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A New Procedure of Variance Estimation Using Linear Combination of Conventional and Non- Conventional Parameters

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ABSTRACT

In this paper, we have envisaged an efficient class of estimator using conventional and non- conventional parameters for finite population variance of the study variable in simple random sampling. Asymptotic expressions of the bias and mean square error of the proposed class of estimators have been obtained. Asymptotic optimum estimator in the proposed class of estimators has been identified with its mean square error formula. In the support of the theoretically results we have given an empirical study.

Keywords*: *Auxiliary variable, Study variable, Mean-square error, Bias, Simple random sampling, Tri-mean, Quartiles.

I.INTRODUCTION

In this article, an improved class of estimators is proposed in estimating the finite population variance under simple random sampling. in simple random sampling. Variations are present everywhere in our day to day life. It is the law of nature that no two things or individuals are exactly alike. For instance, a physician needs a full understanding of variations in the degree of human blood pressure, body temperature, and pulse rate for adequate prescription. A manufacturer needs constant knowledge of the level of variations in people's reaction to his product to be able to know whether to reduce or increase his price, or improve the quality of his product. An agriculturist needs an adequate understanding of the variations in climatic factors especially from place to place (or time to time) to be able to plan on when, how and where to plant his crop. Many more situations can be encountered in practice where the estimation of population variance Various fields of life like genetics, biology and medical studies have been facing the problem in estimating the finite population variance. A fair understanding of variability is vitally important for better results in different walks of life. With the increasing growth in the number and diverse uses of sample surveys worldwide, it is often desired to analyze and interpret the resulting voluminous data by swifter methods [1]. let us consider a finite population $U=\{ U_1,U_2,U_3.....U_N \}$ of N distinct and identifiable units. Let Y be a real variable with value Y_i measured on $U_i = i=1,2,3,\ldots,N$ giving a vector $Y=\{Y_1,Y_2,Y_3.......Y_N\}$.The goal is to estimate the population mean $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ and its population variance $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{y})^2$ on the basis of the random sample selected from the population U. [8]

proposed the use of ratio or difference estimators of the variance under a passion sampling design (a design in which each sampling unit is given an independent chance of being selected into the sample without replacement) for the purpose of reducing the effect of the random sample size on the variance estimator. Under a sample design in which one sample unit is selected in each stratum with probability proportional to size (PPS). [11] considered reducing the bias of the estimator of the variance in a one-per stratum design by using a variance estimator as the Yates Grundy variance estimator for two sample units per stratum design with joint inclusion probabilities. The problem of constructing efficient estimators for the population variance has been widely discussed by various authors such as [4], who proposed ratio and regression estimators. [9] considered a ratio type estimator for estimating population variance by improving [4] estimator. [2] proposed a new hybrid class of estimators and showed that in some cases their efficiency is better than the traditional ratio estimators, whereas [5] [6] proposed the modified estimators using coefficient of variation (C.V.) and their linear combinations. [12] improved the already existing estimators by introducing modified estimators with the use of known parameters like C.V., Kurtosis, Median, Quartiles and Deciles. Recently, [10] proposed a modified ratio estimator using non-conventional location parameters because these parameters take care of outliers in the data.

II. NOTATIONS

Let *N* = population size, *n* = sample size, $\gamma = \frac{1}{n}$,*Y*= study variable, *X*= auxiliary variable. , $\overline{X}, \overline{Y}$ = population means, $\overline{x}, \overline{y}$ = sample means, S_x^2, S_y^2 = population variances, s_x^2, s_y^2 = sample variances , S_x, S_y = population standard deviation. C_x, C_y = coefficient of variation, ρ = correlation coefficient, $\beta_1(x)$ = skewness of the auxiliary variable, $\beta_2(x)$ = kurtosis of the auxiliary variable, $\beta_2(y)$ = kurtosis of the study variable, Md = median of the auxiliary variable, *B*(.)=bias of the estimator, *MSE*(.)= Mean square error, $\widehat{S}_R^2$ = ratio type variance estimator, $\widehat{S}_{KC1}^2, \widehat{S}_{KG}^2$, existing modified ratio estimators, $Q_a = \frac{Q_1 + Q_3}{2}$ is population semi-quartile average of the auxiliary variable *x*, $TM = \frac{Q_1 + 2Q_2 + Q_3}{4}$ is a Tri-Mean, S_j = proposed estimator by Showkat and Javaid

In [4] suggested a ratio type variance estimator for population variance S_y^2 when the population variance S_x^2 of an auxiliary variable *X* is known together with its *b*ias and *m*ean *s*quared *e*rror as:

$$\widehat{S}_R^2 = s_y^2 \frac{S_x^2}{s_x^2} \tag{1.1}$$

$$B(\widehat{S}_R) = \gamma S_y^4 \left[\beta_2(x-1) - (\lambda_{22} - 1)\right]$$

$$MSE(\hat{S}_R{}^2) = \gamma S_y^4\left[(\beta_{2(y)} - 1) + (\beta_{2(x)} - 1) - 2(\lambda_{22} - 1)\right]$$

Here γ is the sampling fraction and λ_{22} is covariance between the study and auxiliary variable.

The [5] suggested four ratio type variance estimators using known values of C.V. and coefficient of kurtosis of an auxiliary variable.

$$\hat{S}^2_{KC1} = s_y^2\left[\frac{S_x^2 + C_x}{s_x^2 + C_x}\right] \tag{1.2}$$

$$B(\hat{S}_{KC1}{}^2) = \gamma S_y^4[A_1\beta_2(x-1) - (\lambda_{22} - 1)]$$

$$MSE(\hat{S}_{KC1}{}^2) = \gamma S_y^4\left[(\beta_{2(y)} - 1) + A_1(\beta_{2(x)} - 1) - 2(\lambda_{22} - 1)\right]$$

where $A_1 = \left[\frac{S_x^2}{s_x^2 + C_x}\right]$

$$\hat{S}^2_{KC2} = s_y^2\left[\frac{S_x^2 + \beta_2(x)}{s_x^2 + \beta_2(x)}\right] \tag{1.3}$$

$$B(\hat{S}_{KC2}{}^2) = \gamma S_y^4[A_2\beta_2(x-1) - (\lambda_{22} - 1)]$$

$$MSE(\hat{S}_{KC2}{}^2) = \gamma S_y^4\left[(\beta_{2(y)} - 1) + A_2(\beta_{2(x)} - 1) - 2(\lambda_{22} - 1)\right]$$

Where $A_2 = \left[\frac{S_x^2 + \beta_2(x)}{s_x^2 + \beta_2(x)}\right]$

$$\hat{S}^2_{KC3} = s_y^2\left[\frac{S_x^2\beta_2(x) + C_x}{s_x^2\beta_2(x) + C_x}\right] \tag{1.4}$$

$$B(\hat{S}_{KC3}{}^2) = \gamma S_y^4[A_3\beta_2(x-1) - (\lambda_{22} - 1)]$$

$$MSE(\hat{S}_{KC3}{}^2) = \gamma S_y^4\left[(\beta_{2(y)} - 1) + A_3(\beta_{2(x)} - 1) - 2(\lambda_{22} - 1)\right]$$

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Where $A_3 = \left[\frac{S_x^2 \beta_2(x) + C_x}{s_x^2 \beta_2(x) + C_x} \right]$

$$\hat{S}^2_{KC4} = s_y^2 \left[\frac{S_x^2 C_x + \beta_2(x)}{s_x^2 C_x + \beta_2(x)} \right] \qquad (1.5)$$

$$B(\hat{S}_{KC4}{}^2) = \gamma S_y^4 \left[A_4 \beta_2(x-1) - (\lambda_{22} - 1) \right]$$

$$MSE(\hat{S}_{KC4}{}^2) = \gamma S_y^4 \left[(\beta_{2(y)} - 1) + A_4(\beta_{2(x)} - 1) - 2(\lambda_{22} - 1) \right]$$

Where $A_4 = \left[\frac{S_x^2 C_x + \beta_2(x)}{s_x^2 C_x + \beta_2(x)} \right]$

In [12] proposed a generalized modified ratio type estimator for estimating population variance using the known parameters of the auxiliary variable and their estimator is given as

$$\hat{S}^2_{JG} = s_y^2 \left[\frac{S_x^2 + \alpha w_i}{s_x^2 + \alpha w_i} \right] \qquad (1.6)$$

$$B(\hat{S}_{JG}{}^2) = \gamma S_y^4 A_{JG} \left[A_{JG} \beta_2(x-1) - (\lambda_{22} - 1) \right]$$

$$MSE(\hat{S}_{JG}{}^2) = \gamma S_y^4 \left[(\beta_{2(y)} - 1) + A_{JG}(\beta_{2(x)} - 1) - 2(\lambda_{22} - 1) \right]$$

Where $A_{JG} = \left[\frac{S_x^2}{s_x^2 + \alpha w_i} \right]$

When the study variable Y and the auxiliary variable X are negatively correlated and the population parameters of the auxiliary variable are known, [12] proposed the following generalized modified product type variance estimator as

$$\widehat{S}_{JG1}^{2} = s_y^2 \left[\frac{S_x^2 + \tau w_i}{s_x^2 + \tau w_i} \right] \qquad (1.7)$$

The notations in the estimator $\widehat{S}_{JG1}^{2}$ are explained in detail in [12].

[10] suggested a linear combination of ratio type estimator

$$\widehat{S}_{SJ}^{2} = s_y^2 \left[\frac{S_x^2 + (TM + Q_a)}{s_x^2 + (TM + Q_a)} \right]$$

$$B(\widehat{S}_{SJ}{}^{2}) = \gamma S_y^4 A_{SJ} \left[A_{SJ} \beta_2 (x-1) - (\lambda_{22} - 1) \right]$$

$$MSE(\widehat{S}_{SJ}{}^{2}) = \gamma S_y^4 \left[(\beta_{2(y)} - 1) + A_{SJ} (\beta_{2(x)} - 1) - 2(\lambda_{22} - 1) \right]$$

III. PROPOSED ESTIMATOR

The performance of the estimator of the study variable can be improved by using known population parameters of an auxiliary variable, which are positively correlated with a study variable. A modified ratio type variance estimator using the linear combination of known value of the population Tri-mean and quartile average of an auxiliary variable is proposed; this modified ratio type variance estimator for population variance S_y^2 is defined as

$$\widehat{S}_{SJ1}^{2} = s_y^2 \left[\frac{S_x^2 + (TM + Q_a)}{s_x^2 + (TM + Q_a)} \right]^{\alpha} \qquad (1.8)$$

Let $e_o = \frac{s_y^2 - S_y^2}{s_y^2}$ and $e_1 = \frac{s_x^2 - S_x^2}{s_x^2}$ .Further we can write $s_y^2 = S_y^2(1+e_0)$ and $s_x^2 = S_x^2(1+e_1)$ from the definition of e_o and e_1 we obtain

$$E(e_o) = E(e_1) = 0$$

$E(e_o^2)=\gamma(\beta_{2(y)}-1), E(e_1^2)=\gamma(\beta_{2(x)}-1)$

$E(e_o e_1)=\gamma(\lambda_{22}-1)$

The proposed estimator of $\hat{S}^2_{SJ1}$ is given as

$$\hat{S}^2_{SJ1}=s_y^2\left[\frac{S_x^2+(TM+Q_a)}{s_x^2+(TM+Q_a)}\right]^{\alpha}$$

$$\hat{S}^2_{SJ1}=S_y^2(1+e_0)\left[\frac{S_x^2+B}{S_x^2+S_x^2e_1+B}\right]^{\alpha} \text{ where } B=(TM+Q_a)$$

$$\hat{S}^2_{SJ1}=S_y^2(1+e_0)\left[\frac{1}{1+\theta e_1}\right]^{\alpha} \quad \theta=\frac{S_x^2}{S_x^2+B}$$

$$\hat{S}^2_{SJ1}=S_y^2(1+e_0)[(1+\theta e_1)]^{-\alpha}$$

$$\hat{S}^2_{SJ1}=S_y^2(1+e_0)\left[(1-\alpha\theta e_1+\frac{\alpha(\alpha+1)}{2}\theta^2 e_1{}^2-....)\right]$$

$$\hat{S}^2_{SJ1}=S_y^2(1+e_0)\left[(1-\alpha\theta e_1+\frac{\alpha(\alpha+1)}{2}\theta^2 e_1{}^2)\right] \tag{1.9}$$

Taking expectation on both sides of eq.(1.9), the *Bias* of the proposed estimator

$$E\left(\hat{S}^2_{SJ1}-S_y^2\right)=S_y^2\left[e_0-\alpha\theta e_1+\frac{\alpha(\alpha+1)}{2}\theta^2 e_1{}^2-\alpha\theta e_1 e_0\right]$$

$$Bias\left(\hat{S}^2_{SJ1}\right)=\gamma.S_y{}^2\theta\left[\frac{\alpha(\alpha+1)}{2}\theta\left(\beta_{2(x)}-1\right)-\alpha\left(\lambda_{22}-1\right)\right]$$

Expanding and neglecting higher terms more than power two of eq.(1.9), we get

$$\hat{S}^2_{SJ1}=S_y^2(1+e_0)[(1-\alpha\theta e_1)]$$

$$\hat{S}^2_{SJ1}-S_y^2=S_y^2[e_0-\alpha\theta e_1)]$$

$$\widehat{S}^2_{SJ1} - S^2_y = S^2_y[e_0 - \alpha\theta e_1)]$$

$$E(\widehat{S}^2_{SJ1} - S^2_y) = \gamma S^2_y E[e_0 - \alpha\theta e_1)] \tag{1.10}$$

Squaring both sides to eq (1.10) we get *MSE* of the proposed estimator

$$E(\widehat{S}^2_{SJ1} - S^2_y)^2 = S^4_y[e_0 - \alpha\theta e_1]^2$$

$$MSE\left(\widehat{S}^2_{SJ1}\right) = S^4_y\left[e_0{}^2 + \alpha^2\theta^2 e_1{}^2 - 2\alpha\theta e_0 e_1\right]$$

$$MSE\left(\widehat{S}^2_{SJ1}\right) = \gamma.S^4_y\left[\left(\beta_{2(y)} - 1\right) + \alpha^2\theta^2\left(\beta_{2(x)} - 1\right) - 2\alpha\theta\left(\lambda_{22} - 1\right)\right]$$

$$= \gamma S^4_y\left[(\beta_{2(y)} - 1) + \alpha^2\theta^2(\beta_{2(x)} - 1) - 2\alpha\theta(\lambda_{22} - 1)\right] \tag{1.11}$$

Differentiate eq. (1.11) with respect α we get

$$\alpha = \frac{(\lambda_{22} - 1)}{\theta(\beta_{2(y)} - 1)}$$

Put the value of α in eq. (1.11), we get the *MSE* of the proposed estimator

$$MSE\left(\widehat{S}^2_{SJ1}\right) = \gamma S^4_y\left[(\beta_{2(y)} - 1) - \frac{(\lambda_{22} - 1)^2}{(\beta_{2(x)} - 1)}\right]$$

Which is the usual regression variance estimator having minimum MSE.

IV. NUMERICAL ILLUSTRATION

The performance of the proposed estimator is assessed with that of simple random sampling without replacement (SRSWOR) sample variance and existing estimators. We use the data of [7] in which fixed capital is denoted by X (auxiliary variable) and output of 80 factories are denoted by Y (study variable). We apply the proposed and existing estimators to this data set and the data statistics are given below:ssssss

$N = 80, n = 20, S_x = 8.4542, S_y = 18.3569, C_x = 0.7507, C_y = 0.3542, A_1 = 0.9896, A_2 = 0.9615, A_3 == 0.9964,$

$\overline{X} = 11.2624,\ ß_{2(x)} = 2.8664, A_4 = 0.9493,\ A_{JG} = 0.8763,$

$\rho = 0.9413, \overline{Y} = 51.8264,\ ,\beta_{2(y)} = 2.2667, ß_{1(x)} = 1.05, 22, \lambda_{22} = 2.2209,$

$Md = 7.5750, Q_1 = 9.318,\ Q_2 = 7.5750,\ Q_3 = 16.975, Q_D = 5.9125, Q_a = 11.0625, Q_R = 11.82,$

$TM = 9.318, \text{Tri-mean } (TM) = 9.318$

The results obtained are shown in Table 1.

The Bias and Mean Square Error of the existing and the proposed estimators.

Table 1

Estimators	MSE
$\hat{S}_R^2 = s_y^2 \frac{S_x^2}{s_x^2}$	3925.1622
$\hat{S}_{KC1}^2 = s_y^2 \left[\frac{S_x^2 + C_x}{s_x^2 + C_x}\right]$	3850.1552
$\hat{S}_{KC2}^2 = s_y^2 \left[\frac{S_x^2 + \beta_2(x)}{s_x^2 + \beta_2(x)}\right]$	3658.4051
$\hat{S}_{KC3}^2 = s_y^2 \left[\frac{S_x^2 \beta_2(x) + C_x}{s_x^2 \beta_2(x) + C_x}\right]$	3898.5560
	3580.8342
$\hat{S}_{JG}^2 = s_y^2 \left[\frac{S_x^2 + \alpha w_i}{s_x^2 + \alpha w_i}\right]$	3180.7740
$\hat{S}_{SJ}^2 = s_y^2 \left[\frac{S_x^2 + (TM + Q_a)}{s_x^2 + (TM + Q_a)}\right]$	2820.06
$\hat{S}_{SJ1}^2 = s_y^2 \left[\frac{S_x^2 + (TM + Q_a)}{s_x^2 + (TM + Q_a)}\right]^\alpha$	**2611.02**

V. CONCLUSION

The paper proposes a ratio type variance estimator using known values of an auxiliary variable. From Table 1 we see that the MSE of the existing estimators ranges from 2820.06 to 3925 respectively, while as the proposed estimator has MSE 2611.02. Hence, the proposed estimator may be preferred over existing estimators for use in practical applications.

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