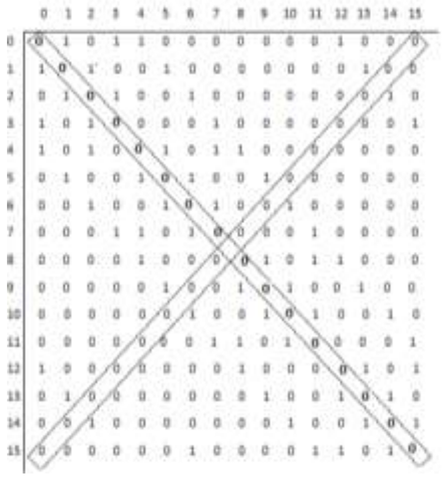
i.e; the diagonals intersect at the centre point of matrix which is blank space because in the hypercube architecture we have always an even number of nodes. In hypercube the upper matrix is not the mirror image of lower matrix and also the left matrix is not the mirror image of right matrix.

Case 3: Adjacency matrix representation of Torus.



(a)



(b)

Fig.3. (a) Torus with 16 nodes and (b) adjacency matrix representation.

From the Fig.3. (b) it is clear that the upper triangular matrix is the mirror image of lower triangular matrix and also if any node fails then the rest of the network functions properly that gives the robustness property of Torus network and also the total number of nodes is always even.

III.THE ANALYSIS OF BAND MATRIX PATTERNS OF DIFFERENT INTERCONNECTION NETWORKS HAVE BEEN PRESENTED IN THE FOLLOWING LEMMAS.

Lemma 3.1: Transpose of the PDN matrix is same as its adjacency matrix.

Proof: In the matrix pattern analysis of Perfect Difference Network we have observed that the transpose of PDN matrix will always give us the adjacency matrix of PDN i.e;

a[ij] = a[ji] ....(1)

This is also illustrated in Fig.5.3.the equation (1) gives us the conclusion that the change does not affect the connectivity of the PDN or in other words we can say that the change will not affect PDN architecture of Perfect Difference Network (PDN).

	0	1	2	3	4	5	6
0	0	1	0	1	1	0	1
1	1	0	1	0	1	1	0
2	0	1	0	1	0	1	1
3	1	0	1	0	1	0	1
4	1	1	0	1	0	1	0
5	0	1	1	0	1	0	1
6	1	0	1	1	0	1	0

(a)

	0	1	2	3	4	5	6
0	0	1	0	1	1	0	1
1	1	0	1	0	1	1	0
2	0	1	0	1	0	1	1
3	1	0	1	0	1	0	1
4	1	1	0	1	0	1	0
5	0	1	1	0	1	0	1
6	1	0	1	1	0	1	0

(b)

Fig.5. (a) Adjacency matrix of PDN and (b) its Transpose

Lemma 3.2: Calculation of the ratio between total number of edges and its multiplication with diameter and total number of nodes gives us a factor (k) which gives a structural relationship between nodes, edges and diameter for different architectures.

Architecture	D	Total no. of nodes	Total no. of edges	k-factor
Hypercube	δ	2^{δ}	$(\delta*2^{\delta})/2$	2
PDN	2	$(\delta^2+\delta+1)$	$\delta(\delta^2+\delta+1)$	2 / δ
Complete Graph	1	$(\delta^2+\delta+1)$	$((\delta^2+\delta)(\delta^2+\delta+1))/2$	$2/(\delta^2+\delta)$

Proof: As we concluded our observation that the ratio of total number of edges and total number of nodes is proportional to the diameter of graph, i.e.

Diameter ∞ total number of edges / total number of nodes

⇒ Diameter = (k* total number of edges) / total number of nodes

⇒ k = (total number of nodes* diameter) / total number of edges

The table below shows the calculation of (k) factor for different architectures.

Table 1

From the Table 1 it is clear that lesser the value of k-factor the connections are more in the architecture and we have observed that the k-factor remains constant for hypercube architecture i.e. 2, because there is a ratio of proportionality between number of nodes and edges.

For PDN and Complete Graph architectures it varies, because diameter remains constant and also the density of edges is more than hypercube.

- For PDN architecture the k-factor is oscillating between (0 to 1), and in case of Complete Graph it oscillates between (0 to 0.17), but it remains constant for hypercube

***Lemma 3.3:* Partitioning of matrix of PDN and Hypercube shows symmetry.**

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IJARSE
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Proof: The matrix of Hypercube can be partitioned into four equal parts which can be further sub divided into four equal parts and so on, because of the property that the total

number of nodes in a hypercube is 2^n (which is always an even). Where as the matrix of PDN cannot be partitioned into equal parts, because the total number of nodes in a Perfect Difference Network (PDN) are $(\delta^2+\delta+1)$ which is an odd.

This is illustrated by the below matrix Fig.6. (a) Adjacency matrix representation of hypercube and (b) Adjacency matrix representation of PDN.

	0	1	2	3	4	5	6	7
0	0	1	0	1	0	0	0	1
1	1	0	1	0	0	0	1	0
2	0	1	0	1	0	1	0	0
3	1	0	1	0	1	0	0	0
4	0	0	0	1	0	1	0	1
5	0	0	1	0	1	0	1	0
6	0	1	0	0	0	1	0	1
7	1	0	0	0	1	0	1	0

(a)

	0	1	2	3	4	5	6
0	1	1	0	1	1	0	1
1	1	1	1	0	1	1	0
2	0	1	1	1	0	1	1
3	1	0	1	1	1	0	1
4	1	1	0	1	1	1	0
5	0	1	1	0	1	1	1
6	1	0	1	1	0	1	1

(b)

Fig.6. (a) Matrix representation of hypercube where n = 3, (b) Matrix representation of PDN where $\delta = 2$

From Fig.6. (a) it is clear that matrix of hypercube can be partitioned into equal square matrices whereas (b) the matrix of PDN cannot be partitioned into equal parts or square matrices.

Lemma 3.4: Calculation of total number of nodes, edges, squares, cubes and hypercubes in a Hypercube.

IJARSE
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Proof: The calculation of edges, nodes, squares, cubes and hypercubes in a Hypercube is shown below.

Case1: The total number of edges in a hypercube is

$$2^{n-1}*n$$

Case2: The total number of squares in a hypercube is

$$2^{n-3}*n\ (n-1)$$

Case3: The total number of cubes in a hypercube is

$$(\ 2^{n-4}*n(n-1)(n-2))/3$$

Case4: The total number of vertices/nodes in a hypercube is

$$(n!/(n-0)!*0!)*2^{n-0}$$

Case5: The total number of hypercubes in a hypercube is

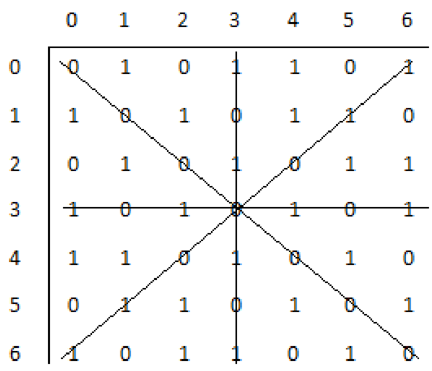
$$(n!/(n-4)!*4!)*2^{n-4}$$

We can use the above formulas for calculating the total number of edges, nodes, squares, cubes and hypercubes for any value of *n* (where *n* is the degree/diameter of a hypercube) or in other words we can say that for higher number of processors the above formulas are beneficial in calculating the edges, nodes, squares, cubes and hypercubes in a Hypercube.

Lemma 3.5: Band matrix representation of different interconnection networks shows binary palindrome.

Proof:

Case 1: Palindrome within PDN



(a)

Fig.7. (a) Palindrome within PDN

The above Figure 5.1.(a) shows the palindrome presentation within PDN. The lines shows the patterns which forms the palindrome within the PDN of order *δ = 2.*

1. The diagonal and cross diagonal shows the palindrome.

International Journal of Advance Research in Science and Engineering
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www.ijarse.com
IJARSE
ISSN: 2319-8354

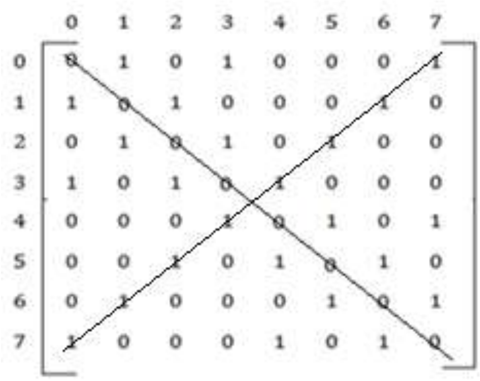
2. The $\lfloor \delta^2 + \delta + 1 \rfloor / 2$ row and column also show the palindrome.

Patterns Involved	Adjacency matrix representation	Palindrome representation
3rd row	1 0 1 0 1 0 1	1 0 1 0 1 0 1
3rd column	1 0 1 0 1 0 1	1 0 1 0 1 0 1
Diagonal	0 0 0 0 0 0 0	0 0 0 0 0 0 0
Cross diagonal	1 1 0 0 0 1 1	1 1 0 0 0 1 1

Table 2 Patterns showing palindrome within PDN

The table 5.1 shows that for PDN of order $\delta = 2, n = \delta^2 + \delta + 1$ patterns forming palindrome within adjacency matrix of PDN.

Case2: Palindrome within Hypercube



(a)

Fig.8. (a) Palindrome within hypercube

Fig.8. (a) Shows the palindrome within the hypercube of order n = 3. The lines represent the Patterns which shows the palindrome.

Patterns Involved	Adjacency matrix representation	Palindrome representation
Diagonal	0 0 0 0 0 0 0	0 0 0 0 0 0 0
Cross diagonal	1 1 1 1 1 1 1	1 1 1 1 1 1 1

Table 3 Patterns showing palindrome within hypercube

IV.CONCLUSION

The analysis of band matrix patterns of different interconnection networks shows us that how these architectures are represented in memory. The formulas presented in this paper give us the total number of nodes, edges, squares, cubes, and hypercubes of a hypercube.

The results of different Boolean operations on a Perfect difference Network (PDN) matrix suggest that PDN can be represented as a lattice. The results of the analysis of band matrix are shown in the form of lemmas. For the constant diameter of architecture the (k) factor varies and the variation in diameter the (k) factor remains constant.

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