

Resemblance of laws for Set Theory and Boolean Algebra

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ABSTRACT

Students of Computer Sciences, Mathematics and related subjects need to study the concepts of Set Theory and Boolean Algebra in subjects like Digital Electronics, Discrete Mathematics, Engineering Mathematics, etc. It has been observed that the basic laws which act as the backbone of these subjects resemble with each other. The resemblance between the laws of these two subjects i.e., Boolean Algebra and Set Theory has been proven in this paper.

Keywords: Associative, Boolean Algebra, commutative, complement, Distribute, identity, Intersection, involution, Set Theory, Idempotent, laws, logical operations, Truth Table, Union,

I.INTRODUCTION

Set Theory and Boolean Algebra are two separate topics and are being studied separately by students of Computer Sciences, Mathematics, etc. In this paper a good effort has been done to make this point clear to the readers that the laws of Set Theory and Boolean Algebra can be studied together because they resemble a lot especially the basic laws of Set Theory and Boolean Algebra are alike. This paper should act as a bridge between Mathematics and Computer Sciences. If a student has learnt Set Theory then it is very easy for him to understand Boolean Algebra and vice versa.

II.FIGURES AND TABLES

Proof by Set Theory	Proof by Boolean Algebra / Truth Table									
Idempotent Laws										
Suppose $A=\{1,2,3,4\}$ (i) $A \cup A = A$ $\Rightarrow \{1,2,3,4\} \cup \{1,2,3,4\} = \{1,2,3,4\} = A$ (ii) $A \cap A = A$ $\Rightarrow \{1,2,3,4\} \cap \{1,2,3,4\} = \{1,2,3,4\} = A$	(i) $A+A = A$ (ii) $A.A = A$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>A</td> <td>A+A</td> <td>A.A</td> </tr> <tr> <td>F</td> <td>F</td> <td>F</td> </tr> <tr> <td>T</td> <td>T</td> <td>T</td> </tr> </table>	A	A+A	A.A	F	F	F	T	T	T
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Suppose $A=\{1,2,3,4\}$, $B=\{4,5,6,7\}$ and universal set $D = \{1,2,3,4,5,6,7,8,9\}$	(i) $(A+B)' = A'.B'$ (ii) $(A.B)' = A'+B'$																																																								
(i) $(A \cup B)' = A' \cap B'$ $\Rightarrow (\{1,2,3,4\} \cup \{4,5,6,7\})' = \{1,2,3,4\}' \cap \{4,5,6,7\}'$ $\Rightarrow \{1,2,3,4,5,6,7\}' = \{5,6,7,8,9\} \cap \{1,2,3,8,9\}$ $\Rightarrow \{8,9\} = \{8,9\}$ $\Rightarrow \text{LHS} = \text{RHS}$	<table border="1" style="margin: 0 auto; border-collapse: collapse; font-size: small;"> <tr> <th>A</th><th>B</th><th>A'</th><th>B'</th><th>A.A'</th><th>A.B</th><th>A.B'</th><th>A'.B</th><th>A'.B'</th><th>(A.B)'</th><th>(A'+B)'</th> </tr> <tr> <td>F</td><td>F</td><td>T</td><td>T</td><td>F</td><td>F</td><td>F</td><td>T</td><td>T</td><td>T</td><td>T</td> </tr> <tr> <td>F</td><td>T</td><td>T</td><td>F</td><td>T</td><td>F</td><td>F</td><td>F</td><td>F</td><td>T</td><td>T</td> </tr> <tr> <td>T</td><td>F</td><td>F</td><td>T</td><td>T</td><td>F</td><td>F</td><td>F</td><td>F</td><td>T</td><td>T</td> </tr> <tr> <td>T</td><td>T</td><td>F</td><td>F</td><td>T</td><td>T</td><td>F</td><td>F</td><td>F</td><td>F</td><td>F</td> </tr> </table>	A	B	A'	B'	A.A'	A.B	A.B'	A'.B	A'.B'	(A.B)'	(A'+B)'	F	F	T	T	F	F	F	T	T	T	T	F	T	T	F	T	F	F	F	F	T	T	T	F	F	T	T	F	F	F	F	T	T	T	T	F	F	T	T	F	F	F	F	F	(ii) $(A \cap B)' = A' \cup B'$ $\Rightarrow (\{1,2,3,4\} \cap \{4,5,6,7\})' = \{1,2,3,4\}' \cup \{4,5,6,7\}'$ $\Rightarrow \{4\}' = \{5,6,7,8,9\} \cup \{1,2,3,8,9\}$ $\Rightarrow \{1,2,3,5,6,7,8,9\} = \{1,2,3,5,6,7,8,9\}$ $\Rightarrow \text{LHS} = \text{RHS}$
A	B	A'	B'	A.A'	A.B	A.B'	A'.B	A'.B'	(A.B)'	(A'+B)'																																															
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III.CONCLUSION

The laws of Boolean Algebra and Set theory since resemble with each other, they have been proven by their respective procedures. The laws of Boolean Algebra have been proven by Truth Tables. It has been observed that a Union Operation (U) resembles with an OR operation (+), Intersection (∩) resembles with And (.) operation, Compliment is similar to Negation in Set Theory and Boolean Algebra respectively.

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