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New Generalized Entropy Measure and its Corresponding Code-word Length and Their Characterizations

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ABSTRACT

In this manuscript we develop a new generalized entropy measure and corresponding to this measure we also develop a new generalized average code-word length and find the bounds of new generalized average code-word length in terms of new generalized entropy measure. Also we show that the measures defined in this communication are the generalizations of some well-known measures in the subject of coding and information theory. The bounds found in this paper for discrete channel have been verified by considering Huffman and Shannon-Fano coding schemes by taking an empirical data. The important properties of the new entropy measure have also been discussed.

Keywords: Shannon's entropy, Mean code-word length, Kraft's inequality, Holder's inequality, Huffman and Shannon-Fano codes.

I.INTRODUCTION

The concept of entropy was introduced by C. E Shannon [1] in his paper "A Mathematical Theory of Communication" Wikipedia defines entropy as "a measure of the uncertainty associated with a random variable. Shannon entropy, quantifies the expected value of the information contained in a message, usually in units such as bits and a 'message' means a specific realization of the random variable. Equivalently, the Shannon's entropy is a measure of the average information content one is missing when one does not know the value of the random variable. Entropy laid the foundation for a comprehensive understanding of communication theory, Shannon's entropy can be considered as one of the most important breakthroughs over the past fifty years in the literature on probabilistic uncertainty. The concept of entropy has been applied in a wide variety of fields such as statistical thermodynamics, urban and regional planning, business, economics, finance, operations research, queuing theory, spectral analysis, image reconstruction, biology and manufacturing.

Let X be a discrete random variable taking finite number of possible values $x_1, x_2, \dots, x_n$, with respective probabilities $p_1, p_2, \dots, p_n$, $p_i \geq 0 \forall i = 1, 2, 3, \dots, n$, and $\sum_{i=1}^{n} p_i = 1$, we denote

$$E = \begin{bmatrix} x_1, x_2, \dots, x_n \\ p_1, p_2, \dots, p_n \end{bmatrix} \quad (1)$$

and we call the scheme (1) as the finite information scheme. Shannon [1] proposed the following measure of uncertainty or measure of information associated with a finite information scheme (1) and calls it as entropy.

$$H(P) = E(I(x_i)) = H(p_1, \ p_2, \dots, p_n) = -\sum_{i=1}^{n} p_i \log p_i \quad (2)$$

The measure (2) serves as a suitable measure of entropy. Let $p_1, p_2, p_{3,\dots,} p_n$ be the probabilities of n codewords to be transmitted and let their lengths $l_1, l_2, \dots, l_n$, satisfy Kraft [2] inequality,

$$\sum_{i=1}^{n} D^{-l_i} \leq 1 \quad (3)$$

Where, D is the size of code alphabet.

For uniquely decipherable codes, Shannon [1] showed that for all codes satisfying (3), the lower bound of the mean code-word length,

$$L = \sum_{i=1}^{n} p_i l_i \quad (4)$$

lies between $H(P)$ and $H(P) + 1$.Where $H(P)$ is defined in (2). Campbell [3] considered the more general exponentiated mean code-word length as

$$L_\alpha = \frac{\alpha}{1-\alpha} log_D \left[\sum_{i=1}^{n} p_i D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right], \alpha > 0, \alpha \neq 1 \quad (5)$$

and showed that subject to (3), the minimum value of (5) lies between $R_\alpha(P)$ and $R_\alpha(P) + 1$, where

$$R_\alpha(P) = \frac{1}{1-\alpha} log_D \left[\sum_{i=1}^{n} p_i^\alpha\right], \alpha > 0, \alpha \neq 1 \quad (6)$$

is Renyi's [4] entropy of order α.

Various researchers have considered different generalized entropy measures and corresponding to these entropy measures they also develop the generalized code-word lengths and develop the coding theorems under the condition of uniquely decipherability; see for instance the published articles of Nath [5] Inaccuracy and coding theory. Longo [6] obtained minimum value of useful mean code-word length in terms of weighted entropy given by Belis and Guiasu [7]. Guiasu and Picard [8] develop a noiseless coding theorem by obtaining the minimum value of another useful average code-word length. Gurdial and Pessoa [9] also extended the theorem by finding the lower bounds for useful average code-word length of order α; also various authors like Jain and Tuteja [10], Taneja et al [11], Bhatia [12], Hooda and Bhaker [13], Khan et al [14], Bhat and Baig [15, 16, 17, 18] have developed different generalized coding theorems by taking into consideration different generalized information measures under the condition of uniquely decipherable codes.

II.BOUNDS OF NEW AVERAGE CODE-WORD LENGTH IN TERMS OF NEW GENERALIZED ENTROPY MEASURE

Define a new generalized entropy measure as:

$$H_\alpha^\beta(P) = \frac{1}{\beta(1-\alpha)} \sum_{i=1}^n p_i^{\frac{\alpha}{\beta}} \tag{7}$$

Where, $0 < \alpha < 1,\ \beta \geq 1, p_i \geq 0\ \forall i = 1,2,\dots,n, \sum_{i=1}^n p_i = 1.$

Remarks for (7)

I. When $\beta = 1$, (7) reduces to entropy, i.e.,

$$H_\alpha(P) = \frac{1}{1-\alpha} \sum_{i=1}^n p_i^\alpha, 0 < \alpha < 1. \tag{8}$$

II. When $\beta = 1$, and $\alpha \to 1$, (7) reduces to Shannon's [1] entropy, i.e.,

$$H(P) = -\sum_{i=1}^n p_i \log p_i$$

Corresponding to (7) we define a new generalized average code-word length and is given by

$$L_\alpha^\beta(P) = \frac{1}{\beta(1-\alpha)} \left[\sum_{i=1}^n p_i^{\frac{1}{\beta}} D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^\alpha, 0 < \alpha < 1, 0 < \beta \leq 1, \beta > \alpha. \tag{9}$$

Where, D is the size of code alphabet.

Remarks for (9)

I. For $\beta = 1$, (9) reduces to code-word length corresponding to entropy (8)

i.e., $L_\alpha(P) = \frac{1}{1-\alpha}\left[\sum_{i=1}^n p_i D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^\alpha$

II. For $\beta = 1$, and $\alpha \to 1$, (9) reduces to optimal code-word length corresponding to Shannon [1] entropy,

$$\text{i.e., } \sum_{i=1}^n p_i l_i \tag{10}$$

Now we found the bounds of (9) in terms of (7) under the condition

$$\sum_{i=1}^n D^{-l_i} \leq 1. \tag{11}$$

This is Kraft's [2] inequality, where D is the size of code alphabet.

Theorem 1: For all integers $(D > 1)$ the sequence of code-word lengths $l_1, l_2, \dots, l_n$ satisfies the condition (11), then the generalized average code-word length (9) satisfies the inequality

$$L_\alpha^\beta(P) \geq H_\alpha^\beta(P), 0 < \alpha < 1, \ \beta \geq 1. \quad (12)$$

Where equality holds good iff

$$l_i = -log\left[\frac{p_i^{\frac{\alpha}{\beta}}}{\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}}}\right] \quad (13)$$

Proof: By Holder's inequality we have

$$\sum_{i=1}^n x_i y_i \geq \left(\sum_{i=1}^n x_i^p\right)^{\frac{1}{p}}\left(\sum_{i=1}^n y_i^q\right)^{\frac{1}{q}} \quad (14)$$

For all $x_i, y_i > 0, \ i = 1, 2, 3, \dots, n$ and $\frac{1}{p} + \frac{1}{q} = 1, \ p < 1(\neq 0), \ q < 0$ or $q < 1(\neq 0), p < 0$.

We see the equality holds iff there exists a positive constant c such that

$$x_i^p = c y_i^q \quad (15)$$

Making the substitution

$$x_i = p_i^{\frac{\alpha}{\beta(\alpha-1)}} D^{-l_i}, \qquad y_i = p_i^{\frac{\alpha}{\beta(1-\alpha)}}, \qquad p = \frac{\alpha-1}{\alpha} \qquad \text{and} \qquad q = 1-\alpha$$

Using these values in (14), we get

$$\sum_{i=1}^n D^{-l_i} \geq \left[\sum_{i=1}^n p_i^{\frac{1}{\beta}} D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\frac{\alpha}{\alpha-1}}\left[\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}}\right]^{\frac{1}{1-\alpha}} \quad (16)$$

Now using the inequality (11) we get,

$$\left[\sum_{i=1}^n p_i^{\frac{1}{\beta}} D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\frac{\alpha}{\alpha-1}}\left[\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}}\right]^{\frac{1}{1-\alpha}} \leq 1. \quad (17)$$

Or, equivalently (17), can be written as

$$\left[\sum_{i=1}^n p_i^{\frac{1}{\beta}} D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\frac{\alpha}{\alpha-1}} \leq \left[\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}}\right]^{\frac{1}{\alpha-1}} \quad (18)$$

Here following cases arise

Case 1:

As, $0 < \alpha < 1$, then $(\alpha - 1) < 0$, raising both sides to the power $(\alpha - 1) < 0$, to inequality (18), we get

$$\left[\sum_{i=1}^{n} p_i^{\frac{1}{\beta}} D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\alpha} \geq \left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}\right] \quad (19)$$

As $0 < \alpha < 1$, $\beta \geq 1$ then $\beta(1-\alpha) > 0$ and $\frac{1}{\beta(1-\alpha)} > 0$, multiply inequality 19), throughout by $\frac{1}{\beta(1-\alpha)} > 0$, we get

$$\frac{1}{\beta(1-\alpha)}\left[\sum_{i=1}^{n} p_i^{\frac{1}{\beta}} D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\alpha} \geq \frac{1}{\beta(1-\alpha)}\left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}\right] \quad (20)$$

Or, equivalently we can write

$L_{\alpha}^{\beta}(P) \geq H_{\alpha}^{\beta}(P)$. Hence the result for $0 < \alpha < 1$, $\beta \geq 1$.

Case 2:

From equation (13), we have

$$l_i = -log\left[\frac{p_i^{\frac{\alpha}{\beta}}}{\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}}\right]$$

Or, equivalently we can write the above equation as

$$D^{-l_i} = \frac{p_i^{\frac{\alpha}{\beta}}}{\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}} \quad (21)$$

Raising both sides to the power $\left(\frac{\alpha-1}{\alpha}\right)$, to equation (21) and after suitable simplification, we get

$$D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)} = p_i^{\frac{\alpha-1}{\beta}}\left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}\right]^{\frac{1-\alpha}{\alpha}} \quad (22)$$

Multiply equation (22) both sides by $p_i^{\frac{1}{\beta}}$, then summing over $i = 1,2,\dots,n$, and after suitable simplifications, we get

$$\sum_{i=1}^{n} p_i^{\frac{1}{\beta}} D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)} = \left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}\right]^{\frac{1}{\alpha}} \quad (23)$$

Raising both sides to the power α to equation (23), then multiply both sides by $\frac{1}{\beta(1-\alpha)}$, we get

$L_{\alpha}^{\beta}(P) = H_{\alpha}^{\beta}(P)$. Hence the result.

Theorem 2: For every code with lengths $l_1, l_2, \dots, l_n$, satisfies Kraft's inequality, $L_\alpha^\beta(P)$, can be made to satisfy the inequality,

$$L_\alpha^\beta(P) < H_\alpha^\beta(P) D^{(1-\alpha)}. \text{ Where, } 0 < \alpha < 1,\ \beta \geq 1 \tag{24}$$

Proof: From the theorem (1), we have

$$L_\alpha^\beta(P) = H_\alpha^\beta(P).$$

Holds if and only if

$$D^{-l_i} = \frac{p_i^{\frac{\alpha}{\beta}}}{\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}}}, 0 < \alpha < 1,\ \beta \geq 1$$

Or, equivalently we can write the above equation as

$$l_i = -\log_D p_i^{\frac{\alpha}{\beta}} + \log_D \left[\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}}\right],$$

We choose the code-word lengths $l_i,\ i = 1,2, \dots, n$, in such a way that they satisfy the inequality

$$-\log_D p_i^{\frac{\alpha}{\beta}} + \log_D \left[\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}}\right] \leq l_i < -\log_D p_i^{\frac{\alpha}{\beta}} + \log_D \left[\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}}\right] + 1 \tag{25}$$

Consider the interval

$$\delta_i = \left[-\log_D p_i^{\frac{\alpha}{\beta}} + \log_D \left[\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}}\right], -\log_D p_i^{\frac{\alpha}{\beta}} + \log_D \left[\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}}\right] + 1\right]$$

Of length 1. In every δ_i, there lies exactly one positive integer l_i, such that,

$$0 < -\log_D p_i^{\alpha\beta} + \log_D \left[\sum_{i=1}^n p_i^{\alpha\beta}\right] \leq l_i < -\log_D p_i^{\alpha\beta} + \log_D \left[\sum_{i=1}^n p_i^{\alpha\beta}\right] + 1 \tag{26}$$

We will first show that the sequence $l_1, l_2, \dots, l_n$, thus defined satisfies the Kraft [2] inequality. From the left inequality of (26), we have

$$-\log_D p_i^{\frac{\alpha}{\beta}} + \log_D \left[\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}}\right] \leq l_i$$

Or, equivalently we can write the above expression as

$$D^{-l_i} \leq \frac{p_i^{\frac{\alpha}{\beta}}}{\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}}} \tag{27}$$

Taking summation over $i = 1,2,\ldots,n,$ on both sides to the inequality (27), we get

$$\sum_{i=1}^{n} D^{-l_i} \leq 1$$

This is Kraft's [2] inequality.

Now the last inequality of (26) gives

$$l_i < -\log_D p_i^{\alpha\beta} + \log_D \left[\sum_{i=1}^{n} p_i^{\alpha\beta}\right] + 1$$

Or, equivalently we can write the above expression as

$$D^{l_i} < \left[\frac{p_i^{\frac{\alpha}{\beta}}}{\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}}\right]^{-1} D \qquad (28)$$

As, $0 < \alpha < 1$, then $(1-\alpha) > 0$, and $\left(\frac{1-\alpha}{\alpha}\right) > 0$, raising both sides to the power $\left(\frac{1-\alpha}{\alpha}\right) > 0$, to inequality (28), we get

$$D^{l_i\left(\frac{1-\alpha}{\alpha}\right)} < \left[\frac{p_i^{\frac{\alpha}{\beta}}}{\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}}\right]^{\frac{\alpha-1}{\alpha}} D^{\frac{1-\alpha}{\alpha}}$$

Or, equivalently we can write the above expression as

$$D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)} < p_i^{\frac{\alpha-1}{\beta}} \left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}\right]^{\frac{1-\alpha}{\alpha}} D^{\frac{1-\alpha}{\alpha}} \qquad (29)$$

Multiply inequality (29), both sides by $p_i^{\frac{1}{\beta}}$, then summing over $i = 1,2,\ldots,n,$ and after suitable simplifications, we get

$$\sum_{i=1}^{n} p_i^{\frac{1}{\beta}} D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)} < \left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}\right]^{\frac{1}{\alpha}} D^{\frac{1-\alpha}{\alpha}} \qquad (30)$$

As, $0 < \alpha < 1, \quad \beta \geq 1$ then $\beta(1-\alpha) > 0$ and $\frac{1}{\beta(1-\alpha)} > 0$, raising both sides to the power α to inequality (30), then multiply the resulted expression both sides by $\frac{1}{\beta(1-\alpha)} > 0$, we get

$L_\alpha^\beta(P) < H_\alpha^\beta(P) D^{(1-\alpha)}$. Hence the result for $0 < \alpha < 1, \beta \geq 1$.

Thus from above two coding theorems we have shown that

$$H_\alpha^\beta(P) \leq L_\alpha^\beta(P) < H_\alpha^\beta(P) D^{(1-\alpha)}. \text{ Where, } 0 < \alpha < 1, \beta \geq 1.$$

In the next section noiseless coding theorems for discrete channel proved above are verified by considering Huffman and Shannon-Fano coding schemes on taking an empirical data.

III.ILLUSTRATION

In this section we illustrate the veracity of the theorems 1 and 2 by taking an empirical data as given in table 1 and 2 on the lines of Hooda et.al [19]. Using Huffman coding scheme the values of $H_\alpha^\beta(P)$, $H_\alpha^\beta(P)D^{(1-\alpha)}$, $L_\alpha^\beta(P)$ and η for different values of α and β are shown in the following table:

Table 1: Using Huffman Coding Algorithm values of $H_\alpha^\beta(P)$,$H_\alpha^\beta(P)D^{(1-\alpha)}$, $L_\alpha^\beta(P)$ and η for different values of α and β. Here D=2 in this case, as we use here binary code.

Probabilities P_i	*Huffman Code words*	l_i	α	β	$H_\alpha^\beta(P)$	$L_\alpha^\beta(P)$	$\eta = \frac{H_\alpha^\beta(P)}{L_\alpha^\beta(P)}$	$H_\alpha^\beta(P) \times I$
0.3846	0	1	0.9	1	11.6466	11.7002	99.5416%	12.4826
0.1795	100	3	0.5	1	4.3725	4.7714	91.6388%	6.1836
0.1538	101	3						
0.1538	110	3						
0.1282	111	3						

Now using Shannon-Fano coding scheme the values of $H_\alpha^\beta(P)$, $H_\alpha^\beta(P)D^{(1-\alpha)}$, $L_\alpha^\beta(P)$ and η for different values of α and β are shown in the following table:

Table 2: Using Shannon-Fano Coding Algorithm values of $H_\alpha^\beta(P)$, $H_\alpha^\beta(P)D^{(1-\alpha)}$, $L_\alpha^\beta(P)$ and η for different values of α and β. Here D=2 in this case, as we use here binary code.

Probabilities P_i	*Shannon Fano Code-words*	l_i	α	β	$H_\alpha^\beta(P)$	$L_\alpha^\beta(P)$	$\eta = \frac{H_\alpha^\beta(P)}{L_\alpha^\beta(P)}$	$H_\alpha^\beta(P) \times D^{(}$
0.3846	0	1	0.9	1	11.6466	11.8035	98.6707%	12.4826
0.1795	10	2	0.5	1	4.3725	5.3776	81.3091%	6.1836

0.1538	110	3						
0.1538	1110	4						
0.1282	1111	4						

From the tables 1 and 2 we infer the following:

1. Theorems 1 and 2 hold in both the cases of Shannon-Fano codes and Huffman codes. i.e.

   $H_\alpha^\beta(P) \le L_\alpha^\beta(P) < H_\alpha^\beta(P)D^{(1-\alpha)}$. Where, $0 < \alpha < 1,\ \beta \ge 1$

2. Mean code-word length $L_\alpha^\beta(P)$ is less in case of Huffman coding scheme as compared to Shannon-Fano coding scheme.
3. Coefficient of efficiency of Huffman codes is greater than coefficient of efficiency of Shannon-Fano codes i.e. it is concluded that Huffman coding scheme is more efficient than Shannon-Fano coding scheme.

In the next section the important properties of new generalized entropy measure $H_\alpha^\beta(P)$ have been studied.

IV.PROPERTIES OF NEW GENERALIZED ENTROPY MEASURE

In this section we will discuss some properties of the new generalized entropy measure $H_\alpha^\beta(P)$ given in equation (7).

Property 1: $H_\alpha^\beta(P)$ is non-negative.

Proof: From (7) we have

$$H_\alpha^\beta(P) = \frac{1}{\beta(1-\alpha)}\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}},\ 0 < \alpha < 1,\ \beta \ge 1$$

It is easy to see that for given values of α and β, $\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}} \ge 1$, also we have $0 < \alpha < 1,\ \beta \ge 1$, and $\frac{1}{\beta(1-\alpha)} > 0$, therefore we conclude that $\frac{1}{\beta(1-\alpha)}\left[\sum_{i=1}^n p_i^{\frac{\alpha}{\beta}}\right] \ge 0.$

Property 2: $H_\alpha^\beta(P)$ is a symmetric function on every $p_i,\ i = 1, 2, 3, \dots, n.$

Proof: It is obvious that $H_\alpha^\beta(P)$ is a symmetric function on every $p_i,\ i = 1, 2, 3, \dots, n.$

i.e., $$H_\alpha^\beta(p_1, p_2, \dots, p_{n-1}, p_n) = H_\alpha^\beta(p_n, p_1, p_2, \dots, p_{n-1})$$

Property 3: $H_\alpha^\beta(P)$ is maximum when $\beta = 1, \alpha \to 1,$ and all the events have equal probabilities.

Proof: When $p_i = \frac{1}{n} \forall i = 1,2,\dots,n$ and $\beta = 1, \alpha \to 1.$ Then $H_\alpha^\beta(P) = \log n,$ which is maximum entropy.

Property 4: For $\alpha \to 1$ and $\beta = 1,$ $H_\alpha^\beta(P)$ is concave downward function for $p_1, p_2, \dots, p_n.$

Proof: From equation (7) we have

$$\text{That } \frac{1}{\beta(1-\alpha)}\left[\sum_{i=1}^{n} p_i^{\frac{\alpha}{\beta}}\right], 0 < \alpha < 1,\ \beta \geq 1$$

If $\alpha \to 1$ and $\beta = 1,$ then the first derivative of (7) with respect to p_i is given by

$$\left[\frac{d}{dp_i} H_\alpha^\beta(P)\right]_{\substack{\beta=1\\ \alpha\to1}} = -1 - \log p_i$$

And the second derivative is given by

$$\left[\frac{d^2}{dp_i^2} H_\alpha^\beta(P)\right]_{\substack{\beta=1\\ \alpha\to1}} = -\left(\frac{1}{p_i}\right) < 0. \text{ For all } p_i \epsilon [0,1] \text{ and } i = 1, 2, \dots, n.$$

Since the second derivative of $H_\alpha^\beta(P)$ with respect to p_i is negative on given interval $p_i \epsilon [0,1] i = 1, 2, \dots, n.$ as $\beta = 1$ and $\alpha \to 1$, therefore,

$H_\alpha^\beta(P)$ is concave downward function for $p_1, p_2, \dots, p_n.$

V.CONCLUSION

In this Paper we develop a new generalized entropy measure and corresponding to this measure we also develop a new generalized average code-word length and find the bounds of new generalized average code-word length in terms of new generalized entropy measure. Also we show that the measures defined in this communication are the generalizations of some well-known measures in the subject of coding and information theory. The bounds found in this paper for discrete channel have been verified by considering Huffman and Shannon-Fano coding schemes by taking an empirical data. The important properties of the new entropy measure have also been discussed.

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