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Construction Methodology and enumeration of the Steiner Triple Systems of Order 29

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ABSTRACT

A Steiner triple system of order n is a collection of subsets of size three, taken from the n-element set {0, 1, ..., n−1}, such that every pair is contained in exactly one of the subsets. The subsets are called triples, and a block-intersection graph is constructed by having each triple correspond to a vertex. If two triples have a non-empty intersection, an edge is inserted between their vertices. It is known that there are 345,147 Steiner triple systems of order 29 up to isomorphism. In this paper, we attempt to distinguish the all isomer groups of the STS systems.

Keywords: Steiner triple systems, order, combinatorics, construction, permutation

I.INTRODUCTION

In 1847, Reverend Thomas P. Kirkman published "On a Problem in Combinations" [11] that first answered questions on the existence of what would later be called Steiner triple systems. The problem he considered was to find a collection of subsets of size three, taken from the n-element set {0, 1, ..., n − 1}, with the property that every pair is contained in exactly one of the subsets.

This problem was a specific case of a more general question on combinations posed by W.S.B. Woodhouse in 1844. Several years later, Jakob Steiner, apparently unaware of Kirkman's article, similarly asked about the existence of various designs, including the one solved by Kirkman. It was Steiner's name that was eventually used for the systems, although Steiner himself did not offer any solutions. Such systems were an early example of what are now known as balanced incomplete block designs in the field of combinatorial design. Kirkman's name did become associated with a related topic that originated with "Kirkman's schoolgirl problem," in which he wanted to arrange fifteen girls into sets of three such that each girl was paired with every other girl once.

This describes a Steiner triple system, but Kirkman added the condition that the thirty-five sets of three be expressible as seven partitions of {0, 1, ..., 14}. The triple system would then describe a week-long schedule for 1 daily walks where the fifteen girls were in different sets each day. In the same year his question was published, Kirkman offered a solution, as did Arthur Cayley [5]. Kirkman also asked about orders other than 15 for which an analogous problem could be solved. These designs are a subtype of Steiner triple systems often called Kirkman triple systems, and we will revisit this idea below.

Here, we are going to introduce the general enumeration and construction of the STS systems of order 29.

II. INTERSECTION GRAPH

From an STS(n), different graphs can be formed. For example, if the vertex set is {0, 1, ..., n − 1} and n is an admissible order, then defining an STS(n) is equivalent to decomposing Kn into triangles (copies of K3). But a different sort of graph can be constructed by taking the vertex set as B, that is, every triple in the STS(n) corresponds to a vertex. If two triples have a non-empty intersection then an edge is inserted between their vertices. The resulting graph is known as the block-intersection graph. It is this graph that will be considered as a possible way to distinguish Steiner triple systems of the same order. By construction, the block-intersection graphs are finite and simple.

Given a Steiner triple system, a matrix is built to 4 serve as an adjacency matrix for the BIG. The code cycles through the list of triples, entering a “1” in position ij if triple i shared a common element with triple j and “0” otherwise (for i = j). A function in SAGE to create a graph on such an adjacency matrix is then applied to each matrix corresponding to an STS(13) and STS(15).

We need to generalize the concepts in the occupancy in the process of the obtaining STS(29). We need to complete enumerate the data analysis in this field.

III.PROPERTIES OF GRAPHS

Some of the very basic graph invariants will be identical for BIGs of the same order. The number of vertices is equal to the number of triples, which as mentioned above is n(n−1) 6 . To calculate the degree of a vertex, note that any element of V is in n−1 2 triples, so for an arbitrary element a in a triple, a is in n−1 ,2 − 1 other triples. Now multiply by three for the three elements in each triple, and we obtain the degree of each vertex: 3(n−1 2 − 1) = 3(n−3) 2 . Thus each BIG is a regular graph. Because the BIGs are regular, their total number of edges is straightforward to compute using the Handshaking Lemma: (number of vertices) × (degree) / 2.

3.1. PROPOSITION: The block-intersection graph of an STS(n) for n ≥ 9 is strongly regular with parameters (n(n−1) 6 , 3(n−3) 2 , n+3 2 , 9).

We know the number of vertices in a BIG is n(n−1) 6 and the graph is regular of degree 3(n−3) 2 . Now we determine the parameters e and f. Consider two adjacent vertices in a BIG. They represent triples B1 and B2 with one common element a; suppose B1 = (a, b, c) and B2 = (a, d, e). Then their common neighbors must intersect both B1 and B2, so four of these neighbors will have the form (b, d,),(b, e,),(c, d,), and (c, e,), where denotes any other permissible element of V .

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The other neighbors in common will be the triples also containing a, which appears in a total of n−1 2 triples. Because B1 and B2 both already contain a, the remaining number is n−1 2 − 2 = n−5 2 . Hence the number of common neighbors is n−5 2 + 4 = n+3 2 . Let B1 and B2 now correspond to non-adjacent vertices.

They must be of the form B1 = (a, b, c) and B2 = (d, e, f). A common neighbor of both of these triples must contain an element from each. For instance, we know that the pair {a, d} must occur in exactly one triple, so one of the neighbors in common has the form (a, d,). Similarly, two other common neighbors are (a, e,) and (a, f,).

3.2. Proposition: The block-intersection graph of an STS(n) is distance-regular.

The basic properties of the block-intersection graphs offer a good overview of the highly structured nature of these graphs. Because the invariants we have covered in this section are the same for BIGs corresponding to STS(n) on the same n, we will consider other graph invariants in further chapters. First, though, we discuss how to obtain the Steiner triple systems from which the BIGs are formed.

IV.METHODS OF CONSTRUCTION

4.1. Proposition: If there is an STS(n), then there is an explicit method for producing an STS(2n+1) for any n ≥ 3.

Because we necessarily have n ≡ 1, 3 mod 6, in either case it is true that 2n + 1 ≡ 1, 3 mod 6; hence there is an STS(2n+1). The triples of the larger system are those from the original system, plus triples containing elements from {n, n + 1, ..., 2n − 1, 2n}. For each (a, b, c) of the STS(n), we create (a+n, b+n, c), (a+n, b, c+n), and (a, b+n, c+n). Finally, 10 add the triple (a, a + n, a + 2n) for all a ∈ {0, ..., n − 1}. This process yields three types of triples that total to n(n−1) 6 + 3(n(n−1) 6) + n = (2n)(2n+1) 6 , the right number for an STS(2n+1). Now we verify that each pair of elements of {0, ..., 2n} appears in exactly one triple. For any pair in {0, ..., n − 1}, there is a unique triple of the STS(n) containing it. A pair with both elements in {n, ..., 2n−1} or one element in {0, ..., n−1}, one element in {n, ..., 2n−1} will occur in a triple of the second type. Finally, any pair including the element 2n occurs in the third type of triple.

4.2. Proposition: If n = 2k − 1 for k ≥ 2, then there exists an STS(n) produced by the lexicographically-least, backtrack-free method.

V.SKOLEMIZATION

One of the uses of Skolemization is automated theorem proving. For example, in the method of analytic tableaux, whenever a formula whose leading quantifier is existential occurs, the formula obtained by removing that quantifier via Skolemization may be generated.

This form of Skolemization is an improvement over "classical" Skolemization in that only variables that are free in the formula are placed in the Skolem term. This is an improvement because the semantics of tableau may implicitly place the formula in the scope of some universally quantified variables that are not in the formula itself; these variables are not in the Skolem term, while they would be there according to the original definition of Skolemization. Another improvement that may be used is applying the same Skolem function symbol for formulae that are identical up to variable renaming.[2]

VI.RESULTS & DISCUSSION

Depending on the number of elements in a set, we may use a different method for constructing a Steiner triple system. For number of vertices v >= 7, if v = 3 (mod 6) we use the Bose construction; if v = 1 (mod 6) we use the Skolem construction. We present a different method which works for either case of v. The method is based on a different coloring of the vertices from the traditional coloring. A matrix with particular characteristics is created that in turn is used to create a tree. If this tree is traversed from the root to a leaf, a Steiner triple system of a given order is generated. A matrix for v = 29 is shown below.

$$\begin{pmatrix}
9&0&0&0&0&0&0&0&0&0&0\\
1&8&0&0&0&0&0&0&0&0&0\\
1&0&8&0&0&0&0&0&0&0&0\\
1&1&1&6&0&0&0&0&0&0&0\\
1&1&1&0&6&0&0&0&0&0&0\\
1&1&1&1&0&5&0&0&0&0&0\\
1&1&1&0&1&1&4&0&0&0&0\\
1&1&1&1&1&0&0&4&0&0&0\\
1&1&1&1&1&1&0&0&3&0&0\\
1&1&1&1&1&0&1&1&0&2&0\\
1&1&1&0&1&1&1&1&0&0&2\\
1&1&1&1&1&1&1&1&0&1&0\\
1&1&1&1&1&1&1&1&0&1&0\\
1&1&1&1&1&0&1&1&1&1&0\\
1&1&1&1&0&1&1&1&1&1&0\\
1&1&1&1&1&1&1&0&1&0&1\\
1&1&1&1&1&1&1&0&1&0&1\\
1&1&1&1&1&1&0&1&1&0&1\\
1&1&1&1&1&1&0&1&1&0&1
\end{pmatrix}$$

REFERENCES

[1] R. C. Bose, Strongly regular graphs, partial geometries and partially balanced designs, Paci_c J. Math. 13 (1963), 389{419. MR 28:1137

[2] G. Butler, Fundamental Algorithms for Permutation Groups, Lecture Notes in Computer Science, Vol. 559, Springer-Verlag, Berlin, 1991. MR 94d:68049

[3] C. J. Colbourn, S. S. Magliveras, and D. R. Stinson, Steiner triple systems of order 19 with nontrivial automorphism group, Math. Comp. 59 (1992), 283{295. MR 92k:05022

[4] C. J. Colbourn and A. Rosa, Triple Systems, Clarendon Press, Oxford, 1999. MR

[5] F. N. Cole, L. D. Cummings, and H. S.White, The complete enumeration of triad systems in 15 elements, Proc. Nat. Acad. Sci. U.S.A. 3 (1917)

[6] G. P. Egorychev, The solution of van der Waerden's problem for permanents, Adv. in Math.42 (1981), 299{305. MR 83b:15002b

[7] D. I. Falikman, Proof of the van der Waerden conjecture on the permanent of a doubly stochastic matrix (in Russian), Mat. Zametki29 (1981), 931{938, 957. MR 82k:15007

[8] M. J. Grannell, T. S. Griggs, and E. Mendelsohn, A small basis for four-line con_gurations in Steiner triple systems, J. Combin. Des.3 (1995), 51{59. MR 95j:05041

[9] B. D. Gray and C. Ramsay, On the number of Pasch configurations in a Steiner triple system, Bull. Inst. Combin. Appl. 24 (1998), 105{112. MR 99c:05030

[10] M. Hall, Jr. and J. D. Swift, Determination of Steiner triple systems of order 15, Math. Tables Aids Comput.9 (1955), 146{152. MR 18:192d