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On w-α-𝔗 sets and w-α-𝔗 functions

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ABSTRACT

In this paper we will give various properties of w-α-𝔗-open and w-α-𝔗-closed sets. Also w-α-𝔗-open and w-α-𝔗-closed mappings are discussed.

Key Words and phrases: **w- α- 𝔗- open, w-semi-𝔗-open, w-pre-𝔗-open.**

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I.INTRODUCTION

In [2], Dontchev introduced the concept of pre- 𝔗-open sets and in [3] Hatir and Noiri introduced the notion of semi-𝔗-open sets, α- 𝔗-open sets and β- 𝔗-open sets. Further in [1], A.Acikgoz et al. Obtain several characterizations of α-𝔗-continuous functions and introduced the concept of α-𝔗-open functions in ideal topological spaces and obtain their properties. The subject of ideals in topological spaces were introduced by Kuratowski[4] and further studied by Vaidyanathaswamy[5]. Corresponding to an ideal a new topology $\tau^*(\mathfrak{T}, \tau)$ called the *-topology was given which is generally finer than the original topology having the kuratowski closure operator $cl^*(A) = A \cup A^*(\mathfrak{T},\tau)$[6], where $A^*(\mathfrak{T},\tau) = \{x \in X : U \cap A \notin \mathfrak{T}$ for every open subset U of x in X called a local function of A with respect to 𝔗 and τ. We will write τ^* for $\tau^*(\mathfrak{T}, \tau)$.

The following section contains some definitions and results that will be used in our further sections.

Definition 1.1.[4]: Let (X, τ) be a topological space. An ideal 𝔗 on X is a collection of non-empty subsets of X such that (a) $\phi \in \mathfrak{T}$ (b) $A \in \mathfrak{T}$ and $B \in \mathfrak{T}$ implies $A \cup B \in \mathfrak{T}$ (c) $B \in \mathfrak{T}$ and $A \subset B$ implies $A \in \mathfrak{T}$.

Definition 1.2 : Let(X,τ,𝔗) be an ideal space and A be any subset of X. Then A is said to be

a.) semi-𝔗-open[3] if $A \subset cl^*(int(A))$.
b.) pre-𝔗-open[2] if $A \subset int(cl^*(A))$.
c.) α- 𝔗-open[3] if $A \subset int(cl^*(int(A)))$.
d.) β-𝔗-open[3] if $A \subset cl(int(cl^*(A)))$.

II.RESULTS

Definition 2.1: Let (X,τ,𝔗) be an ideal space and A be any subset of X. Then A is said to be

a.) w- α- 𝔗- open if $A \subset int(cl(int^*(A)))$.

b.) w-semi-$\mathfrak{T}$-open if $A \subset cl(int^*(A))$.

c.) w-pre-$\mathfrak{T}$-open if $A \subset int^*(cl(A)$.

Definition 2.2: Let $(X,\tau,\mathfrak{T})$ be an ideal space and A be any subset of X. Then A is said to be

a.) α^*- $\mathfrak{T}$- open if $A \subset int^*(cl^*(int^*(A)))$.

b.) semi*-$\mathfrak{T}$-open if $A \subset cl^*(int^*(A))$.

c.) Pre*-$\mathfrak{T}$-open if $A \subset int^*(cl^*(A)$.

Lemma 2.3: Let $(X,\tau,\mathfrak{T})$ be an ideal space and U and V be two τ^*-open subsets of X. Then prove that

$cl^*(U) \cap V \subset cl^*(U \cap V)$.

Proof: Let $x \in cl^*(U) \cap V$. To prove $x \in cl^*(U \cap V)$. Let W be any τ^*-open set containing x. Then $x \in V$ and V is τ^*-open set implies that $V \cap W$ is also τ^*-open set containing x. Now $x \in cl^*(U)$ implies that $V \cap W \cap U \neq \phi$ and so $W \cap (U \cap V) \neq \phi$ implies that $x \in cl^*(U \cap V)$.

Hence $cl^*(U) \cap V \subset cl^*(U \cap V)$.

Theorem 2.4: Let $(X,\tau, \mathfrak{T})$ be an ideal space and A be any subset of X. Then prove that A is w-α-$\mathfrak{T}$-open if and only if A is w-pre-$\mathfrak{T}$-open and A is w-semi-$\mathfrak{T}$-open subset of X.

Proof: Firstly, let A is A is w-α-$\mathfrak{T}$-open subset of X then $A \subset int(cl(int^*(A)))$ and so $A \subset cl(int^*(A))$. Also

$A \subset int^*(cl(A))$. This implies that A is w-pre-$\mathfrak{T}$-open and w-semi-$\mathfrak{T}$-open subset of X.

Conversely, let A is w-pre-$\mathfrak{T}$-open and w-semi-$\mathfrak{T}$-open subset of X. Then A is w-pre-$\mathfrak{T}$-open implies that

$A \subset int^*(cl(A))$ and further A is w-semi-$\mathfrak{T}$-open implies that $A \subset int^*(cl(cl(int^*(A)))) = int^*(cl(int^*(A)))$.

Hence A is A is w-α-$\mathfrak{T}$-open subset of X.

Proposition 2.5 : Let $(X,\tau, \mathfrak{T})$ be an ideal space.

(a) If V is semi*-$\mathfrak{T}$-open and A is α^*-$\mathfrak{T}$-open subset of X then $V \cap A$ is semi*-$\mathfrak{T}$-open subset of X.

(b) If V is pre-$\mathfrak{T}$-open and A is α^*-$\mathfrak{T}$-open subset of X then $V \cap A$ is pre*-$\mathfrak{T}$-open subset of X.

Proof: (a): Let V is semi*-$\mathfrak{T}$-open and A is α^*-$\mathfrak{T}$-open subset of X.

Then $V \cap A \subset cl^*(int^*(V)) \cap int^*(cl^*(int^*(A)))$

$\subset cl^*(int^*(V) \cap int^*(cl^*(int^*(A))))$ using Lemma 2.3.

$\subset cl^*(int^*(V) \cap cl^*(int^*(A)))$

⊂ cl*(cl*(int*(V)⋂int*(A)))

= cl*(int*(V)⋂int*(A))

⊂ cl(int*(V⋂A)).

Hence V⋂A is semi*-𝔗-open subset of X.

(b): Let V be pre*-𝔗-open and A be α*-𝔗-open subset of X.

Then V⋂A ⊂ int*(cl*(V))⋂int*(cl*(int*(A)))

= int*(int*(cl(V))⋂cl(int*(A)))

⊂ int*(cl(int*(cl*(V))⋂int*(A)))

⊂ int*(cl(cl(V)⋂int*(A)))

⊂ int*(cl*(cl*(V⋂int*(A)))

= int*(cl*(V⋂int*(A))

⊂int*(cl*(V⋂A)).

Hence V⋂A is pre*-𝔗-open subset of X.

Corollary 2.6: Let (X,τ, 𝔗) be an ideal space.

(a) If V is semi*-𝔗-open and A is τ*-open subset of X then V⋂A is semi*-𝔗-open.
(b) If V is pre*-𝔗-open and A is τ*-open subset of X then V⋂A is semi*-𝔗-open.

Proof: Proof follows from the above Theorem and the fact that every τ*-open subset of X is α*-𝔗-open subset of X.

Theorem 2.7: Let (X,τ, 𝔗) be an ideal space. Then a subset B of X is w-α-𝔗-open iff there exist τ*-open subset U of X such that U ⊂ B ⊂ int*(cl(U)).

Proof: Firstly, let B be w-α-𝔗-open subset of X. Then B ⊂ int*(cl(int*(B))). Let U = int*(B). Since we know that int*(B) is τ*-open so U is τ*-open subset of X such that U ⊂ B ⊂ int*(cl(U)).

Conversely, let there exist τ*-open subset U of X such that U ⊂ B ⊂ int*(cl(U)). Now U ⊂ B implies that int*(U) ⊂ int*(B) and so U ⊂ int*(B). Therefore, B ⊂ int*(cl(U)) implies that B ⊂ int*(cl(int*(B))).

Hence B is w-α-𝔗-open subset of X.

Theorem 2.8: If A is w-semi-$\mathfrak{I}$-open subset of an ideal space (X,τ,$\mathfrak{I}$) and be any subset of X such that

A ⊂ B ⊂ int*(cl(A)) then prove that B is also w-α-$\mathfrak{I}$-open.

Proof: Let A be any w-α-$\mathfrak{I}$-open subset of X and B be any subset of X such that A ⊂ B ⊂ int*(cl(A)). Now A is w-α-$\mathfrak{I}$-open subset of X so by the above Theorem 2.7 there exist τ*-open subset G of X such that G ⊂ A ⊂ int*(cl(G)) and so G ⊂ A ⊂ B ⊂ int*(cl(A)) ⊂ int*(cl(int*(cl(G)))) ⊂ int*(cl(cl(G)))

= int*(cl(G)).

Therefore, G ⊂ B ⊂ int*(cl(G)).

Hence B is w-α-$\mathfrak{I}$-open.

Theorem 2.9: Let (X,τ, $\mathfrak{I}$) be an ideal space. Then prove the following:

(a) If $\{U_\alpha\}_{\alpha\in\Delta}$ be a family of w-α-$\mathfrak{I}$-open subsets of X. Then prove that $\bigcup_\alpha U_\alpha$ is also a w-α-$\mathfrak{I}$-open set.
(b) If U is w-α-$\mathfrak{I}$-open subset of X and V is τ-open subset of X then prove that U⋂V is also a w-α-$\mathfrak{I}$-open set.

Proof: (a) Since ∀ α ϵ Δ, U_α is w-α-$\mathfrak{I}$-open subset of X. So $U_\alpha \subset$ int*(cl(int*(U_α))).

Now $\bigcup_\alpha U_\alpha \subset \bigcup_\alpha \text{int}*(\text{cl}(\text{int}*(U_\alpha)))$ and so $\bigcup_\alpha U_\alpha \subset$ int*($\bigcup_\alpha$ cl(int $*$ (U_α))) since

$\bigcup_\alpha int(A_\alpha) \subset$ int($\bigcup_\alpha A_\alpha$). Further, $\bigcup_\alpha cl(A_\alpha) \subset$ cl($\bigcup_\alpha A_\alpha$) implies that

$\bigcup_\alpha U_\alpha \subset$ int*($cl(\bigcup_\alpha int * (U_\alpha))$) and so

$\bigcup_\alpha U_\alpha \subset$ int*($cl(int * (\bigcup_\alpha(U_\alpha)))$).

Hence $\bigcup_\alpha U_\alpha$ is w-α-$\mathfrak{I}$-open subset of X.

(b)Let U be w-α-$\mathfrak{I}$-open subset of X and V be τ-open subset of X. Then U ⊂ int*(cl(int*(U))). Now

U⋂V ⊂ int*(cl(int*(U))) ⋂ V = int*(cl(int*(U))⋂int(V)) since V is τ-open subset of X and so

U⋂V ⊂ int*(cl(int*(U)⋂ int(V))) using Lemma 2.3 .But int*(A)⋂int*(B) = int*(A ⋂ B). Therefore,

U⋂V ⊂ int*(cl(int*(U⋂V))).

Hence U⋂V is w-α-$\mathfrak{I}$-open.

Next we introduce w-α-$\mathfrak{I}$-closed sets.

Definition 2.10: Let (X,τ,$\mathfrak{I}$) be an ideal space. Then a subset F of X is called w-α-$\mathfrak{I}$-closed if its complement X-F is w-semi-$\mathfrak{I}$-open.

Theorem 2.11: If a subset F of an ideal space (X,τ,$\mathfrak{I}$) is w-semi-$\mathfrak{I}$-closed then prove that cl*(int(cl*(F) ⊂ F.

Proof: Let F be any w-α-$\mathfrak{I}$-closed subset of X. Then X-F is w-α-$\mathfrak{I}$-open subset of X. Therefore,

X-F ⊂ int*(cl(int*(X-F))) = int*(cl(X-cl*(F))) using int*(X-A) = X-cl*(A) or X-int*(A) = cl*(X-A) for any subset A of X and so X-F ⊂ int*(X-int(cl*(F)))

= X-cl*(int(cl*(F)))

Therefore, cl*(int(cl*(F))) ⊂ F.

Definition 2.12: Let (X,τ,$\mathfrak{I}$) and (Y,σ, $\mathcal{J}$) with $\mathcal{J}$=f($\mathfrak{I}$) be two topological spaces. Then a map f : (X,τ,$\mathfrak{I}$) → (Y,σ, $\mathcal{J}$) is said to be w-α-$\mathfrak{I}$-open(closed) if image of every open(closed) set in X is w-α-$\mathfrak{I}$-open(closed) in Y.

i.e. f is w-α-$\mathfrak{I}$-open(closed) if ∀ G ϵ τ, f(G) is w-α-$\mathfrak{I}$-open(closed) subset of Y.

Theorem 2.13: Let (X,τ,$\mathfrak{I}$) and (Y,σ,$\mathcal{J}$) be two ideal topological spaces and f : (X,τ,$\mathfrak{I}$) → (Y,σ, $\mathcal{J}$) be any map. Then prove that the following are equivalent:

a) f is w-α-$\mathfrak{I}$-open map.
b) For each x ϵ X and any open set U containing x, there exist w-α-$\mathfrak{I}$-open subset V in Y containing f(x) such that V ⊂ f(U).

Proof: Firstly, let f be w-α-$\mathfrak{I}$-open map. Let x ϵ X be any element and U be any open set in X containing x. This implies that there exist basic open set say B such that x ϵ B ⊂ U. Now f is w-α-$\mathfrak{I}$-open map and B is open subset of X implies that f(B) is w-α-$\mathfrak{I}$-open subset of Y containing f(x). Let V = f(B) then B ⊂ U implies that f(B) ⊂ f(U). Hence there exist w-α-$\mathfrak{I}$-open subset V of Y containing f(x) such that V ⊂ f(U).

Conversely, let G be any open subset in X. We have to prove that f(G) is w-α-$\mathfrak{I}$-open subset of Y.

Now there can be two possibilities:

Case(i) If G = ϕ, then f(G) = ϕ. Hence we have nothing to prove.

Case(ii) If G ≠ ϕ, then f(G) ≠ ϕ. Let y ϵ f(G) and so there exist x ϵ G such that y = f(x). Now, G is the open subset of X containing x so there exist w-α-$\mathfrak{I}$-open subset V in Y containing f(x) such that V ⊂ f(G).

So, ∀ y ϵ f(G) there is w-α-$\mathfrak{I}$-open subset V in Y containing y such that y ϵ V ⊂ f(G). This implies that f(G) is union of w-α-$\mathfrak{I}$-open subsets of Y. Hence f(G) is w-α-$\mathfrak{I}$-open subset of Y.

Hence f is w-α-𝔗-open map.

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