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EFFECTS OF SURFACE TENSION ON THE STABILITY OF TWO SUPERPOSED VISCOELASTIC FLUIDS IN A MAGNETIC FIELD

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ABSTRACT

The effect of surface tension on the stability of two superposed fluids can be described in a universal way by a non-dimensional ' surface tension number' is which provides a measure of the relative importance of surface tension and viscosity. When both fluids extend to infinity, the problem can be reduced to the finding of the roots of a quartic equation. The character of these roots is first analysed so as to obtain all possible modes of stability or instability. Two illustrative cases are then considered in further detail: an unstable case for which the density of the lower fluid is zero and a stable case for which the density of the upper fluid iszero, the latter case corresponding to gravity waves. Finally, the variational principle derived by Chandrasekhar for problems of this type is critically discussed and it is shown to be of less usefulness than had been thought, especially in those cases where periodic modes exist.

I INTRODUCTION

The instability of the interface between two superposed fluids has been studied by many writers. By considering a periodic disturbance of the interface with wave-number *k,* Rayleigh (8) showed that the amplitude of the disturbance would grow like e^{nt}, where

$$n^2 = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} gk$$

*g*is the acceleration of gravity, and *p1*and *p2* are the densities of the lower and upper fluids, respectively. For *p2> p1,* $n^2$is positive and the situation is therefore unstable; for p2 <*P1* however, $n^2$is negative and equation (1) is then simply the well-known relation for gravity waves.

The effect of a constant acceleration on the system has also been considered by Taylor (9) who showed that, in this case, equation (1) must be modified by replacing *g* by $g + g_1 = g^*$(say), where *gx*is the imposed acceleration. Thus, if the imposed acceleration is directed from the lighter towards the heavier fluid, it will have a destabilizing effect on the system.

For large values of *k,* corresponding to small wavelengths ¥ (= *2nlk),* the effects of surface tension and viscosity become important. These effects have been studieby Bellman and Pennington(i) who showed that when surface tension alone is considered, equation (1) assumes the form

$$n^2 = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} g_* k - \frac{T}{\rho_2 + \rho_1} k^3, \qquad (2)$$

where*T* is the coefficient of surface tension of the interface. Thus, if *g** is positive and *P2>P1,*then there is instability only for 0 <*k* <*kc,* where *kc*is the cut-off wavenumber given by

$$k_c = \sqrt{[(\rho_2 - \rho_1) g^*/T]}. \qquad (3)$$

Bellman and Pennington further showed that the cut-off wave-number defined by equation (3) remains unchanged when viscous effects are also taken into account.The case where viscous effects alone are included has been discussed in greater detail by Chandrasekhar (3) for the case in which the kinematic viscosities of the two fluids are the same, and he has shown how the required (k, *n)* relationship can be obtained from a quartic algebraic equation.

One of the purposes of the present paper, therefore, is to show how the combined effects of surface tension and viscosity can be taken into account in a completely general way. Some approximate results for this more general situation have been given by Tchen(10). He first obtained exact solutions for small and large values of the wave-number and in the neighborhood of the branch point. By introducing suitable interpolation formulae he was then able to obtain approximate results in the intermediate wave-number ranges.

A complete discussion of the characteristic quartic (§3), however, reveals the existence of other admissible modes. While these other modes are of a damped character and are thus not directly relevant to the stability question, they must be included when considering the related initial value problem as discussed by Carrier and Chang (2). The results presented in § 3 thus make possible a more explicit discussion of the problem from this latter point of view.

The problem of the instability of the melted surface of an ablating body has recently been considered by Feldman (5) who has shown that some aspects of this problem can be discussed from the point of view of the stability of superposed fluids. In his work the effects of both surface tension and viscosity are considered, and a comparison of some of his results with those of § 4 show them to be in close agreement.

Finally, in § 6, the variational principle derived by Chadrasekhar (3) for problems of this type and later used by Hide ((6), (7)) to obtain approximate solutions to a number of difficult problems is critically discussed. From this discussion it would appear thatan important term is missing from Hide's equations and that when this term is restored the method is less useful than had been thought, particularly in those cases where oscillatory modes are present.

2. *The characteristic equation for the,case v1 = v2.* When the kinematic viscosities of the two fluids are assumed to be the same, the analysis is considerably simplified without, at the same time, loss of the essential features of the problem. If a solution of the linearized equations of motion is sought which is proportional to

$$\exp(ik_x x + ik_y y + nt),$$

then one is led to a characteristic equation relating the wave-number *k* [= *](kx+ ky)]* to the amplification factor *n*, in which the effects of surface tension appear in a certainparametric form. Thus, following the notation of Chandrasekhar, we let

$$y = \sqrt{(1 + n/k^2\nu)} \quad \text{and} \quad Q = g_*/k^3\nu^2, \tag{4}$$

where we have set *v± = v2 — v.* In terms of these variables, the characteristic equation of the problem can be written in the form [cf. (l), equation (4-5)]

$$y^4 + 4\alpha_1\alpha_2 y^3 + 2(1 - 6\alpha_1\alpha_2)y^2 - 4(1 - 3\alpha_1\alpha_2)y + (1 - 4\alpha_1\alpha_2) + Q(\alpha_1 - \alpha_2) + Q^{\frac{1}{3}}S = 0, \tag{5}$$

where

$$\alpha_1 = \frac{\rho_1}{\rho_1 + \rho_2} \quad \text{and} \quad \alpha_2 = \frac{\rho_2}{\rho_1 + \rho_2} \quad (\alpha_1 + \alpha_2 = 1). \tag{6}$$

Equation (5) differs from the result given by Chandrasekhar [(3), equation (53)] only in the addition of the term involving the 'surface tension number' *S*.The surface tension number is usually denned (see, for example, (4), p. 95) by a relation of the form

$$S = \frac{T}{|g_*|(\rho_1 + \rho_2)L^2}, \tag{7}$$

where*L* is a characteristic length of the system. For the present purpose, the most convenient length to use is the one denned by

$$L = (\nu^2/|g_*|)^{\frac{1}{3}}. \tag{8}$$

$$S = \frac{T}{|g_*|^{\frac{1}{3}}(\rho_1 + \rho_2)\nu^{\frac{4}{3}}} \tag{9}$$

and this is the meaning of *S* in equation (5). Thus, *S* is a measure of the relative importance of surface tension and viscosity. For small values of *S*, viscous effects dominate and the term containing *S* in equation (5) can then be neglected. This corresponds to the cases treated by Chandrasekhar. For large values of *S*, the effect of viscosity is negligible compared to that of surface tension; this limiting case can be treated more conveniently, however, in an alternative manner that will be described later.

$$(|g_*|/\nu^2)^{\frac{1}{3}}\,\mathrm{cm}^{-1} \quad \text{and} \quad (|g_*|^2/\nu)^{\frac{1}{3}}\,\mathrm{sec.}^{-1}, \tag{10}$$

respectively, so that [cf. equations (4)]

$$k = Q^{-\frac{1}{3}} \quad \text{and} \quad n = (y^2-1)\,Q^{-\frac{2}{3}}. \tag{11}$$

Properties of the characteristic quartic. When either ax or a2 vanish, the terms independent of *Q* in equation (5) then have the same form. The resulting quartic

$$y^4 + 2y^2 - 4y + 1 = Y(y) \quad (\text{say}), \tag{12}$$

plays an important role in the subsequent analysis, and many features of the solution can be understood in terms of its properties.

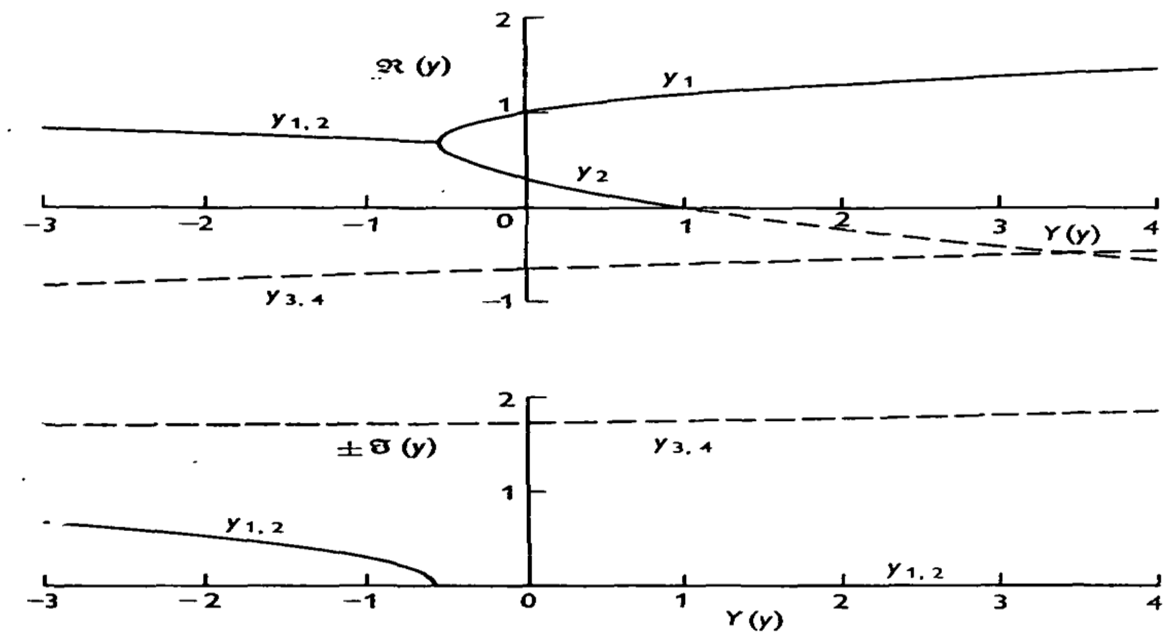

Figure 1. The roots of the characteristic quartic $y^4 + 2y^2 - 4y + 1 = Y(y)$.

Table 1 ***The roots of the characteristic quartic***

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$Y(y)$	y_1	y_2	y_3, y_4
4·0	+1·4623	−0·5649	−0·4487 ± 1·8520*i*
3·0	+1·3870	−0·4092	−0·4889 ± 1·8123*i*
2·0	+1·2961	−0·2239	−0·5361 ± 1·7755*i*
1·0	+1·1795	0·0000	−0·5898 ± 1·7445*i*
0·0	+1·0000	+0·2956	−0·6478 ± 1·7214*i*
−0·5	+0·8080	+0·5470	−0·6775 ± 1·7132*i*
−0·5814	+0·6823	+0·6823	−0·6823 ± 1·7121*i*

$Y(y)$	y_1, y_2	y_3, y_4
−0·5814	+0·6823 ± 0·0000*i*	−0·6823 ± 1·7121*i*
−0·75	+0·6923 ± 0·1869*i*	−0·6923 ± 1·7099*i*
−1·0	+0·7071 ± 0·2929*i*	−0·7071 ± 1·7071*i*
−1·5	+0·7363 ± 0·4290*i*	−0·7363 ± 1·7030*i*
−2·0	+0·7649 ± 0·5258*i*	−0·7649 ± 1·7007*i*
−2·5	+0·7926 ± 0·6054*i*	−0·7926 ± 1·7000*i*
−3·0	+0·8194 ± 0·6716*i*	−0·8194 ± 1·7006*i*

being two real roots and become complex conjugate roots. In the physical problem, this corresponds to a change from aperiodically damped modes to damped oscillatory modes.

In the neighborhood of the critical point is it possible to obtain simple analytical results for the behaviour of the roots that will prove useful in our later discussion. For this purpose let

$$y = y_r + iy_i. \tag{13}$$

$$(y_r^4 - 6y_r^2 y_i^2 + y_i^4) + 2(y_r^2 - y_i^2) - 4y_r + 1 = Y \tag{14}$$

$$4y_i[y_r(y_r^2 - y_i^2) + y_r - 1] = 0. \tag{15}$$

$$y_i = 0 \quad \text{for} \quad Y \geqslant Y_c \tag{16}$$

$$y_i^2 = (y_r^3 + y_r - 1)/y_r \quad \text{for} \quad Y \leqslant Y_c. \tag{17}$$

$$Y = Y_c + Y_c'(y - y_c) + \tfrac{1}{2} Y_c''(y - y_c)^2 + O[(y - y_c)^3], \tag{18}$$

$$Y_c' = 0 \quad \text{and} \quad Y_c'' = 4(3y_c^2 + 1). \tag{19}$$

$$y = y_c \pm \sqrt{[(2/Y_c'')(Y - Y_c)]} + O(Y - Y_c), \tag{20}$$

$$Y = y_r^{-2} - 4y_r^2 - 4y_r^4, \tag{21}$$

$$y_r(Y) = y_r(Y_c) + y_r'(Y_c)(Y - Y_c) + O[(Y - Y_c)^2] \tag{22}$$

$$y_r'(Y_c) = -\frac{1}{16y_c^3 + 8y_c + 2/y_c^3} = -0{\cdot}05939. \tag{23}$$

$$y_i(Y) = \pm\sqrt{\left[\frac{3y_c^2+1}{y_c} y_r'(Y_c)(Y - Y_c)\right]} + O(Y - Y_c). \tag{24}$$

$$Y(y) = \frac{1}{k^3} - \frac{S}{k}. \tag{25}$$

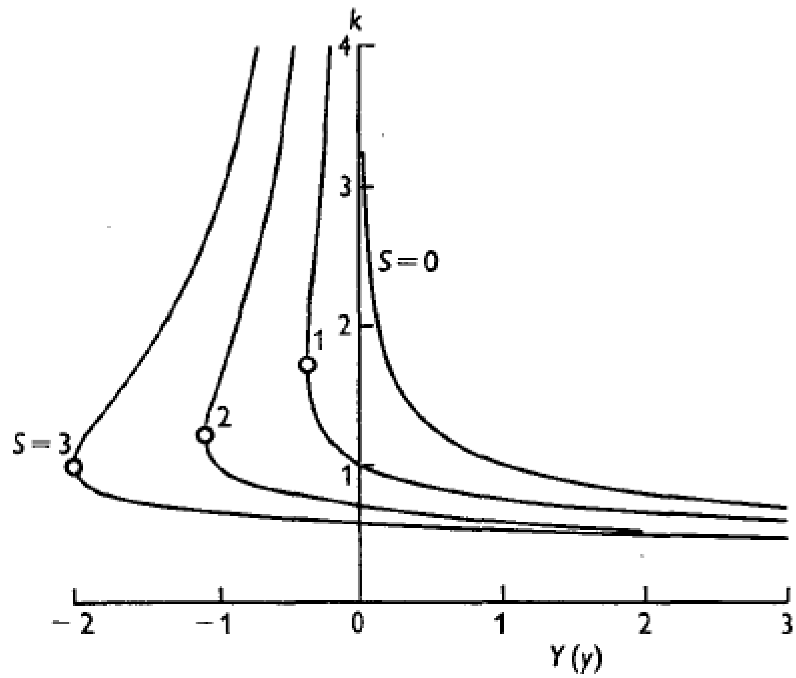

Figure 2.The graphs of *Y(y) + l/k^2-S/k* for the case p_1= 0

II DISCUSSION

In discussing the stability of two superposed fluids with constant properties both of which extend to infinity, the problem canin all cases be reduced to the finding of the roots of the algebraic equation (5) and the solution can then be discussed along the lines given above. In more general cases, when the fluid properties are a function of position or when the fluid is of limited depth, it is difficult to obtain an exact solution since the characteristic equation will not thenbe algebraic.This variational principle has been used by Hide (6) to obtain approximate solutions to a number of problems.It would appear, however, that an important term is missing from some of Hide'sequations and that when this term is restored the usefulness of the method is considerably reduced.

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