Optimizing Inventory Policy for Seasonal Products with Trade Credit and Price Discounts for Shelf Life items and Backorders

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ABSTRACT

In this paper an inventory model for items with fixed shelf life period and seasonal demand is presented. Items are sold out before shelf life (expiry period) of items offering price discount in decreasing phase of demand. Shortages are assumed with partial backlogging. To reduce the lost sales backordered quantity is enhanced by declaration of offering price discount. Further a credit period is allowed by the supplier to the retailer to earn interest on the accumulated sales revenue till the credit period. The retailer pays interest on the amount not sold till the credit period. Numerical illustrations with tables, graphs and sensitivity analysis of the optimal solution are presented.

Keywords: Shelf life, Partial backlogging, Seasonal demand, credit period, Price discounting.

I. INTRODUCTION

Deterioration of items is a common aspect in any inventory control system. It’s happen when the items of inventory become outworn, debase, decay or damaged associating on the type of goods. As an upshot of the deterioration shortages may happen. Hence deterioration portion has to be given so much importance while obtaining the optimal policy for an inventory model. Ghare and Schrader (1963) were the first to include the deterioration factor while determining inventory policy. Weibull distribution deterioration factor was initiating first time in EOQ model by Covert and Philip(1973). Since then many researchers presented their models using deterioration factor in different ways like Datta and Pal (1990b), Goyal and Giri (2001). Pandey and vaish (2017) proposed an optimal inventory policy with demand depending on seasonal factor for deteriorating items. Many products have finite shelf life like packed food items (biscuits, jam and bread etc.) fashionable items and medicines etc. Because of limited shelf-life or expiry period in order to make the balance in inventory cost at optimum level, a perfect inventory model is required which suits to meet the actual demand in the market. Avinadev (2009) developed an EOQ model with fixed shelf life for declining demand rate. Ukil et al (2015) developed an EPQ model for products having finite shelf life. Muriana (2016) introduced an inventory policy with fixed shelf life for perishable products under stochastic demand. Dave and Patel (1981) first introduced the linear time dependent demand in inventory formulating model. Deng et al (2007), Darzanou and Skouri (2011) and Wu et al (2014) developed inventory models assuming ramp type...
demand. Jaggi (2013) introduced inspection ordering policy for deteriorating items with time-dependent demand under inflationary conditions.

Price discount on unit selling price of goods attracts the customers to buy more and more. Thus price discount is an important factor which enhances the demand which in turn increases the total profit per unit time. There are so many authors who used price discount factor with some other related factors in developing their inventory models like Ardalan (1994) with temporary price discount, Pan and Hsiao (2005) and Lee et al (2007) with controllable lead time, Sana and Chaudhari (2008) with increasing profit, Hsu and Yu (2009) with imperfect items, Lin (2009) with integrated vendor buyer inventory policy with back order, Cardanas-Barron et al (2010) with one-time discount offer and back orders, Garg, Vaish and Gupta (2012) with ramp type demand and Vaish, B and Agarwal, D (2012) for non-instantaneous deterioration with quadratic demand rate. Heydari (2014) developed supply chain inventory model with time based policy for temporary price discounting. Liying et al (2016) developed a price discount policy for coordinating supplier, retailer and carrier. Pandey and Vaish (2017) developed an inventory policy with seasonal demand and price discounting on back orders.

Normally, the payment for any consignment is done by retailer to the supplier immediately. But in present days, due to tough competition in market and for attracting more customers a credit period has to be allotted to retailer by the suppliers to attain the maximum profit. No interest has to pay during this time credit period but beyond it, the retailer has to be pay an interest on the amount in stock. Before the end of time period of trade credit limit, the retailer can accumulate the revenue and earn so much profit and interest. This is termed as permissible delay in payment. Hence it is more profitable to the retailer to delay his payment till the very last day of the settlement period. First time inventory model with permissible delay in payment was developed by Goyal (1985) using suitable conditions. After that many authors presented inventory models using permissible delay factor like Chand and Ward (1987), Aggarwal and Jaggi (1995) and Hwang and Shinn (1997) Jamal et al. (1997), Chang and Dye (2001), Ouyang et al. (2005), Tsao and Sheen (2008) and Teng et al (2011). Jaggi (2013a) described an inventory model with effects of inspection on retailer’s ordering policy for deteriorating items with time-dependent demand. Jun li (2014) introduced an inventory model with credit period. Khanna (2017) developed an inventory policy for deteriorating imperfect quality items including permissible delay in payment.

In many real situations stock ends before the arrival of new consignment and many customers do not want wait for the next consignment. This is called as partial backlogging. This results in a greater amount of lost sales which in turn has a profit loss. While formulating an inventory model with shortages considering the partial backlogging is necessary. Many authors presented their models with shortages using factor of partial backlogging like Montgomery et al. (1973), Skouri (2003), Chang and Dye (1999), Dye et al. (2007) and Pandey et al (2016).

There are items which do not deteriorate slowly but have certain shelf life period (expiry period) from the date of their manufacturing beyond which they become obsolete and cannot be sold. The customer buys these products seeing their expiry date. Thus as the expiry date comes nearer some customers are not willing to purchase these items and thus the demand goes in decreasing phase. Therefore to attract customers and to sell all inventories before expiry date of the product, the inventory manager offers a price discount in the decreasing phase of demand. Further in the case of partial backlogging a declaration of offering a price discount on
backordered quantity reduces the lost sales to a great extant. Lastly it is more profitable for the retailer to earn 
interest on the accumulated sales revenue till the permissible delay payment period. The present paper is 
prepared considering the above mentioned features.

**ASSUMPTIONS and NOTATIONS:**
The following assumptions are used to develop the model:

1. The demand rate is $at(T-t)$, $a > 0$

2. It is assumed that product has a certain shelf life period $T$ and after that it becomes obsolete and cannot be 
sold. So after receiving the inventory at time $u$ ($T/2 < u < T$) (decreasing phase of demand) a price discount $d_1$ 
on selling price of each unit is offered by the retailer to customer to boost up the demand by 
$\alpha_1(1-d_1)^n$ where $n_1$ is a positive real number. As a result all inventories are sold out before expiry 
period. 

3. Shortages are allowed and are partially backlogged. A declaration of offering price discount $d_2$ on back 
ordered quantity increases the demand by $\alpha_2(1-d_2)^n$ where $n_2$ is a positive real number.

- $c$: Purchasing cost per unit
- $T$: Cycle length and shelf life period of item
- $t_1$: The time at which inventory level becomes zero
- $Q_1$: Initial inventory level at the beginning of each cycle and
- $Q_2$: Backordered quantity
- $Q$: Ordered Quantity ($Q_1+Q_2$)
- $I_p$: Rate of interest paid
- $I_e$: Rate of interest earned
- $M$: The retailer’s trade credit period offered by supplier
- $h$: Holding cost per unit per unit time
- $s$: Shortage cost per unit per unit time
- $l$: Lost sale cost per unit per unit time
- $O$: Ordering cost per order
- $\varepsilon$: Rate of backlogging $0 < \varepsilon < 1$
- $SR$: Sales revenue per replenishment cycle
- $I(t)$: The inventory level at time $t$.
- $F(t_1)$: Profit per unit time

**II. MODEL FORMULATION AND ANALYSIS**
The equations governing the inventory level $I(t)$ at any time $t$ during the inventory cycle $T$ are given 
as follows:

$$\frac{dI(t)}{dt} = -at(T-t) \quad 0 \leq t \leq u \quad \ldots (1)$$
\[\frac{dI(t)}{dt} = -\alpha \cdot u(T-t) \quad u \leq t \leq t_i \quad \ldots (2)\]

\[\frac{dI(t)}{dt} = -\alpha \cdot \varepsilon \cdot u(T-t) \quad t_i \leq t \leq T \quad \ldots (3)\]

With boundary condition \[I(0) = Q_1, I(t_i) = 0 \quad \ldots (4)\]

the solutions of above equations are given by:

\[I(t) = -\frac{a}{2} \int_0^t (t_i^3 - t^3) + \frac{a}{3} (t_i^3 - t_i^3) \quad 0 \leq t \leq u \quad \ldots (5)\]

\[I(t) = \alpha_1 \left[ \frac{a}{2} \left( t_i^3 - t^3 \right) - \frac{a}{3} (t_i^3 - t_i^3) \right] \quad u \leq t \leq t_i \quad \ldots (6)\]

\[I(t) = \alpha_2 \left[ \frac{a}{2} \left( t_i^3 - t^3 \right) - \frac{a}{3} (t_i^3 - t_i^3) \right] \quad t_i \leq t \leq T \quad \ldots (7)\]

Equating the value of \( I(u) \) from equation (5) and equation (6) then the value of \( Q_1 \) can be obtained as

\[Q_1 = \alpha_1 \left[ \frac{a}{2} \left( t_i^3 - t^3 \right) + \frac{a}{3} (t_i^3 - t_i^3) \right] (1 - \alpha) \quad \ldots (8)\]

\( Q_2 \) can be obtained as \( Q_2 = -1 \) (T)

\[Q_2 = a \alpha_\varepsilon \left[ \frac{T^3}{6} - \frac{T_i^3}{2} + \frac{t_i^3}{3} \right] \quad \ldots (9)\]

The Purchasing cost \( PC \) is given by

\[PC = c(Q_1 + Q_2)\]

\[PC = c\left[ \alpha_1 \left[ \frac{a}{2} \left( t_i^3 - t^3 \right) + \frac{a}{3} (t_i^3 - t_i^3) \right] (1 - \alpha) + \alpha_\varepsilon \left[ \frac{T^3}{6} - \frac{T_i^3}{2} + \frac{t_i^3}{3} \right] \right] \quad \ldots (10)\]

The holding cost \( HC \) is given by

\[HC = h \int_0^T I(t) \, dt + \int_u^T I(t) \, dt + \int_I(t) \, dt\]

\[HC = ah \left[ \frac{T^3}{3} \left( 1 - \alpha_c \right) + \frac{T_i^3}{3} \left( \alpha_c - 2 \varepsilon c_i \right) \right] \left( \alpha_c - \alpha_c \varepsilon \right) - \frac{u^4}{4} \left( 1 - \alpha \right) + \alpha_\varepsilon T^2 \frac{T_i^3}{2} + \left( 6t^2_i - T^2 \right) \quad \ldots (11)\]

The shortage cost \( SC \) is given by

\[SC = -s \int_0^T I(t) \, dt\]

\[SC = -sa_\varepsilon \left[ \frac{T^3}{2} \left( t_i^3 - t^3 \right) - \frac{1}{3} (t_i^3 - t_i^3) \right] \, dt\]

\[SC = sa_\varepsilon \left[ \frac{2T_i^3}{3} + \frac{T^4}{4} - \frac{T_i^4}{2} - \frac{T_i^2 t_i^2}{2} \right] \quad \ldots (12)\]
The lost sale LSC cost for the system can be calculated as

\[
LSC = a \int_{0}^{T} [t(T-t) - \alpha \epsilon t(T-t)] dt \\
LSC = a(l - \alpha \epsilon) \left( \frac{T^3}{6} - \frac{T^2 \epsilon}{2} + \frac{\epsilon^3}{3} \right)
\]  

... (13)

The Sales Revenue SR is given by

\[
SR = p \left[ \int_{0}^{u} a(t(T-t) dt + (1-d_1) \int_{u}^{M} a(T(t-T)dt + Q_1(1-d_2)) \right]
\]

\[
SR = ap\left\{ \frac{T^3}{2} - \frac{u^3}{3} + (1-d_1)\alpha \epsilon (t_1^3 - u^3) - (1-d_1)\alpha \epsilon (t_1^3 - u^3) + (1-d_2)\alpha \epsilon \left( \frac{T^3}{6} - \frac{T^2 \epsilon}{2} + \frac{\epsilon^3}{3} \right) \right\}
\]  

... (14)

The Present model also assumes the permissible delay in payments, therefore subjected to the credit period, three possible cases arise for retailer’s total profit \( M \leq u \leq t_1, u \leq M \leq t_1 \) and \( t_1 \leq M \leq T \)

**Case-1:** \( M \leq u \leq t_1 \) in this case the retailer can use sales revenue to earn interest during the period \([0, M]\) and the annual interest will be paid for the amount of inventory not sold till the credit period M. The total interest earned and the total interest paid will be as follows:

**Interest Earned**

\[
IE_i = pI_a \left( \int_{0}^{M} at(T-t)dt + (1-d_1)MQ_1 \right)
\]

\[
IE_i = apI_a \left\{ \frac{TM^3}{6} - \frac{M^4}{12} + (1-d_1)\alpha \epsilon M \left( \frac{T^3}{6} - \frac{T^2 \epsilon}{2} + \frac{\epsilon^3}{3} \right) \right\}
\]  

... (15)

**Interest Paid**

\[
IP_i = cI_a \left( \int_{0}^{M} I(t)dt + \int_{u}^{M} I(t)dt \right)
\]

\[
IP_i = acI_a \left\{ \frac{TM^3}{6} - \frac{M^4}{12} - (1-\alpha_1)(\frac{T^3}{6} - \frac{u^3}{12}) + \frac{T^3 \epsilon}{3} - \frac{u^3 \epsilon}{3} + \frac{T^2 \epsilon}{2} - \frac{u^2 \epsilon}{2} \right\}
\]

... (16)

The profit function per unit time for this case can be calculated as follows:
\[ F(t_i) = \frac{1}{T} [S.R. + IE_r - IP_r - P.C. - H.C. - S.C. - L.S.C. - O.C.] \]

\[ F(t_i) = \frac{1}{T} \left( ap \left( \frac{M_i^4}{6} - \frac{T_i^4}{12} + \frac{u}{3} \right) + \left( 1 - d_i \right) \alpha T \left( t_i^2 - u^2 \right) - \left( 1 - d_i \right) \alpha_i \left( t_i^2 - u^2 \right) + \left( 1 - d_i \right) \alpha_i \epsilon \left( \frac{T}{2} (T^2 - t_i^2) \right) \right) \]

\[ - \frac{1}{3} \left( T^3 - t_i^3 \right) + \frac{MT_i^3}{6} + \left( 1 - d_i \right) \alpha_i \epsilon M \left( \frac{M^3}{6} - \frac{T_i^3}{2} + \frac{t_i^3}{3} \right) \]

\[ - \alpha c \left( \frac{M_i^3}{6} - \frac{T_i^3}{12} - \left( 1 - \alpha \right) \frac{Tu}{6} + \frac{u}{3} \right) + \left( 1 - \alpha \right) \frac{Tu}{2} + \frac{u}{3} \left( \alpha_i \left( u - M \right) - u \right) \left( 1 - \alpha_1 \right) \left( u - M \right) \left( T \frac{u}{2} - \frac{u^3}{3} \right) \]

\[ - \epsilon \alpha \left( \frac{T_i^4}{2} + \frac{t_i^4}{6} \right) \left( 1 - \alpha_2 \epsilon \right) \left( \frac{T}{2} - \frac{T_i^2}{2} + \frac{t_i^2}{3} \right) - a \left( \frac{T}{6} - \frac{T_i^2}{2} + \frac{t_i^2}{3} \right) \]

\[ \ldots (17) \]

**Case-2:** \( u \leq M \leq t_1 \) in this case the retailer can use sales revenue to earn interest during the period \([0, M]\) and the annual interest will be paid for the amount of inventory not sold till the credit period \( M \). The total interest earned and the total interest paid will be as follows:

**Interest Earned**

\[ IE_2 = pl \left[ \int_0^u at(T - t)dt + \int_0^u \left( M - u \right) \right] \left[ \int_0^M \left( 1 - d_i \right) \alpha_i \epsilon \left( T - t \right) dt + MQ_2 \right] \]

\[ IE_2 = ap \left( \frac{u_i^4}{4} - \frac{T + M - u_i}{3} + \frac{MT_i^3}{12} + \frac{M_i^4}{6} - \frac{T_i^4}{12} + \frac{u}{3} \right) \]

\[ + \left( 1 - d_i \right) \alpha_i \epsilon M \left( \frac{T_i^3}{6} - \frac{T_i^4}{12} + \frac{t_i^4}{6} \right) \]

\[ \ldots (18) \]

**Interest Paid**

\[ IP_2 = cI_1 \left[ \int_0^1 I(t)dt \right] \]

\[ IP_2 = \alpha c \left( \frac{MT_i^3}{6} - \frac{M_i^4}{12} + \frac{T_i^4}{3} + \frac{t_i^4}{6} + \frac{MT_i^3}{3} \right) \]

\[ \ldots (19) \]

The profit function per unit time for this case can be calculated as follows:
Case-3: \( u \leq t_1 \leq M \leq T \) in this case the retailer can use sales revenue to earn interest during the period \([0, M]\) and as there is no inventory on hand beyond the credit period \(M\) so the retailer has to pay no interest. The total interest earned will be as follows:

**Interest Earned**

\[
IE_p = \int_0^T \left[ \frac{u^3}{2} \left( 1 + (M - u) \right) - \frac{u^3}{2} \left( 1 + (M - u) \right) \right] dt + \int_{M}^{t_1} \left[ \frac{u^2}{2} \left( 1 + (M - u) \right) - \frac{u^2}{2} \left( 1 + (M - u) \right) \right] dt
\]

(21)

The profit function per unit time for this case can be calculated as follows:

\[
F(t_1) = \frac{1}{T} \left[ S.R. + IE_p - IP_e - P.C. - H.C. - S.C. - L.S.C. - O.C. \right]
\]

\[
F(t_1) = \frac{1}{T} \left[ \alpha T \frac{u^3}{2} + (1 - d_t) \alpha T (t_1^2 - u^2) - \frac{1}{3} (1 - d_t) \alpha (t_1^3 - u^3) \right] + \frac{1}{2} \frac{TM^3}{6} - \frac{u^3}{3} \frac{MTu^3}{2} + \left( \frac{T M}{6} - \frac{u^3}{3} \frac{MTu^3}{2} \right)
\]

\[
-\frac{1}{3} \left( T^3 - t_1^3 \right) \right) + \frac{T M}{6} + (1 - d_t) \alpha \varepsilon M \left( \frac{T^3}{6} - \frac{u^3}{3} \frac{MTu^3}{2} \right)
\]

\[
-\frac{1}{3} \left( T^3 - t_1^3 \right) \right) + \frac{T M}{6} + (1 - d_t) \alpha \varepsilon M \left( \frac{T^3}{6} - \frac{u^3}{3} \frac{MTu^3}{2} \right)
\]

\[
-\frac{1}{3} \left( T^3 - t_1^3 \right) \right) + \frac{T M}{6} + (1 - d_t) \alpha \varepsilon M \left( \frac{T^3}{6} - \frac{u^3}{3} \frac{MTu^3}{2} \right)
\]

(22)
To find optimal values for profit function $F(t_1)$

The optimal value of $t_1$ is obtained by solving the equation

$$\frac{dF(t_1)}{dt_1} = 0$$

... (23)

Provided $\frac{d^2F(t_1)}{dt_1^2} < 0$  

(24)

**Numerical Illustration for case 1: $M \leq u \leq t_1$**

$T=1$ year, $u=0.55$, $M=0.50$ units, $p=45$ rs, $s=0.2$ rs/unit/time, $l=0.2$ rs/unit/time, $a=50000$, $h=0.8$ rs/unit/time, $n_1=5$, $c=0.6$, $c=40$ rs, $\alpha = (1-0.20)^2$, $\sigma =200$ rs/order, $I_c=0.05$, $I_o=0.07$, $b=0.8$, $d_1=0.12$, $d_2=0.20$. Applying the solution procedure described above the optimal values obtained are as follows:

$t_1^* = 0.765436$ days, $F^*(t_1) = 12875.8$ rs, $Q^* = 10393.7$, $\frac{d^2F^*(t_1)}{dt_1^2} = -33695.1$

**Numerical Illustration for case 2: $u \leq M \leq t_1$**

$T=1$ year, $u=0.55$, $M=0.75$ units, $p=45$ rs, $s=0.2$ rs/unit/time, $l=0.2$ rs/unit/time, $a=50000$, $h=0.8$ rs/unit/time, $n_1=5$, $n_2=2$, $c=0.6$, $c=40$ rs, $\alpha = (1-0.20)^2$, $\sigma =200$ rs/order, $I_c=0.05$, $I_o=0.07$, $d_1=0.12$, $d_2=0.20$. Applying the solution procedure described above the optimal values obtained are as follows:

$t_1^* = 0.823708$ days, $F^*(t_1) = 210435$ rs, $Q^* = 108482$, $\frac{d^2F^*(t_1)}{dt_1^2} = -454455$

**Numerical Illustration for case 3: $t_1 \leq M \leq T$**

$T=1$ year, $u=0.55$, $M=0.95$ units, $p=45$ rs, $s=0.2$ rs/unit/time, $l=0.2$ rs/unit/time, $a=50000$, $h=0.8$ rs/unit/time, $n_1=5$, $c=0.6$, $c=40$ rs, $\alpha = (1-0.20)^2$, $\sigma =200$ rs/order, $I_c=0.05$, $I_o=0.07$, $b=0.8$, $d_1=0.12$, $d_2=0.20$. Applying the solution procedure described above the optimal values obtained are as follows:

$t_1^* = 0.935795$ days, $F^*(t_1) = 257845$ rs, $Q^* = 114102$, $\frac{d^2F^*(t_1)}{dt_1^2} = -141353$

For the case 2 the effects of various parameters on total profit per unit are obtained as follows;
Effects of parameter "u" on Total Profit per Unit Time

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Table-1

Effects of parameter "M" on Total Profit per Unit Time

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Table-2

Effects of parameter "I_p" on Total Profit per Unit Time

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Fig-2
Table-3

Effects of parameter "Ie" on Total Profit per Unit Time

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</tbody>
</table>

Table-4

Effects of parameter "h" on Total Profit per Unit Time

<table>
<thead>
<tr>
<th>%change in h</th>
<th>h</th>
<th>t1</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>-15%</td>
<td>0.68</td>
<td>0.85742</td>
<td>216932</td>
</tr>
<tr>
<td>-10%</td>
<td>0.72</td>
<td>0.846043</td>
<td>214714</td>
</tr>
<tr>
<td>-5%</td>
<td>0.76</td>
<td>0.834807</td>
<td>212548</td>
</tr>
<tr>
<td>0</td>
<td>0.8</td>
<td>0.823708</td>
<td>210435</td>
</tr>
<tr>
<td>5%</td>
<td>0.84</td>
<td>0.812744</td>
<td>208377</td>
</tr>
<tr>
<td>10%</td>
<td>0.88</td>
<td>0.801913</td>
<td>206376</td>
</tr>
<tr>
<td>15%</td>
<td>0.92</td>
<td>0.791211</td>
<td>204434</td>
</tr>
</tbody>
</table>

Sensitivity Analysis
III. OBSERVATIONS

1. Table (1) reveals that as the parameter \(u\) decreases, profit per unit time of the system increases.
2. From table (2) it is observed parameter \(M\) increases, profit per unit time of the system also increases.
3. Table (3) shows that as the interest paid \(I_p\) increases, profit per unit time of the system also decreases.
4. From table (4) it is observed that the interest earn \(I_e\) increases, the unit time profit of the system increases.
5. Table (5) reveals that the holding cost \(h\) decreases, the unit time profit of the system also increases.
6. From sensitivity table (6) it has been observed \(F(t_1)\) is negligible sensitive to \(I_p\), moderately sensitive to \(I_e\) and \(h\) and fairly sensitive to \(M\) and \(u\).
IV. CONCLUSION

In the present paper an inventory model is developed with realistic features of seasonal demand and partial backlogging. The most important feature of the modal is the product shelf life or expiry period of items. These items have no selling value after their date of expiry. So it is necessary to sell all inventory of such type of items before their expiry date. Considering this fact a price discount is offered to the customers in decreasing phase of demand to sell out all inventories before the expiry period and to reduce the loss. Further declaration of price discount at the start of shortage period enhances the backorder quantity. The most common feature of present market is that the suppliers allot the retailers a credit period to settle the account. In this period the retailers earn interest on the accumulated sales revenue and pays interest on the amount not sold till credit period. The present paper is prepared considering the above mentioned real features. Three possible cases for credit period are discussed and the numerical illustrations, tables, graph and sensitivity analysis represents the reality of the model to practical situations of the market. The present study can be further studied for some other situations of demand and partial backlogging useful for inventory problems.

REFERENCES


[23.] Jun Li, Hairong Feng, Yinlian Zeng (2014) Inventory games with permissible delay in payments, European Journal of Operational Research, 234(3) 694-700


