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Comparative Study of complete graph and Perfect Difference Network (PDN)

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ABSTRACT

In this paper we have analyzed the architectures of Complete Graph and Perfect Difference Network (PDN). We have also shown how Perfect Difference Network (PDN) can be derived from the Complete Graph of ($\delta^2+\delta+1$) nodes by removing the skip diagonal links of Complete Graph. On the removal of diagonal links of Perfect Difference Network (PDN) we have seen symmetry in the architecture.

Keywords: Perfect Difference Set (PDS), Perfect Difference Network (PDN), Perfect Difference Graph (PDG), Complete Graph, Circuit.

I.INTRODUCTION

An analysis of Complete Graph and Perfect Difference Network of ($\delta^2+\delta+1$) nodes is done. The removal of skip links from the Complete Graph results the formation of Perfect Difference Network (PDN) which is presented in the form of different results.

1.1 Perfect Difference Set

The Perfect Difference Sets were first discussed by J. Singer in 1938 in terms of points and lines in a projective plane of a Galois Field (GF) [10], [11].

Definition 1: Perfect Difference Set - If the set S of δ+1 distinct integers $S_0, S_1 ..., S_\delta$ has the property that the $\delta^2 + \delta$ differences $S_i - S_j$ ($0\leq i, j \leq \delta, i\neq j$) are distinct modulo $\delta^2 + \delta + l$, S is called a perfect difference set mod $\delta^2 + \delta + l$.

The existence of perfect difference sets seems intuitively improbable, at any rate for large δ, but in 1938 J. Singer proved that, whenever δ is a prime or power of prime, say $\delta = p^n$, a perfect difference set mod $p^{2n} + p^n + 1$ exists. [2], [3], [4]

From now we on, let δ denote p^n and we write that $n = \delta^2 + \delta + l = p^{2n} + p^n + 1$. $S = \{s : |s_i - s_j| \bmod n$, where $0\leq i, j\leq \delta, i \neq j$, δ is a prime or power of prime and $n = \delta^2 + \delta + l\}$ [4].

1.2. Perfect Difference Network

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Perfect Difference Network architecture, based on a PDS is designed where each ith node is connected via direct links to nodes i ± 1 and i ± s_j(mod n), for $2 \le j \le \delta$. Each link is bidirectional and the preceding connectivity leads to a chordal ring of δ in-degree and δ out-degree (total degree of a node d(v) = 2δ) and diameter D = 2 [1], [3]. PDN has already been studied for, high performance communication and parallel processing network [3] and some topological properties of PDNs and parallel algorithms [4], [5], [6], [9] were suggested. It was shown that an n-node PDN can emulate the complete network with optimal slow down and balanced message traffic.

Although other interconnection architectures with topological and performance characteristics similar to PDNs exist, we view PDNs as worthy additions to the repertoire of computer system designers.

Alternative network topologies offer additional design points that can be exploited to accommodate the needs of new and emerging technologies. Further study is needed to resolve some open questions and to cost/performance comparisons for PDNs.

1.3. Perfect Difference Graph

In a PDN for each link from node I to node j, the reverse link j to i also exists, so the network corresponds to an undirected Perfect Difference Graph (PDG), G = (V, E) consisting of a set of vertices V and a set of edges E connecting pairs of vertices in V [7], [8].

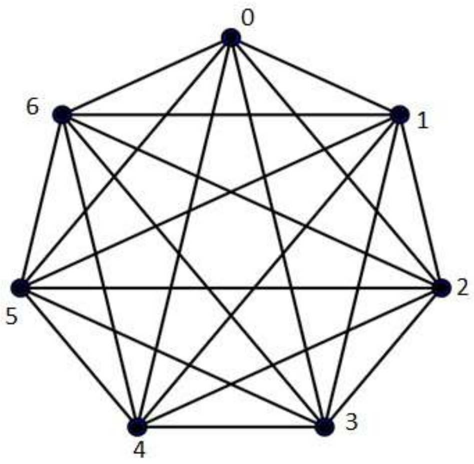
1.4. Complete Graph

A Complete Graph is a simple graph G = (V, E) where for all vertices v_i, v_j€V, $v_i \neq v_j$, there exists an edge (v_i, v_j) [12].

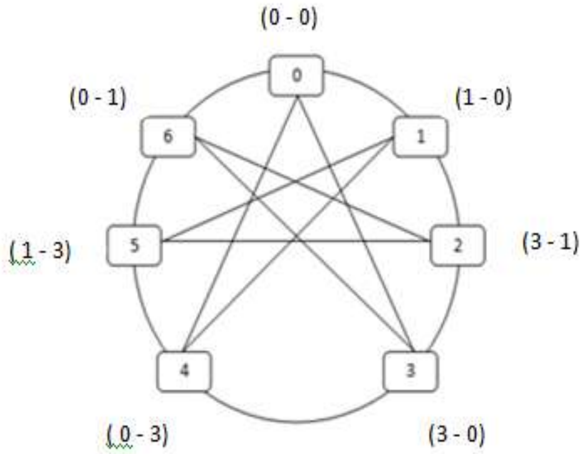
In other words, in a Complete Graph every vertex is connected to every other vertex i.e. every pair of different vertices are adjacent.

1.5. Circuit: A circuit is a closed trial or a closed walk in which no edge is repeated. Vertices may however be repeated. i.e. the sequence $v_0, e_1, v_1, e_2, \ldots, e_n, v_n$ is a circuit if $v_0 = v_n$ and all e_i's are distinct [12].

II. COMPARATIVE STUDY OF COMPLETE GRAPH AND PERFECT DIFFERENCE NETWORK (PDN)



Fig(2.1)



Fig(2.2)

Fig(2.1)complete graph of ($\delta^2+\delta+1$) Fig(2.2)PDN of ($\delta^2+\delta+1$)

Complete graph is the super set of PDN. Let us take a complete graph of 7 nodes given as (0,1,2,…,6) the power set of the above set will be 2^7 =128 shown below

Pairs of 1 :{ {ϕ},{0},{1},{2},{3},{4},{5},{6}} =8

Pairs of 2 :{{01},{02},{03},{04},{05},{06},{12},{13},{14},{15},{16},{23}
{24},{25}{26},{34},{35},{36},{45},{46},{56}} =21

Pairs of
3:{{012},{013},{014},{015},{016},{023},{024},{025},{026},{034},{035},{036}{045},{046},{056},{123},{124},{125},{126},{134},{135},{136},{145},{146},{156},{234},{235},{236},{245}.{246},{256},{345},{356},{456}} =35

Pairs of 4: {{0123},{0124},{0125},{0126},{0134},{0135},{0136},{0145}
{0146},{0156},{0234},{0235},{0236},{0245},{0246},{0256},{0345},{0346}{0356},{0456},{1234},{1235},
{1236}{1245},{1246},{1256},{1345},{1346},{1356},{1456},{2345},{2346},{2356},{2456},{3456}}
=35

Pairs of 5 : {{01234},{01235},{01236},{01245},{01246},{01256},{01345}
{01346},{01356},{01456},{02345},{02346},{02356}{02456},{03456},{12345},{12346}
{12356},{12456},{13456},{23456}} =21

Pairs of 6 :{ {012345},{012356},{012356},{012456},{013456},{023456} {123456} }
=7

Pairs of 7: {{0123456}} =1

Circuits are calculated starting from 3 i.e., minimum no of nodes involved in a circuit is 3(self loops and pairs of 2 are excluded) in PDN.

2.1 Matrix representation of complete graph and PDN

	0	1	2	3	4	5	6
0	0	1	1	1	1	1	1
1	1	0	1	1	1	1	1
2	1	1	0	1	1	1	1
3	1	1	1	0	1	1	1
4	1	1	1	1	0	1	1
5	1	1	1	1	1	0	1
6	1	1	1	1	1	1	0

Fig 2.1.1 Complete Graph

	0	1	2	3	4	5	6
0	0	1	0	1	1	0	1
1	1	0	1	0	1	1	0
2	0	1	0	1	0	1	1
3	1	0	1	0	1	0	1
4	1	1	0	1	0	1	0
5	0	1	1	0	1	0	1
6	1	0	1	1	0	1	0

Fig 2.1.2 PDN

In the above figure, the upper or lower triangular matrix gives the total number of edges in a Complete Graph, we can manipulate this as, 1+2+3+4+5+6+ …+n = n (n-1)/2 (where n is the total number of nodes) Where as both (upper + lower) triangular matrix gives the total number of links in Complete Graph. So we can manipulate this as fallows.

$$2(n\,(n-1))/2$$
$$= n\,(n-1)$$
$$= n^2 - n$$

Hence the total links of Complete Graph is (n^2 - n)

2.2 Removal of diagonals in complete graph that are in PDN

On the removal of the diagonal edges of the complete graph that are the PDN diagonals the symmetry is seen as shown below in the below table

Diagonal links	Total no of nodes involved in circuit				Total
	3 nodes	4 nodes	5 nodes	6 nodes	
(0-3)	5	4	1	-	10
(0-4)	5	4	1	-	10
(1-4)	5	4	1	-	10
(1-5)	5	4	1	-	10
(2-5)	5	4	1	-	10
(2-5)	5	4	1	-	10
(2-6)	5	4	1	-	10
(3-6)	5	4	1	-	10

Table 2.2.1

2.3 Removal of skip diagonals in complete graph that are not in PDN

Diagonal links	Total no of nodes involved in circuit				Total
	3 nodes	4 nodes	5 nodes	6 nodes	
(1-3)	5	6	4	1	16
(1-6)	5	6	4	1	16
(2-4)	5	6	4	1	16
(3-5)	5	6	4	1	16
(4-6)	5	6	4	1	16
(0-2)	5	6	4	1	16
(0-5)	5	6	4	1	16

Table 2.3.1

1. As we see in the above table (table 4.1) the complete graph shows the symmetry in architecture which is due to the repetition of edges in different circuits.
2. Symmetry is due to the repetition of edges for different circuits.
3. The power set gives the total no of unique circuits with repetition of edges in different circuits.

2.4 Removal of repetition of edges in the circuits of complete graph

On the removal of the skip diagonal edges in complete graph at each node $\delta(\delta^4+2\delta^3+2\delta^2+\delta)-(\delta^2+\delta+1))$ circuits gets removed and the remaining graph forms a PDN which has the circuits of $(2\delta^3+\delta^2+\delta)$. Which is the combination of 3,4,5 and 7 nodes as shown below.

Combination of three nodes = {(014),(034),(036)(125)(145)(236)(256)}

Combination of four nodes = {(0123),(0126),(0156),(0456),(1234),(2345) (3456)}

Combination of five nodes= {(01234),(01236),(01256),(01456),(03456) (12345),(23456)}

Combination of seven nodes= {(0123456)}.

The table below shows the removal of diagonals and with them the no of circuits removed.

Diagonal links	Total no of nodes involved in circuit				Total circuits removed
	δ+1 nodes	δ+2 nodes	δ+3 nodes	δ+4 nodes	
(0-2)	5	6	4	1	16
(0-5)	4	4	3	1	12
(1-3)	5	5	3	1	14

(1-6)	4	4	3	1	12
(2-4)	4	3	1	1	9
(3-5)	3	3	0	1	7
(4-6)	2	3	0	1	6
Total no of circuits removed					76

Table 2.4.1

1. After the removal of these diagonal links the remaining graph forms the PDN of $(2\delta^3+\delta^2+\delta)-1$ circuits.
2. Now if we remove the diagonal links of PDN the remaining graph will behave as a ring topology with only one circuit of length $(\delta^2+\delta+1)$ and will function as a ring topology network.
3. The below table shows the removal of diagonal links from PDN and the circuits removed with that diagonal link.

$S_i - s_j$	No of nodes involved in circuit				Circuits removed
	δ+1 nodes	δ+2 nodes	δ+3 nodes	δ+4 nodes	
(0-3)	1	1	1	0	3
(0-4)	1	1	1	0	3
(1-4)	1	1	1	0	3
(1-5)	1	1	1	0	3
(2-5)	1	1	1	0	3
(2-6)	1	1	1	0	3
(3-6)	1	1	1	0	3
Total circuits $(\delta+1)(\delta^2+\delta+1)$					21

Table 2.4.2

4. After the removal of diagonal links from PDN the remaining graph will be a chordal ring of only one circuit of length $(\delta^2+\delta+1)$.

III.CONCLUSION

The comparative study of Complete Graph and Perfect Difference Network (PDN) of $(\delta^2+\delta+1)$ nodes has been done. We have found that the Perfect Difference Network (PDN) can be derived from the Complete Graph by removing the skip diagonal links from the Complete Graph of $(\delta^2+\delta+1)$ nodes. We have also shown that the total links of Complete Graph can be calculated from the matrix of Complete Graph.

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