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BAYESIAN ANALYSIS FOR GENERALIZED RAYEIGH DISTRIBUTION: LINDLEY'S APPROXIMATION

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ABSTRACT

The non-central chi-square distribution regarded as generalized Rayleigh distribution can be used in mathematical physics, communication etc. In this paper the estimates of the parameters of an extension of Rayleigh distribution have been obtained by using the maximum likelihood estimation and Lindley's approximation technique using informative and non-informative priors under different loss functions.

Keywords: Bayes estimation, Loss function, Lindley's approximation, Newton Raphson method.

I.INTRODUCTION

The Rayleigh distribution has many real life applications in testing lifetime of an object. This distribution has got valuable attention in the field of reliability theory and survival analysis, probability theory and operations research, in different fields of physics such as wave heights, in lifetime of hours of tubes, resistors, networks, crystals, transformers, relays and capacitors in aircraft radar sets, and to study the wind speeds over a year at wind turbine sites. In recent years, new classes of models have been proposed based on modifications of the existing model, see Bilal et al. [1]. Surles and Padgett [2] introduced two-parameter Burr Type X distribution and named it as the generalized Rayleigh distribution. Kundu and Raqab [3] and Raqab and Kundu [4] have discussed the different methods of estimation of the parameters and other properties of Generalized Rayleigh distribution. Recently, Shankar et al. [5], Aslam et al. [6] studied its properties in Bayesian framework.

Bilal et al. introduced a new model, an extension of Rayleigh distribution whose pdf is given as

$$f(x)=\frac{\alpha\,\beta}{\theta}\left(\frac{\alpha x-\lambda}{\theta}\right)^{\beta-1}\exp\left[-\left\{\frac{\alpha x-\lambda}{\theta}\right\}^{\beta}\right]\ ;\ \frac{\lambda}{\alpha}<x<\infty\ ;\beta,\theta>0\ ;\alpha\neq 0$$

(1)

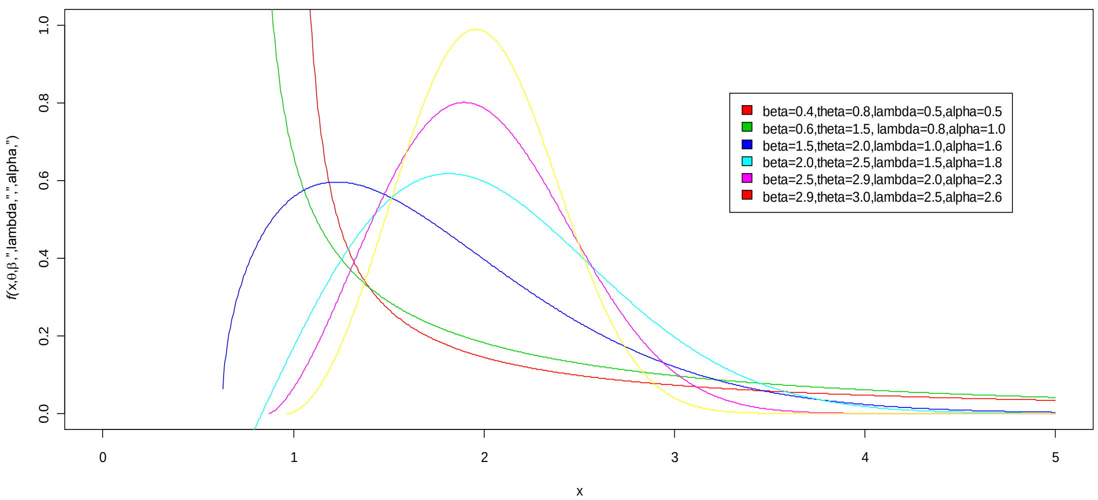

Fig 1.1: Pdf of extension of Rayleigh distribution under different values to parameters

The corresponding cumulative distribution function is given as

$$F(x) = 1 - \exp\left[-\left\{\frac{\alpha x - \lambda}{\theta}\right\}^{\beta}\right]$$

The survival function and hazard function is as

$$R(x) = \exp\left[-\left\{\frac{\alpha x - \lambda}{\theta}\right\}^{\beta}\right] \quad \& \quad h(x) = \frac{\alpha\,\beta}{\theta}\left\{\frac{\alpha x - \lambda}{\theta}\right\}^{\beta-1}$$

The likelihood function of (1) is given by

$$L(x\,|\,\theta) = \left(\frac{\alpha\,\beta}{\theta}\right)^{n} \exp\left[-\sum_{i=1}^{n}\left\{\frac{\alpha x_i - \lambda}{\theta}\right\}^{\beta}\right]\prod_{i=1}^{n}\left(\frac{\alpha x_i - \lambda}{\theta}\right)^{\beta-1}$$

$$\therefore \qquad \hat{\theta} = \left[\frac{\sum_{i=1}^{n}(\alpha x_i - \lambda)^{\beta}}{n}\right]^{1/\beta}$$

(2)

Assuming α & λ to be known.We propose to solve $\hat{\beta}$ by using the Newton-Raphson method. Therefore $\hat{\beta}$ is obtained by choosing initial value for β_i and iterating the process till it converges as.

$$\beta_{i+1} = \beta_i + \frac{\dfrac{n}{\beta} - \dfrac{n\sum_{i=1}^{n} (\alpha x_i - \lambda)^{\beta} \ln(\alpha x_i - \lambda)}{\sum_{i=1}^{n} (\alpha x_i - \lambda)^{\beta}} + \sum_{i=1}^{n} \ln(\alpha x_i - \lambda)}{\dfrac{n}{\beta^2} + \dfrac{n\sum_{i=1}^{n} (\alpha x_i - \lambda)^{\beta} \ln^2\left(\dfrac{(\alpha x_i - \lambda) n^{1/\beta}}{\sum_{i=1}^{n} (\alpha x_i - \lambda)^{\beta}}\right)}{\sum_{i=1}^{n} (\alpha x_i - \lambda)^{\beta}}}$$

(3)

When $\hat{\beta}$ is obtained then $\hat{\theta}$ can be determined.

II.LINDLEY'S APPROXIMATION UNDER UNIFORM PRIOR AND GAMMA PRIOR:

It may be noted that posterior distribution of (θ, β) takes a ratio that involves integration in the denominator and cannot be reduced to closed form. Hence the evaluation of the posterior expectation for obtaining the Bayes estimators of θ and β will be tedious. Thus, we propose the use of Lindley's approximation method [7] for obtaining Bayes estimates. Lindley developed an asymptotic approximation to the ratio

$$I = \frac{\int_{\Omega} w(\theta) e^{L(\theta)} d\theta}{\int_{\Omega} v(\theta)\, e^{L(\theta)} d\theta}$$

(4)

Where $\theta = (\theta_1, ..., \theta_m), L(\theta)$ is the logarithmic of likelihood function, $w(\theta)$ & $v(\theta)$ are arbitrary functions of θ & Ω represents the space range of θ. If $w(\theta) = u(\theta) v(\theta)$ is the prior distribution of $v(\theta)$ then $I = E\{u(\theta) \mid x\}$ can be evaluated as

$$I = u(\lambda, \tau) + \frac{1}{2}(u_{11}\sigma_{11} + u_{22}\sigma_{22}) + \rho_1 u_1 \sigma_{11} + \rho_2 u_2 \sigma_{22} + \frac{1}{2}(L_{30} u_1 \sigma_{11}^2 + L_{03} u_2 \sigma_{22}^2 + (L_{12} u_1 L_{21} u_2 +)\sigma_{11}\sigma_{22})$$

(5)

Where

$$u_1 = \frac{\partial u(\theta, \beta)}{\partial \theta}, u_{11} = \frac{\partial^2 u(\theta, \beta)}{\partial \theta^2}, u_2 = \frac{\partial u(\theta, \beta)}{\partial \beta}, u_{22} = \frac{\partial^2 u(\theta, \beta)}{\partial \beta^2}, \rho = \ln g(\theta, \beta)$$

$$\rho_1 = \frac{\partial \rho}{\partial \theta}, \rho_2 = \frac{\partial \rho}{\partial \beta}, L_{20} = \frac{\partial^2 \ln L}{\partial \theta^2}, L_{02} = \frac{\partial^2 \ln L}{\partial \beta^2}, L_{21} = \frac{\partial^3 \ln L}{\partial \beta\, \partial \theta^2}, L_{12} = \frac{\partial^3 \ln L}{\partial \theta\, \partial \beta^2},$$

$$L_{30}=\frac{\partial^3 \ln L}{\partial\theta^3}, L_{03}=\frac{\partial^3 \ln L}{\partial\beta^3},\ \sigma_{11}=(-L_{20})^{-1}, \sigma_{22}=(-L_{02})^{-1}$$

$$L_{20}=-\frac{n}{\beta^2}-\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\theta}\right)^{\beta}\ln^2\left(\frac{\alpha x_i-\lambda}{\theta}\right), L_{02}=\frac{n\beta}{\theta^2}-\frac{\beta(\beta+1)}{\theta^2}\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\theta}\right)^{\beta},$$

$$L_{30}=-\frac{2n}{\beta^3}-\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\theta}\right)^{\beta}\ln^3\left(\frac{\alpha x_i-\lambda}{\theta}\right);\ L_{03}=-\frac{2n\beta}{\theta^3}+\frac{\beta(\beta+1)(\beta+2)}{\theta^3}\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\theta}\right)^{\beta}$$

$$L_{21}=\frac{\beta}{\theta}\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\theta}\right)^{\beta}\ln^2\left(\frac{\alpha x_i-\lambda}{\theta}\right)+\frac{2}{\theta}\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\theta}\right)^{\beta}\ln\left(\frac{\alpha x_i-\lambda}{\theta}\right)$$

$$L_{12}=\frac{n}{\theta^2}-\frac{\beta(\beta+1)}{\theta^2}\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\theta}\right)^{\beta}\ln\left(\frac{\alpha x_i-\lambda}{\theta}\right)$$

$$\sigma_{11}=\left[\frac{n}{\beta^2}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\theta}\right)^{\beta}\ln^2\left(\frac{\alpha x_i-\lambda}{\theta}\right)\right]^{-1}, \sigma_{22}=\left[-\left(\frac{n\beta}{\theta^2}-\frac{\beta(\beta+1)}{\theta^2}\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\theta}\right)^{\beta}\right)\right]^{-1}$$

III.BAYESIAN ESTIMATION FOR $\hat{\theta}_{LLF}$, $\hat{\beta}_{LLF}$ UNDER THE LINEX LOSS FUNCTION USING LINDLEY'S APPROXIMATION:

Bayesian approach synthesis two sources of information about the unknown parameters of interest. The first of these is the simple data, expressed formally by likelihood function. The second is the prior distribution, which represents additional information that is available to investigator. When no information about the parameter is available one can make use of the non-informative prior like uniform prior defined as $g(\theta,\beta)\propto 1$ (See Sultan et al. [8]). The linex loss function is an asymmetric loss function, introduced by Klebanov [9] and used by Varian [10] in the context of real estate assessment. The linex loss function is defined as $L(\hat{\theta},\theta)=\exp(c(\hat{\theta}-\theta))-c(\hat{\theta}-\theta)-1$where $\hat{\theta}$ is the estimate of θ .the constant c determines the shape of the loss function. Under the linex loss function the Baye's estimate $\hat{\theta}_{LLF}$ of θ is given by

$$\hat{\theta}_{LLF}=\frac{-1}{c}\ln E_{\theta}(e^{-c\theta})$$

where E_{θ} is posterior expectation. Thus Baye's estimator $\hat{\theta}_{LLF}$ of a function $u(\theta,\beta)=[\exp(-c\theta)]$ under uniform prior using the Lindley's approximation can be obtained as

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$$u(\theta,\beta)=e^{-c\theta},\ u_1=-ce^{-c\theta}, u_{11}=c^2e^{-c\theta}, u_2=u_{22}=0, \rho_1=\rho_2=0$$

$$\therefore\ \hat{\theta}_{LLF}=e^{-c\hat{\theta}}+\frac{ce^{-c\hat{\theta}}}{2\left[\frac{n}{\hat{\beta}^2}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)\right]}\left[c+\frac{\frac{2n}{\hat{\beta}^3}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^3\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{\frac{n}{\hat{\beta}^2}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}+\frac{n-\hat{\beta}(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{n\beta-\hat{\beta}(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}}\right]$$

(6)

Following the same procedure as for the Baye's estimation of $\hat{\theta}_{LLF}$.

The Baye's estimate for $\hat{\beta}_{LLF}$ can be obtained as. Here

$$u(\theta,\beta)=e^{-c\theta},\ u_1=-ce^{-c\theta}, u_{22}=c^2e^{-c\theta}, u_1=u_{11}=0$$

$$\therefore\ \hat{\beta}_{LLF}=e^{-c\hat{\beta}}+\frac{ce^{-c\hat{\beta}}\hat{\theta}^2}{2\hat{\beta}\left[n-(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\right]}\left[c-\frac{2n+(\hat{\beta}+1)(\hat{\beta}+2)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}}{\hat{\theta}\left[n-(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\right]}-\frac{\hat{\beta}\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)+2\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{\hat{\theta}\left[\frac{n}{\hat{\beta}^2}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)\right]}\right]$$

(7)

Under gamma prior $g(\theta,\beta)\propto\theta^{s-1}\beta^{p-1}e^{-(r\theta+q\beta)}$; $p,q,r,s>0$ are hyper parameters.

$$\rho=(s-1)\ln\theta+(p-1)\ln\beta-r\theta-q\beta\ ;\ \rho_1=\frac{s-1}{\theta}-r;\ \rho_2=\frac{p-1}{\beta}-q$$

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$$\therefore\ \hat{\theta}_{LLF}=e^{-c\hat{\theta}}+\frac{ce^{-c\hat{\theta}}}{\left[\frac{n}{\hat{\beta}^2}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)\right]}\left[\frac{c}{2}-\frac{s-1}{\hat{\theta}}+r+\frac{1}{2}\left\{\frac{\frac{2n}{\hat{\beta}^3}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^3\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{\frac{n}{\hat{\beta}^2}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}-\frac{n-\hat{\beta}(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{n\beta-\hat{\beta}(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}}\right\}\right]$$

(8)

and

$$\therefore\ \hat{\beta}_{LLF}=e^{-c\hat{\beta}}-\frac{ce^{-c\hat{\beta}}\hat{\theta}^2}{\hat{\beta}\left[n-(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\right]}\left[\frac{c}{2}-\frac{p-1}{\hat{\beta}}+q-\frac{1}{2}\left\{\frac{2n-(\hat{\beta}+1)(\hat{\beta}+2)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}}{\hat{\theta}\left[n-(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\right]}+\frac{\hat{\beta}\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)+2\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{\hat{\theta}\left[\frac{n}{\hat{\beta}^2}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)\right]}\right\}\right]$$

(9)

IV.BAYESIAN ESTIMATION FOR $\hat{\theta}_{GELF}, \hat{\beta}_{GELF}$ UNDER THE GENERALIZED ENTROPY LOSS FUNCTION USING LINDLEY'S APPROXIMATION:

The generalized entropy loss function is the generalization of the entropy loss function which is given as $L(\hat{\theta}-\theta)\propto\left(\frac{\hat{\theta}}{\theta}\right)^{a}-a\ln\left(\frac{\hat{\theta}}{\theta}\right)-1$ and the Bayes estimate under the generalized entropy loss function is given by $\hat{\theta}_{GELF}=[E_{\theta}(\theta^{-a})]^{-1/a}$ provided $E_{\theta}(\theta^{-a})$ exists and is finite. Thus the Bayes estimator for $\hat{\theta}_{GELF}$ of a function $u(\theta,\beta)=[(\theta)^{-a}]$ under uniform prior using the Lindley's approximation can be obtained as

$$u(\theta,\beta)=(\theta)^{-a},\ u_1=-a\theta^{-a-1},u_{11}=a(a+1)\theta^{-a-2},u_2=u_{22}=0$$

$$\therefore\ \hat{\theta}_{GELF} = \hat{\theta}^{-a} + \frac{a\,\theta^{-(a+1)}}{2\left[\frac{n}{\hat{\beta}^{2}}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^{2}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)\right]}\left[\frac{a+1}{\hat{\theta}}+\frac{\frac{2n}{\hat{\beta}^{3}}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^{3}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{\frac{n}{\hat{\beta}^{2}}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^{2}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)} + \frac{n-\hat{\beta}(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{n\beta-\hat{\beta}(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}}\right]$$

(10)

Here $u(\theta,\beta)=(\beta)^{-a}$, $u_2=-a\beta^{-a-1}, u_{22}=a(a+1)\,\beta^{-a-2}, u_1=u_{11}=0$

Thus the Bayesian estimator for $\hat{\beta}_{GELF}$ is obtained as

$$\therefore\ \hat{\beta}_{GELF} = \hat{\beta}^{-a} + \frac{a\,\beta^{-(a+1)}\hat{\theta}^{2}}{2\hat{\beta}\left[n-(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\right]}\left[\frac{a+1}{\hat{\beta}}-\frac{2n-(\hat{\beta}+1)(\hat{\beta}+2)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}}{\hat{\theta}\left[n-(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\right]} - \frac{\hat{\beta}\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^{2}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)+2\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{\hat{\theta}\left[\frac{n}{\hat{\beta}^{2}}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^{2}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)\right]}\right]$$

(11)

Under gamma prior

$$\therefore\ \hat{\theta}_{GELF} = \hat{\theta}^{-a} + \frac{a\,\theta^{-(a+1)}}{\left[\frac{n}{\hat{\beta}^{2}}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^{2}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)\right]}\left[\frac{a+1}{2\hat{\theta}}-\frac{s-1}{\hat{\theta}}+r+\frac{1}{2}\left\{\frac{\frac{2n}{\hat{\beta}^{3}}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^{3}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{\frac{n}{\hat{\beta}^{2}}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^{2}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)} + \frac{n-\hat{\beta}(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{n\beta-\hat{\beta}(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}}\right\}\right] \quad (12)$$

and

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$$\therefore \hat{\beta}_{GELF} = \hat{\beta}^{-a} + \frac{a\,\beta^{-(a+2)}\hat{\theta}^2}{\left[n-(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\right]}\left[\frac{a+1}{2\hat{\beta}}-\frac{p-1}{\hat{\beta}}+q-\frac{1}{2}\left\{\frac{2n-(\hat{\beta}+1)(\hat{\beta}+2)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}}{\hat{\theta}\left[n-(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\right]}+\frac{\hat{\beta}\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)+2\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{\hat{\theta}\left[\frac{n}{\hat{\beta}^2}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)\right]}\right\}\right] \quad (13)$$

V.BAYESIAN ESTIMATION FOR $\hat{\theta}_{SELF}$, $\hat{\beta}_{SELF}$ UNDER THE SQUARED ERROR LOSS FUNCTION USING LINDLEY'S APPROXIMATION:

The squared error loss function (SELF) which is symmetric loss function was proposed by Legendre [11] and Gauss [12] to develop least squares theory and gives equal weightage to the losses for overestimation and under estimation. The squared error loss function (SELF) is given as $L(\hat{\theta},\theta)=(\hat{\theta}-\theta)^2$. Thus the Bayes estimator for $\hat{\theta}_{SELF}$ of a function $u=u(\theta,\beta)$ under the uniform prior using the Lindley's approximation can be obtained as

$$u(\theta,\beta)=\theta,\ u_1=1, u_{11}=u_2=u_{22}=0$$

$$\therefore \hat{\theta}_{SELF} = \hat{\theta} - \frac{1}{2\left[\frac{n}{\hat{\beta}^2}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)\right]}\left[\frac{\frac{2n}{\hat{\beta}^3}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^3\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{\frac{n}{\hat{\beta}^2}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}+\frac{n-\hat{\beta}(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{n\beta-\hat{\beta}(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}}\right]$$

(14)

Following the same procedure the Bayes estimator for $\hat{\tau}_{SELF}$ of a function $u=u(\lambda,\tau)$ under uniform prior using the Lindley's approximation can be obtained as

Here $u(\theta,\beta)=\beta,\ u_2=1, u_1=u_{11}=u_{22}=0$

$$\therefore \hat{\beta}_{SELF} = \hat{\beta} - \frac{\hat{\theta}^2}{2\hat{\beta}\left[n-(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\right]}\left[\frac{2n-(\hat{\beta}+1)(\hat{\beta}+2)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}}{\hat{\theta}\left[n-(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\right]} + \frac{\hat{\beta}\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)+2\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{\hat{\theta}\left[\frac{n}{\hat{\beta}^2}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)\right]}\right]$$

(15)

Under gamma prior

$$\therefore \hat{\theta}_{SELF} = \hat{\theta} + \frac{1}{\left[\frac{n}{\hat{\beta}^2}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)\right]}\left[\frac{s-1}{\hat{\theta}} - r - \frac{1}{2}\left\{\frac{\frac{2n}{\hat{\beta}^3}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^3\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{\frac{n}{\hat{\beta}^2}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)} + \frac{n-\hat{\beta}(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{n\beta-\hat{\beta}(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}}\right\}\right]$$

(16)

and

$$\therefore \hat{\beta}_{SELF} = \hat{\beta} - \frac{\hat{\theta}^2}{\hat{\beta}\left[n-(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\right]}\left[\frac{p-1}{\hat{\beta}} - q + \frac{1}{2}\left\{\frac{2n-(\hat{\beta}+1)(\hat{\beta}+2)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}}{\hat{\theta}\left[n-(\hat{\beta}+1)\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\right]} + \frac{\hat{\beta}\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)+2\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)}{\hat{\theta}\left[\frac{n}{\hat{\beta}^2}+\sum_{i=1}^{n}\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)^{\hat{\beta}}\ln^2\left(\frac{\alpha x_i-\lambda}{\hat{\theta}}\right)\right]}\right\}\right]$$

(17)

Where $\hat{\theta}, \hat{\beta}$ are the maximum likelihood estimators of the parameters of generalized gamma distribution and $\hat{\theta}_{LLF}$, $\hat{\beta}_{LLF}$; $\hat{\theta}_{GELF}$, $\hat{\beta}_{GELF}$; $\hat{\theta}_{SELF}$, $\hat{\beta}_{SELF}$ are the Baye's estimators of $\hat{\theta}, \hat{\beta}$ obtained by using Lindley's approximation.

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