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Topological properties of Complete Graph and Perfect Difference Network (PDN)

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ABSTRACT

In this paper we have explored the topological properties of complete Graph and Perfect Difference Network (PDN), of ($\delta^2+\delta+1$) nodes *and their relation with Euler Graph. In this paper we have also found the mathematical relation for calculating the total number of Hamiltonian paths in a Complete Graph and Perfect Difference Network (PDN).*

Keywords: Perfect Difference Set (PDS), Perfect Difference Network (PDN), Perfect Difference Graph (PDG), Complete Graph, Circuit, Euler Graph, Hamiltonian Graph.

I.INTRODUCTION

A study of topological properties of Complete Graph and Perfect Difference Network of ($\delta^2+\delta+1$) nodes is done. The relation of Complete Graph and Perfect Difference Network with Euler and Hamiltonian Graph's is presented in the form of Lemma's.

1.1 Perfect Difference Set

The Perfect Difference Sets were first discussed by J. Singer in 1938 in terms of points and lines in a projective plane of a Galois Field (GF) [10], [11].

Definition 1: Perfect Difference Set - If the set S of $\delta+1$ distinct integers $S_0, S_1 ..., S_\delta$ has the property that the $\delta^2 + \delta$ differences $S_i - S_j$ ($0\leq i, j \leq \delta$, $i\neq j$) are distinct modulo $\delta^2 + \delta + l$, S is called a perfect difference set mod $\delta^2 + \delta + l$.

The existence of perfect difference sets seems intuitively improbable, at any rate for large δ, but in 1938 J. Singer proved that, whenever δ is a prime or power of prime, say $\delta = p^n$, a perfect difference set mod $p^{2n} + p^n + 1$ exists. [2], [3], [4]

From now we on, let δ denote p^n and we write that $n = \delta^2 + \delta + l = p^{2n} + p^n + 1$. $S = \{s : |s_i - s_j| \ mod \ n$, where $0\leq i, j\leq \delta$, $i \neq j$, δ is a prime or power of prime and $n = \delta^2 + \delta + l\}$ [4].

1.2. Perfect Difference Network

Perfect Difference Network architecture, based on a PDS is designed where each ith node is connected via direct links to nodes i ± 1 and i ± s_j (mod n), for $2 \le j \le \delta$. Each link is bidirectional and the preceding connectivity leads to a chordal ring of δ in-degree and δ out-degree (total degree of a node $d(v) = 2\delta$) and diameter D = 2 [1], [3]. PDN has already been studied for, high performance communication and parallel processing network [3] and some topological properties of PDNs and parallel algorithms [4], [5], [6], [9] were suggested. It was shown that an n-node PDN can emulate the complete network with optimal slow down and balanced message traffic.

Although other interconnection architectures with topological and performance characteristics similar to PDNs exist, we view PDNs as worthy additions to the repertoire of computer system designers.

Alternative network topologies offer additional design points that can be exploited to accommodate the needs of new and emerging technologies. Further study is needed to resolve some open questions and to cost/performance comparisons for PDNs.

1.2. Perfect Difference Graph

In a PDN for each link from node I to node j, the reverse link j to i also exists, so the network corresponds to an undirected Perfect Difference Graph (PDG), G = (V, E) consisting of a set of vertices V and a set of edges E connecting pairs of vertices in V [7], [8].

1.3. Complete Graph

A Complete Graph is a simple graph G = (V, E) where for all vertices v_i, v_j € V, $v_i \neq v_j$, there exists an edge (v_i, v_j) [12].

In other words, in a Complete Graph every vertex is connected to every other vertex i.e. every pair of different vertices are adjacent.

1.4. Circuit

A circuit is a closed trial or a closed walk in which no edge is repeated. Vertices may however be repeated. i.e. the sequence $v_0, e_1, v_1, e_2, \ldots, e_n, v_n$ is a circuit if $v_0 = v_n$ and all e_i's are distinct [12].

II.TOPOLOGICAL PROPERTIES OF COMPLETE GRAPH AND PERFECT DIFFERENCE NETWORK (PDN) -

some of the topological properties of Complete Graph and Perfect Difference Network (PDN) of ($\delta^2+\delta+1$) nodes are presented below in the form of Lemma's.

Lemma 1: Calculation of total number of edges in a Complete Graph of ($\delta^2+\delta+1$) nodes.

Proof:

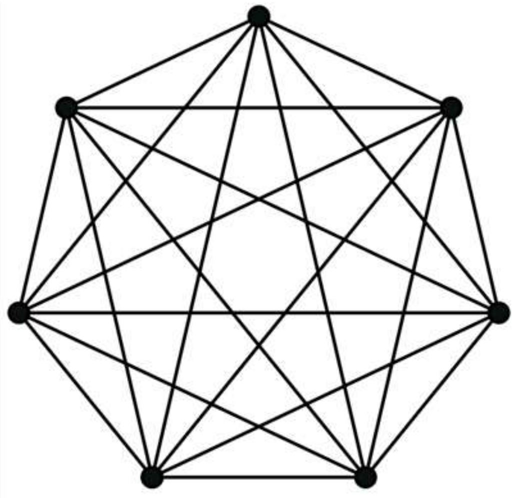

Fig 1.1 Complete Graph with **(δ^2+δ+1)**

Consider a complete graph of ($\delta^2+\delta+1$) nodes, where δ is prime or power of prime, then it give the following conclusion that the total degree of a single node in a Complete Graph is

$$(\delta^2+\delta+1) - 1$$

$$= \delta^2+\delta$$

Now we know that the total degree of a single node in a Complete Graph is ($\delta^2+\delta$), so the total degree of a Complete Graph will be,

$$\delta^2+\delta\ (\delta^2+\delta+1)$$

$$= \delta^4 + \delta^3 + \delta^2 + \delta^3 + \delta^2 + \delta$$

$$= \delta^4 + 2\delta^3 + 2\delta^2 + \delta \text{ ……………………………………….. (1.1)}$$

The total degree of a Complete Graph also gives the total links of a Complete Graph. Then the total edges of a Complete Graph of ($\delta^2+\delta+1$) nodes will be.

Total edges of a Complete Graph = Total degree of Complete Graph/2.

$$= (\delta^4 + 2\delta^3 + 2\delta^2 + \delta)/2 \text{ ……… (1.2)}$$

Lemma 2: The Complete Graph of (δ^2+δ+1) nodes and PDN possesses a property of Euler tour.

Proof: It has been proved that an undirected graph is Euler graph if and only if it is connected and has either zero or two vertices with an odd degree.

We know that the degree of each vertex in a Complete Graph of (δ^2+δ+1) nodes is (δ^2+δ) which is an even, (i.e. zero vertices have an odd degree). So we can say that the Complete Graph of (δ^2+δ+1) nodes possesses a property of Euler tour. And the Euler Path length is,

$$((\delta^2+\delta)\ (\delta^2+\delta+1))/2 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (2.1)$$

Where as for a PDN the degree of each vertex is 2δ which is also an even, (i.e. zero vertices have an odd degree). So we say that the PDN also possesses an Euler tour property and the path length is.

$$((2\delta)\ (\delta^2+\delta+1))/2$$

$$= \quad \delta\ (\delta^2+\delta+1) \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (2.2)$$

Hence Complete Graph of (δ^2+δ+1) nodes and PDN possesses a property of Euler tour.

Lemma 3: The Complete Graph of (δ^2+δ+1) nodes and PDN possesses a property of Hamiltonian path and the path length is n-1, (where n= δ^2+δ+1).

Proof: In a Complete Graph of (δ^2+δ+1) nodes and PDN, each vertex of a Complete Graph and PDN are visited exactly once without the repetition of edges, hence we say that the Complete Graph of (δ^2+δ+1) nodes and PDN possesses a property of Hamiltonian tour. And the Hamiltonian path length in both Complete Graph of (δ^2+δ+1) nodes and PDN is same, because the total number of nodes in a Complete Graph and a PDN is (δ^2+δ+1).

The Hamiltonian path length is = total number of nodes – 1

$$= (\delta^2+\delta+1) - 1$$

$$= (\delta^2+\delta) \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (3.1)$$

Hence the Hamiltonian path length for a Complete Graph and a PDN is (δ^2+δ).

Lemma 4: Calculation of total number of Hamiltonian paths in a PDN and Complete Graph of (δ^2+δ+1).

Proof: For the calculation of total number of Hamiltonian paths in a PDN and a Complete Graph we have the fallowing facts

- The total number of edges in a PDN is δ (δ^2+δ+1).

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- The total number of edges in a Complete Graph is $((\delta^2+\delta)(\delta^2+\delta+1))/2$
- The total number of vertices in a PDN and a Complete Graph is $(\delta^2+\delta+1)$.

The total number of Hamiltonian paths in a PDN = total number of edges in a PDN / total number of nodes in a PDN.

$= (\delta(\delta^2+\delta+1)) / (\delta^2+\delta+1)$.

$= \delta$.

The total number of Hamiltonian paths in a Complete Graph = total number of edges of a Complete Graph / total number of nodes in Complete Graph.

$= (((\delta^2+\delta)(\delta^2+\delta+1))/2) / (\delta^2+\delta+1)$

$= ((\delta^2+\delta)(\delta^2+\delta+1)) / 2*(\delta^2+\delta+1)$

$= (\delta^2+\delta) / 2$.

The below table shows the total number of unique Hamiltonian paths (in which all edges of the path are different from each other) of a Complete Graph and a PDN for different values of (δ)

δ	Total nodes $(\delta^2+\delta+1)$	Total no. of edges in Complete Graph is $(\delta^4+2\delta^3+2\delta^2+\delta)/2$	Total no. of Hamiltonian paths in a Complete Graph is $(\delta^2+\delta) / 2$	Total number of edges in a PDN is $\delta(\delta^2+\delta+1)$	Total number of Hamiltonian paths in a PDN is δ.
2	7	21	3	14	2
3	13	78	6	39	3
4	21	210	10	84	4
5	31	465	15	155	5
7	57	1596	28	398	7
8	73	2628	36	584	8
9	91	4095	45	819	9
11	133	8778	66	1463	11
13	183	16653	91	2379	13
16	273	37128	136	4368	16

Table 4.1

Lemma 5: The total number of nodes in a Perfect Difference Network (PDN) is always odd.

Proof: We know that the total number of nodes in a PDN is $(\delta^2+\delta+1)$ where δ is prime or power of prime.

Let x be an even number

Put, $(\delta^2+\delta) = x$

$\Rightarrow$ $\delta(\delta+1) = x$

$\Rightarrow$ $(\delta^2+\delta) = x$ ………………………….. (5.1)

The equation (5.1) gives us the conclusion that square of any number with addition to the same number is always even.

Now $(\delta^2+\delta)$ is even and we know that any even number with addition to 1 becomes an odd number. Hence $(\delta^2+\delta+1)$ is an odd number (i.e. the total number of nodes in a PDN is always odd).

III.CONCLUSION

The comprehensive study of topological properties of Complete Graph and Perfect Difference Network (PDN) of $(\delta^2+\delta+1)$ nodes has been done. We have found the relation of complete Graph and Perfect Difference Network (PDN) with Euler Graph and Hamiltonian Graph mathematically by evaluating the total links of Complete Graph and Perfect Difference Network (PDN).

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