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Bayesian analysis of Generalized Exponentiated Moment Exponential Distribution with Applications to Medical Science and Engineering

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ABSTRACT

In this paper, we propose to obtain the Bayesian estimators of unknown shape parameter of a three parameter generalized exponentiated moment exponential (GEME) distribution, based on non-informative (Quasi and Extension of Jeffery's) priors using three different loss functions. Two real life data sets have been used to compare the performance of the estimates under different loss functions. The expression for survival function has also been established under Quasi prior and extension of Jeffrey's prior.

Keywords: Baye's estimation, GEME distribution, Loss functions, Priors and Survival function.

I.INTRODUCTION

The three-parameter generalized exponentiated moment exponential (GEME) distribution will be quite effectively used in analyzing several lifetime data, particularly in place of three-parameter gamma distribution, three parameter Weibull distribution or three-parameter exponentiated exponential distribution. Moment distributions have a vital role in mathematics and statistics, in particular in probability theory, in the perspective research related to ecology, reliability, biomedical field, econometrics, survey sampling and in life-testing. Hasnain [11] developed an exponentiated moment exponential (EME) distribution and discussed some of its important properties. One of such distributions is the two-parameter weighted exponential distribution introduced by Gupta and Kundu [10]. Dara and Ahmad [4] proposed a distribution function of moment exponential distribution and developed some basic properties like moments, skewness, kurtosis, moment generating function and hazard function. Bayes estimators for the weighted exponential distribution (WED) was considered by Farahani and Khorram [8] while S.Dey et al. [15] considered the estimation of the parameters of weighted exponential distribution. Recently, Devendera Kumar [5] obtained the moments and estimation of the exponentiated moment exponential distribution. They obtained Bayes estimation under symmetric and asymmetric loss functions using gamma prior for both shape and scale parameters. They also compared the classical method with Bayesian method through Monte Carlo simulation.

As given in Zafar Iqbal et *al* [12], the cumulative distribution function (cdf) of three parameter generalized exponentiated moment exponential (GEME) distribution is given by

$$F(x)=\left[1-\frac{x^{\gamma}+\beta}{\beta}e^{\frac{-x^{\gamma}}{\beta}}\right]^{\alpha}, \quad x>0. \qquad (1)$$

Where α, β and γ are positive real parameters. The probability density function (pdf) of GEME distribution is defined as

$$f(x) = \frac{\alpha\gamma}{\beta^2}\left[1-\left(1+\frac{x^{\gamma}}{\beta}\right)e^{\frac{-x^{\gamma}}{\beta}}\right]^{\alpha-1} x^{2\gamma-1}e^{\frac{-x^{\gamma}}{\beta}}, \quad x>0, \alpha, \beta, \gamma > 0. \tag{2}$$

Here α and γ are the shape parameters and β is the scale parameter. For $\gamma = 1$, it represents the EME distribution, for $\alpha = \gamma = 1$, it represents the size biased moment exponential distribution and for $\beta = \gamma = 1$, it represents the one parameter exponentiated exponential distribution.

II. SURVIVAL FUNCTION

The branch of statistics that deals with the failure in mechanical systems is called survival analysis. In engineering, it is called reliability analysis or reliability theory. In fact the survival function is the probability of failure by time *y*, where *y* represents survival time. The survival function is given by

$$S(x) = 1-F(x) = 1-\left[1-\left(1+\frac{x^{\gamma}}{\beta}\right)e^{\frac{-x^{\gamma}}{\beta}}\right]^{\alpha}, \quad x>0, \alpha, \beta, \gamma > 0 \tag{3}$$

and the hazard function is

$$h(x) = \frac{\frac{\alpha\gamma}{\beta^2}\left[1-\left(1+\frac{x^{\gamma}}{\beta}\right)e^{\frac{-x^{\gamma}}{\beta}}\right]^{\alpha-1} x^{2\gamma-1}e^{\frac{-x^{\gamma}}{\beta}}}{1-\left[1-\left(1+\frac{x^{\gamma}}{\beta}\right)e^{\frac{-x^{\gamma}}{\beta}}\right]^{\alpha}}; \; 0<x<\infty, \alpha, \beta, \gamma > 0. \tag{4}$$

III. MAXIMUM LIKELIHOOD ESTIMATION OF (GEME) DISTRIBUTION

Theorem 3.1: - Let $\underline{x} = (x_1, x_2, ..., x_n)$ be a random sample of size n having pdf (2); then the maximum likelihood estimator of shape parameter α, when the parameters γ and β are known, is given by

$$\hat{\alpha}_{MLE} = \frac{n}{\sum_{i=1}^{n} \ln\left[1-\left(1+\frac{{x_i}^{\gamma}}{\beta}\right)e^{\frac{-{x_i}^{\gamma}}{\beta}}\right]^{-1}}. \tag{5}$$

Proof: - The likelihood function of the pdf (2) is given by

$$L(\underline{x}\,|\,\alpha)=\frac{\alpha^n\gamma^n}{\beta^{2n}}\prod_{i=1}^{n}\left[1-\left(1+\frac{x_i^{\ \gamma}}{\beta}\right)e^{\frac{-x_i^{\gamma}}{\beta}}\right]^{\alpha-1}x_i^{2\gamma-1}e^{\frac{-x_i^{\gamma}}{\beta}}\ . \qquad (6)$$

The log likelihood function is given by

$$\ln L(\underline{x}\,|\,\alpha)=n\ln\alpha+n\ln\gamma-2n\ln\beta+(2\gamma-1)\sum_{i=1}^{n}\ln x_i-\frac{\sum_{i=1}^{n}x_i^{\ \gamma}}{\beta}+(\alpha-1)\sum_{i=1}^{n}\ln\left[1-\left(1+\frac{x_i^{\ \gamma}}{\beta}\right)e^{\frac{-x_i^{\gamma}}{\beta}}\right].$$

Differentiating log likelihood function with respect to α and equating to zero, we get $\frac{\partial}{\partial\alpha}\ln L(\underline{x}\,|\,\alpha)=\frac{n}{\alpha}+\sum_{i=1}^{n}\ln\left[1-\left(1+\frac{x_i^{\ \gamma}}{\beta}\right)e^{\frac{-x_i^{\gamma}}{\beta}}\right]=0$

$$\Rightarrow\quad \hat{\alpha}_{MLE}=\frac{n}{\sum_{i=1}^{n}\ln\left[1-\left(1+\frac{x_i^{\ \gamma}}{\beta}\right)e^{\frac{-x_i^{\gamma}}{\beta}}\right]^{-1}}. \qquad (7)$$

VI. BAYES ESTIMATOR

In this section, we now derive the Bayes estimator of the shape parameter α in GEMED when the parameters β and γ are assumed to be known. We consider two different priors and three different loss functions.

In our presented study, the loss functions used are defined below:

SELF: Legendre [13] proposed square error loss function and defined it as

$$l(\hat{\alpha},\alpha)=c(\hat{\alpha}-\alpha)^2\,.$$

Al-BLF: The Al-Bayyati's loss function introduced by Al-Bayyati's [1] which is given by

$$l(\hat{\alpha},\alpha)=\alpha^{c_2}(\hat{\alpha}-\alpha)^2\quad ;c_2\in R^+$$

ELF: The entropy loss function established by Dey et al. [7] which is given as

$$L(\delta)=b_1[\delta-\log(\delta)-1]\ ;b_1>0.$$

4.1 Posterior Density Under Quasi Prior

The quasi prior is defined as $g_1(\alpha)\ \propto\ \frac{1}{\alpha^d},\qquad \alpha>0,\quad d>0.$

Combing the quasi prior and the likelihood function (6), then the posterior distribution of parameter α is given by

$$p_{1Q}(\alpha\,|\,\underline{x})=\frac{\beta_1^{\ n-d+1}}{\Gamma(n-d+1)}\alpha^{n-d}e^{-\alpha\beta_1}\ ;\alpha>0, \qquad (8)$$

which is the density kernel of gamma distribution having parameters $\alpha_1 = (n-d+1) and$

$$\beta_1 = \sum_{i=1}^{n} \ln\left[1-\left(1+\frac{x_i^{\gamma}}{\beta}\right)e^{\frac{-x_i^{\gamma}}{\beta}}\right]^{-1}$$

. So the posterior distribution of $(\alpha \mid \underline{x}) \sim G(\alpha_1, \beta_1)$.

4.2 Bayesian Estimation Under Quasi Prior by Using Different Loss Functions

Theorem 4.2.1:- Assuming the loss function $l_{1QS}(\hat{\alpha},\alpha)$, the Bayesian estimator of the shape parameter α, when the parameters β and γ are assumed to be known, is of the form

$$\hat{\alpha}_{1QS} = \frac{(n-d+1)}{\beta_1} \quad ; \beta_1 = \sum_{i=1}^{n} \ln\left[1-\left(1+\frac{x_i^{\gamma}}{\beta}\right)e^{\frac{-x_i^{\gamma}}{\beta}}\right]^{-1}.$$

Proof: - The risk function of the estimator α under the squared error loss function $L_{1QS}(\hat{\alpha},\alpha)$ is given by the formula

$$R(\hat{\alpha},\alpha) = \int_0^{\infty} c(\hat{\alpha}-\alpha)^2 \frac{\beta_1^{n-d+1}}{\Gamma(n-d+1)} \alpha^{n-d} e^{-\alpha\beta_1} \; d\alpha$$

$$R(\hat{\alpha},\alpha) = c\frac{\beta_1^{n-d+1}}{\Gamma(n-d+1)}\left[\hat{\alpha}^2 \int_0^{\infty} \alpha^{n-d+1-1} e^{-\alpha\beta_1} d\alpha + \int_0^{\infty} \alpha^{n-d+3-1} e^{-\alpha\beta_1} d\alpha - 2\hat{\alpha}\int_0^{\infty} \alpha^{n-d+2-1} e^{-\alpha\beta_1} d\alpha\right]$$

$$R(\hat{\alpha},\alpha) = c\left[\hat{\alpha}^2 + \frac{(n-d+2)(n-d+1)}{\beta_1^2} - 2\hat{\alpha}\frac{(n-d+1)}{\beta_1}\right].$$

Minimization of the risk with respect to $\hat{\alpha}$ gives us the optimal estimator as:

$$\hat{\alpha}_{1QS} = \frac{(n-d+1)}{\beta_1} \quad ; \beta_1 = \sum_{i=1}^{n} \ln\left[1-\left(1+\frac{x_i^{\gamma}}{\beta}\right)e^{\frac{-x_i^{\gamma}}{\beta}}\right]^{-1}. \qquad (9)$$

Theorem 4.2.2:- Assuming the loss function $l_{1QE}(\hat{\alpha},\alpha)$, the Bayesian estimator of the shape parameter α, when the parameters β and γ are assumed to be known, is of the form

$$\hat{\alpha}_{1QE} = \frac{(n-d)}{\beta_1} \quad ; \beta_1 = \sum_{i=1}^{n} \ln\left[1-\left(1+\frac{x_i^{\gamma}}{\beta}\right)e^{\frac{-x_i^{\gamma}}{\beta}}\right]^{-1}.$$

Proof: - The risk function of the estimator α under the entropy loss function $l_{1QE}(\hat{\alpha},\alpha)$ is given by the formula

$$R(\hat{\alpha},\alpha)=\int_0^\infty b_1\left(\frac{\hat{\alpha}}{\alpha}-\log\left(\frac{\hat{\alpha}}{\alpha}\right)-1\right)\frac{\beta_1^{\,n-d+1}}{\Gamma(n-d+1)}\alpha^{n-d}e^{-\alpha\beta_1}\,d\alpha.$$

$$R(\hat{\alpha},\alpha)=b_1\left[\frac{\hat{\alpha}\beta_1}{(n-d)}-\log(\hat{\alpha})+\frac{\Gamma'(n-d+1)}{\Gamma(n-d+1)}-1\right].$$

Minimization of the risk with respect to $\hat{\alpha}$ gives us the optimal estimator

$$\hat{\alpha}_{1QE}=\frac{(n-d)}{\beta_1}\quad;\beta_1=\sum_{i=1}^{n}\ln\left[1-\left(1+\frac{x_i^{\,\gamma}}{\beta}\right)e^{\frac{-x_i^{\,\gamma}}{\beta}}\right]^{-1}. \tag{10}$$

Theorem 4.2.3:- Assuming the loss function $l_{1QA}(\hat{\alpha},\alpha)$, the Bayesian estimator of the shape parameter α, when the parameters β and γ are assumed to be known, is of the form

$$\hat{\alpha}_{1QA}=\frac{(c_2+n-d+1)}{\beta_1}\quad;\beta_1=\sum_{i=1}^{n}\ln\left[1-\left(1+\frac{x_i^{\,\gamma}}{\beta}\right)e^{\frac{-x_i^{\,\gamma}}{\beta}}\right]^{-1}.$$

Proof: - The risk function of the estimator α under the Al-Bayyati loss function $l_{1QA}(\hat{\alpha},\alpha)$ is given by the formula

$$R(\hat{\alpha},\alpha)=\int_0^\infty\alpha^{c_2}(\hat{\alpha}-\alpha)^2\frac{\beta_1^{\,n-d+1}}{\Gamma(n-d+1)}\alpha^{n-d}e^{-\alpha\beta_1}\quad d\alpha$$

$$R(\hat{\alpha},\alpha)=\frac{1}{\Gamma(n-d+1)}\left[\hat{\alpha}^2\frac{\Gamma(c_2+n-d+1)}{\beta_1^{\,c_2}}+\frac{\Gamma(c_2+n-d+3)}{\beta_1^{\,c_2+2}}-2\hat{\alpha}\frac{\Gamma(c_2+n-d+2)}{\beta_1^{\,c_2+1}}\right].$$

Minimization of the risk with respect to $\hat{\alpha}$ gives us the optimal estimator as:

$$\hat{\alpha}_{1QA}=\frac{(c_2+n-d+1)}{\beta_1}\quad;\beta_1=\sum_{i=1}^{n}\ln\left[1-\left(1+\frac{x_i^{\,\gamma}}{\beta}\right)e^{\frac{-x_i^{\,\gamma}}{\beta}}\right]^{-1} \tag{11}$$

Remark 1.1 Replacing $c_2=0$, in (11), we get the Baye's estimator under square error loss function with quasi prior which is same as (9).

4.3 Posterior density under Extension of Jeffery's prior

The extension of Jeffery's prior is defined as $g_2(\alpha)\propto\frac{1}{\alpha^{2c_1}},c_1\in R^+$.

Combing the extension of Jeffery's and the likelihood function (6), then the posterior distribution of parameter α is given by

$$p_{2E}(\alpha/\underline{x}) = \frac{\beta_1^{n-2c_1+1}}{\Gamma(n-2c_1+1)}\alpha^{n-2c_1}e^{-\alpha\beta_1};\alpha>0, \quad (12)$$

which is the density kernel of gamma distribution having parameters

$\alpha_1 = (n-2c_1+1)$ and $\beta_1 = \sum_{i=1}^{n}\ln\left[1-\left(1+\frac{x_i^{\gamma}}{\beta}\right)e^{\frac{-x_i^{\gamma}}{\beta}}\right]^{-1}$. So the posterior distribution of

$(\alpha \mid \underline{x}) \sim G(\alpha_2, \beta_1)$.

4.4. Bayesian Estimation under Extension of Jeffery's Prior Using Different Loss Functions

Theorem 4.4.1:- Assuming the loss function $l_{2EJS}(\hat{\alpha},\alpha)$, the Bayesian estimator of the shape parameter α, when the parameters β and γ are assumed to be known, is of the form

$$\hat{\alpha}_{2EJS} = \frac{(n-2c_1+1)}{\beta_1} \quad ;\beta_1 = \sum_{i=1}^{n}\ln\left[1-\left(1+\frac{x_i^{\gamma}}{\beta}\right)e^{\frac{-x_i^{\gamma}}{\beta}}\right]^{-1}.$$

Proof: - The risk function of the estimator α under the squared error loss function $l_{2EJS}(\hat{\alpha},\alpha)$ is given by the formula

$$R(\hat{\alpha},\alpha) = \int_0^{\infty} c(\hat{\alpha}-\alpha)^2 \frac{\beta_1^{n-2c_1+1}}{\Gamma(n-2c_1+1)}\alpha^{n-2c_1}e^{-\alpha\beta_1}d\alpha.$$

$$R(\hat{\alpha},\alpha) = c\left[\hat{\alpha}^2 + \frac{(n-2c_1+2)(n-2c_1+1)}{\beta_1^2} - 2\hat{\alpha}\frac{(n-2c_1+1)}{\beta_1}\right].$$

Minimization of the risk with respect to $\hat{\alpha}$ gives us the optimal estimator

$$\hat{\alpha}_{2EJS} = \frac{(n-2c_1+1)}{\beta_1} \quad ;\beta_1 = \sum_{i=1}^{n}\ln\left[1-\left(1+\frac{x_i^{\gamma}}{\beta}\right)e^{\frac{-x_i^{\gamma}}{\beta}}\right]^{-1}. \quad (13)$$

Theorem 4.4.2:- Assuming the loss function $l_{2EJE}(\hat{\alpha},\alpha)$, the Bayesian estimator of the shape parameter α, when the parameters β and γ are assumed to be known, is of the form

$$\hat{\alpha}_{2EJE} = \frac{(n-2c_1)}{\beta_1} \quad ;\beta_1 = \sum_{i=1}^{n}\ln\left[1-\left(1+\frac{x_i^{\gamma}}{\beta}\right)e^{\frac{-x_i^{\gamma}}{\beta}}\right]^{-1}.$$

Proof: - The risk function of the estimator α under the entropy loss function $l_{2EJE}(\hat{\alpha},\alpha)$ is given by the formula

$$R(\hat{\alpha},\alpha)=\int_0^{\infty} b_1\left(\frac{\hat{\alpha}}{\alpha}-\log\left(\frac{\hat{\alpha}}{\alpha}\right)-1\right)\frac{\beta_1^{n-2c_1+1}}{\Gamma(n-2c_1+1)}\alpha^{n-2c_1}e^{-\alpha\beta_1}\,d\alpha.$$

$$R(\hat{\alpha},\alpha)=b_1\left[\frac{\hat{\alpha}\beta_1}{(n-2c_1)}-\log(\hat{\alpha})+\frac{\Gamma'(n-2c_1+1)}{\Gamma(n-2c_1+1)}-1\right].$$

Minimization of the risk with respect to $\hat{\alpha}$ gives us the optimal estimator

$$\hat{\alpha}_{2EJE}=\frac{(n-2c_1)}{\beta_1}\quad;\beta_1=\sum_{i=1}^{n}\ln\left[1-\left(1+\frac{x_i^{\gamma}}{\beta}\right)e^{\frac{-x_i^{\gamma}}{\beta}}\right]^{-1}\tag{14}$$

Theorem 4.4.3:- Assuming the loss function $l_{2EJA}(\hat{\alpha},\alpha)$, the Bayesian estimator of the shape parameter α, when the parameters β and γ are assumed to be known, is of the form

$$\hat{\alpha}_{2EJA}=\frac{(c_2+n-2c_1+1)}{\beta_1}\quad;\beta_1=\sum_{i=1}^{n}\ln\left[1-\left(1+\frac{x_i^{\gamma}}{\beta}\right)e^{\frac{-x_i^{\gamma}}{\beta}}\right]^{-1}.$$

Proof: - The risk function of the estimator α under the entropy loss function $l_{2EJA}(\hat{\alpha},\alpha)$ is given by the formula

$$R(\hat{\alpha},\alpha)=\int_0^{\infty}\alpha^{c_2}(\hat{\alpha}-\alpha)^2\frac{\beta_1^{n-2c_1+1}}{\Gamma(n-2c_1+1)}\alpha^{n-2c_1}e^{-\alpha\beta_1}\,d\alpha.$$

$$R(\hat{\alpha},\alpha)=\frac{1}{\Gamma(n-2c_1+1)}\left[\hat{\alpha}^2\frac{\Gamma(c_2+n-2c_1+1)}{\beta_1^{c_2}}+\frac{\Gamma(c_2+n-2c_1+3)}{\beta_1^{c_2+2}}-2\hat{\alpha}\frac{\Gamma(c_2+n-2c_1+2)}{\beta_1^{c_2+1}}\right].$$ Minimization of the risk with respect to $\hat{\alpha}$ gives us the optimal estimator

$$\hat{\alpha}_{2EJA}=\frac{(c_2+n-2c_1+1)}{\beta_1}\quad;\beta_1=\sum_{i=1}^{n}\ln\left[1-\left(1+\frac{x_i^{\gamma}}{\beta}\right)e^{\frac{-x_i^{\gamma}}{\beta}}\right]^{-1}.\tag{15}$$

V. ESTIMATOR OF SURVIVAL FUNCTION

5.1 Estimator Under Quasi Prior of Survival Function

By using posterior distribution function, we can find the survival function such that

$$\hat{S}_{1Q}(x)=\int_0^\infty\left\{1-\left(1-\left(1+\frac{x^\gamma}{\beta}\right)e^{\frac{-x^\gamma}{\beta}}\right)^\alpha\right\}\frac{\beta_1^{\,n-d+1}}{\Gamma(n-d+1)}\alpha^{n-d}e^{-\alpha\beta_1}d\alpha$$

$$\hat{S}_{1Q}(x)=\int_0^\infty\frac{\beta_1^{\,n-d+1}}{\Gamma(n-d+1)}\alpha^{n-d}e^{-\alpha\beta_1}d\alpha-\int_0^\infty\left(1-\left(1+\frac{x^\gamma}{\beta}\right)e^{\frac{-x^\gamma}{\beta}}\right)^\alpha\frac{\beta_1^{\,n-d+1}}{\Gamma(n-d+1)}\alpha^{n-d}e^{-\alpha\beta_1}d\alpha$$

$$\hat{S}_{1Q}(x)=1-\left(\frac{\beta_1}{\beta_1-\beta_2}\right)^{n-d+1}.$$

(16) where $\beta_1=\sum_{i=1}^{n}\ln\left[1-\left(1+\frac{x_i^{\gamma}}{\beta}\right)e^{\frac{-x_i^{\gamma}}{\beta}}\right]^{-1}$ and $\beta_2=\ln\left[1-\left(1+\frac{x^\gamma}{\beta}\right)e^{\frac{-x^\gamma}{\beta}}\right].$

5.2 Estimator under Extension Prior Of Survival Function

By using posterior distribution function, we can find the survival function such that

$$\hat{S}_{2E}(x)=\int_0^\infty\left\{1-\left(1-\left(1+\frac{x^\gamma}{\beta}\right)e^{\frac{-x^\gamma}{\beta}}\right)^\alpha\right\}\frac{\beta_1^{\,n-2c_1+1}}{\Gamma(n-2c_1+1)}\alpha^{n-2c_1}e^{-\alpha\beta_1}d\alpha$$

$$\hat{S}_{2E}(x)=\int_0^\infty\frac{\beta_1^{\,n-2c_1+1}}{\Gamma(n-2c_1+1)}\alpha^{n-2c_1}e^{-\alpha\beta_1}d\alpha-\int_0^\infty\left(1-\left(1+\frac{x^\gamma}{\beta}\right)e^{\frac{-x^\gamma}{\beta}}\right)^\alpha\frac{\beta_1^{\,n-2c_1+1}}{\Gamma(n-2c_1+1)}\alpha^{n-2c_1}e^{-\alpha\beta_1}d\alpha$$

$$\hat{S}_{2E}(x)=1-\left(\frac{\beta_1}{\beta_1-\beta_2}\right)^{n-2c_1+1}.$$

(17) where $\beta_1=\sum_{i=1}^{n}\ln\left[1-\left(1+\frac{x_i^{\gamma}}{\beta}\right)e^{\frac{-x_i^{\gamma}}{\beta}}\right]^{-1}$ and $\beta_2=\ln\left[1-\left(1+\frac{x^\gamma}{\beta}\right)e^{\frac{-x^\gamma}{\beta}}\right].$

VI. APPLICATIONS

To compare the performance of the estimates under different loss functions for the generalized exponentiated moment exponential distribution, two real data sets are used and analysis performed with the help of R software.

Data set I: The first data set is given by Gross and Clark [9] which represents the lifetime's data relating to relief times (in minutes) of 20 patients receiving an analgesic. The data are as follows: 1.1, 1.4, 1.3, 1.7, 1.9, 1.8, 1.6, 2.2, 1.7, 2.7, 4.1, 1.8, 1.5, 1.2, 1.4, 3.0, 1.7, 2.3, 1.6, and 2.0

Table 1: Bayes Risk of α under Quasi prior for Data set I

β	γ	d	SELF		ELF		ABLF	
			$c=0.5$	$c=1.0$	$b_1=0.2$	$b_1=0.4$	$c_2=0.5$	$c_2=-0.5$
1.0	1.5	0.3	0.12806	0.25611	0.44425	0.88851	0.39562	0.16777
		1.3	0.12187	0.24374	0.44452	0.88904	0.36763	0.16361
1.5	2.0	0.3	0.07248	0.14496	0.50117	1.00234	0.19423	0.10948
		1.3	0.06898	0.13796	0.50143	1.00287	0.18049	0.10677
2.0	2.5	0.3	0.05948	0.11897	0.52092	1.04185	0.15172	0.09440
		1.3	0.05661	0.11323	0.52119	1.04239	0.14099	0.09206
2.5	3.0	0.3	0.05711	0.11422	0.52501	1.05001	0.14418	0.09155
		1.3	0.05435	0.10871	0.52527	1.05054	0.13398	0.08929

SELF: Squared error loss function, ELF: Entropy loss function and ABLF: Al-Bayyati's loss function.

Table 2: Bayes Risk of α under Extension of Jeffery's prior for Data set I

β	γ	c_1	SELF		ELF		ABLF	
			$c=0.5$	$c=1.0$	$b_1=0.2$	$b_1=0.4$	$c_2=0.5$	$c_2=-0.5$
1.0	1.5	0.4	0.12496	0.24993	0.44438	0.88877	0.38154	0.16571
		1.4	0.11259	0.22518	0.44498	0.88996	0.32695	0.15718
1.5	2.0	0.4	0.07073	0.14146	0.50130	1.00260	0.18732	0.10813
		1.4	0.06373	0.12745	0.50189	1.00379	0.16051	0.10257
2.0	2.5	0.4	0.05805	0.11610	0.52105	1.04211	0.146326	0.09324
		1.4	0.05230	0.10461	0.52165	1.04331	0.12538	0.08844
2.5	3.0	0.4	0.055732	0.11146	0.52513	1.05027	0.13905	0.09043
		1.4	0.05021	0.10043	0.52573	1.05146	0.11916	0.08578

SELF: Squared error loss function, ELF: Entropy loss function and ABLF: Al-Bayyati's loss function.

From Table 1 and 2 shows that squared error loss function provides the minimum posterior risk as compared to the other loss functions particularly as C is (0.5) and the prior Extension of Jeffery's prior provides the less posterior risk than Quasi prior. Moreover, when we increase the true value of parameters $(\beta,\gamma)=c((1.0,1.5),(1.5,2.0)\,\text{and}\,(2.5,3.0))$ and increase the value of $d=c(0.3,1.3)$ and $c_1=c(0.4,1.4)$, the Bayes risk of $\hat{\alpha}$ decreases quite significantly.

Data set II: The second data set studied by Meeker and Escobar [14], which gives the times of failure and running times for a sample of devices from a eld-tracking study of a larger system. At a certain point in time, 30 units were installed in normal service conditions. Two causes of failure were observed for each unit that failed:

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the failure caused by an accumulation of randomly occurring damage from power-line voltage spikes during electric storms and failure caused by normal product wear. The times are:

2.75, 0.13, 1.47, 0.23, 1.81, 0.30, 0.65, 0.10, 3.00, 1.73, 1.06, 3.00, 3.00, 2.12, 3.00, 3.00, 3.00, 0.02, 2.61, 2.93,0.88, 2.47, 0.28, 1.43, 3.00, 0.23, 3.00, 0.80, 2.45, 2.66.

Table 3: Bayes Risk of α under Quasi prior for Data set II

β	γ	d	SELF		ELF		ABLF	
			$c=0.5$	$c=1.0$	$b_1=0.2$	$b_1=0.4$	$c_2=0.5$	$c_2=-0.5$
1.0	1.5	0.3	0.00484	0.00969	0.80938	1.61877	0.00724	0.01307
		1.3	0.00468	0.00937	0.80950	1.61900	0.00689	0.01285
1.5	2.0	0.3	0.00245	0.00491	0.87729	1.75459	0.00310	0.00785
		1.3	0.00237	0.00475	0.87741	1.75483	0.00295	0.00772
2.0	2.5	0.3	0.00155	0.00311	0.92277	1.84554	0.00175	0.00558
		1.3	0.00151	0.00301	0.92289	1.84578	0.00167	0.00549
2.5	3.0	0.3	0.00110	0.00220	0.95754	1.91509	0.00113	0.00430
		1.3	0.00106	0.00213	0.95766	1.91532	0.00108	0.00423

SELF: squared error loss function, ELF: Entropy loss function and ABLF: Al-Bayyati's loss function.

Table 4: Bayes Risk of α under Extension of Jeffery's prior Data set II

β	γ	c_1	SELF		ELF		ABLF	
			$c=0.5$	$c=1.0$	$b_1=0.2$	$b_1=0.4$	$c_2=0.5$	$c_2=-0.5$
1.0	1.5	0.4	0.00476	0.00953	0.80944	1.61888	0.00707	0.01296
		1.4	0.00445	0.00890	0.80969	1.61938	0.00638	0.01252
1.5	2.0	0.4	0.00241	0.00483	0.87735	1.75471	0.00302	0.00779
		1.4	0.00225	0.00451	0.87760	1.75521	0.00273	0.00752
2.0	2.5	0.4	0.00153	0.00306	0.92283	1.84566	0.00171	0.00553
		1.4	0.00143	0.00286	0.92308	1.84616	0.00154	0.00535
2.5	3.0	0.4	0.00108	0.00216	0.95760	1.91521	0.00111	0.00426
		1.4	0.00100	0.00202	0.95785	1.91570	0.00101	0.00412

SELF: squared error loss function, ELF: Entropy loss function and ABLF: Al-Bayyati's loss function.

From Table 3 and 4 shows that squared error loss function provides the minimum posterior risk as compared to the other loss functions particularly as C is (0.5) and the prior Extension of Jeffery's prior provides the less posterior risk than Quasi prior. Moreover, when we increase the true value of parameters $(\beta,\gamma)=c((1.0,1.5),(1.5,2.0)\text{and}(2.5,3.0))$ and increase the value of $d=c(0.3,1.3)$ and $c_1=c(0.4,1.4)$, the Bayes risk of $\hat{\alpha}$ decreases quite significantly.

VII. CONCLUSION

On comparing the Bayes posterior risk of different loss functions, it is observed that SELF has less Bayes posterior risk than other loss functions in both priors. According to the decision rule of less Bayes posterior risk we conclude that SELF is more preferable loss function for different values of parameters.

It is clear from Tables 1& 4 the comparison of Bayes posterior risk under different loss functions using quasi as well as Extension of Jeffery's priors has been made through which we conclude that within each loss function Extension of Jeffery's prior provides less Bayes posterior risk than Quasi prior so it is more suitable for the generalized exponentiated moment exponential distribution. Moreover, when we increase the true value of parameters $(\beta,\gamma)=c((1.0,1.5),(1.5,2.0)\text{and}(2.5,3.0))$ and increase the value of $d=c(0.3,1.3)$ and $c_1=c(0.4,1.4)$, the Bayes risk of $\hat{\alpha}$ decreases quite significantly.

REFERENCES

[1] Al-Bayyti , H.N. Comparing methods of estimating Weibull failure models using simulation, Ph.D. Thesis, College of Administration and Economics, Baghdad University, Iraq, 2002.

[2] Al-Kutubi, H. S., On comparison estimation procedures for parameter and survival function, Iraqi Journal of Statistical Science, 9, 2005, 1-14.

[3] Calabria, R. and Pulcini, G., An engineering approach to Bayes estimation for the Weibull distribution, Microelectronics Reliability, 34, (5), 1994,789-802.

[4] Dara, S.T. and Ahmad, M., Recent Advances in Moment Distributions and their Hazard Rate. Ph.D. Thesis, National College of Business Administration and Economics Lahore, Pakistan, 2012.

[5] Devendera Kumar, Moments and Estimation of the Exponentiated Moment Exponential Distribution, Mathematical Sciences and Applications E-Notes, 4(1), 2016, 94-112.

[6] Dey, D.K. and Liu, Pie-San L., On Comparison of Estimators in a Generalized Life Model, Micro-electron. Reliab., 45(3), 1992, 207-221.

[7] Dey, D.K., Ghosh, M. and Srinivasan, C., Simultaneous Estimation of Parameters under Entropy loss, J. Statist.Plan. and Infer.1987,347-363 .

[8] Farahani, Z.S.M., Khorram, E., Bayesian statistical inference for weighted exponential distribution, Communication in Statistics Simulation and Computation **43**, 2014, 1362-1384.

[9] Gross, A.J. and Clark, V.A., Survival Distributions, Reliability Applications in the Biometrical Sciences, John Wiley, New York, 1975.

[10] Gupta, R.D., Kundu, D., A new class of weighted exponential distribution, Statistics, **43**, 2009, 621-634.

[11] Hasnain, S.A., Exponentiated Moment Exponential Distribution, Ph.D. Thesis, National College of Business Administration and Economics, Lahore, Pakistan, 2013.

[12] Iqbal, Zafar, Hasnain, Syed Anwer, Salman, Muhammad, Ahmad, Munir, Hamedani, G. G, Generalized Exponentiated Moment Exponential Distribution, Pak. J. Statist. 30(4), 2014, 537-554.

[13] Legendre, A., Nouelles Methods Pour La Determination Des Orbites Dis Cometes, Coucier, Pairs, 1805.

[14] Meeker W. Q., Escobar L. A., Statistical Methods for Reliability Data, John Wiley, New York, 1998.

[15] Sanku Dey, Sajid Ali and Chanseaok Park, Weighted exponential distribution properties and different methods of estimation, Journal of Statistical Computation and Simulation, 2015.