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Some Information Measures for Gamma Exponential Distribution

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Abstract

In this paper, a new family of T-X distribution namely Gamma Exponential Distribution(GED) is defined. We study information properties of the GE distribution and provide an assortment of information measures for the GED. The measures include Shannon entropy, Renyi entropy, β-entropy, Generalized entropy, Kullback-Leibler divergence. Some of its properties including limit behaviour, hazard function, survival function, moments, mean deviation, mode, moment generating function, characteristic function are discussed. Parameter estimation of the gamma exponential distribution by the maximum likelihood method is proposed. Fisher information matrix for the gamma exponential distribution is also obtained.

keywords: T-X family, Shannon entropy, moments, estimation, Kullback-Leibler information.

1 Introduction

The exponential distribution is one of the key distributions in the theory and practice of statistics and are commonly employed in the formation of methods of lifetime distributions and stochastic process in general.[1] referred to the exponential distribution, while discussing the sampling distribution of standard deviation, as Pearson's Type X distribution. Applications of the exponential distribution in actuarial, biological and engineering problems were demonstrated subsequently by [2], [3] and [4]. The extension of the exponential distribution in the form of Weibull distribution were studied by [5]. This family of distributions includes the exponential distribution as a special case when the shape parameter equals one.

Reliability theory and reliability engineering also make extensive use of the exponential distribution. Because of the memoryless property of this distribution, it is well suited to model, the constant hazard rate portion of the bathtub curve used in reliability theory. The basic characterization of the exponential distribution based on lack of memory is simply the logarithmic equivalent of the functional equation $f(x+y) = f(x)f(y)$, which is due to [6], [7] and [8]. A complete solution of the logarithmic equivalent of this functional equation for both continuous and discrete cases was provided by [9] and [10]. [11] characterized exponential through Poisson and renewal process, [12] through order statistics and [13] through range and ratios of order statistics.

Numerous authors have derived various generalizations of the distributions. [14],[15] and [16] introduced the two-parameter generalized exponential distribution can be used in analyzing many life time data. Researchers developed and studied new and more flexible distribution. Parameters estimation of gamma-Pareto distribution was considered by [17]. [18] proposed $T - X$ method for generating families of continuous distributions. The problem of estimation of paremeters of Weibull-Pareto distribution by the method of modified maximum likelihood was considered [19]. [20] studied various structural properties of gamma X family specialised their results on gamma-normal distribution. The gamma and exponential distributions are the most widely used in the reliability and survival studies. In addition, the exponential distribution is one of the members of gamma$-X$ family as well. [21] generalizes the method of T-X family by inclusion of an additional parameter c which leads to a new family of exponentiated T-X distribution and studied the properties of exponentiated weibull-exponential distribution. [22] extended the exponentiated weibull-exponential distribution to more general form.

Alzaatreh et al (2013) presented the cumulative distribution function (cdf) of the Transformed-Transformer family or $T - X$ family as follows

$$G(x) = \int_0^{-\log(1-F(x))} r(t)dt \tag{1}$$

Where r(t) is the probability density function (pdf) of random variable T defined over $[0, \infty)$ and $F(x)$ is the cumulative distribution function of random variable X.

The probability density function for the continuous random variable X can be written as

$$g(x) = \frac{f(x)}{1-F(x)} r\left[-\log\left(1-F(x)\right)\right] \quad (2)$$

$$g(x) = h(x)r(H(x)) \quad (3)$$

$g(x)$ shows that the $T-X$ family of distributions has associated with the hazard functions and each generated distribution can be considered as a weighted hazard function of the random variable X.

If $T \sim \gamma(\alpha, \beta)$, then

$$r(t) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} t^{\alpha-1} e^{-\frac{t}{\beta}}; t \geq 0$$

Using (2), the pdf of gamma$-X$ family is

$$g(x) = \left(\Gamma(\alpha)\beta^{\alpha}\right)^{-1} f(x)\left[-\log\left(1-F(x)\right)\right]^{\alpha-1}\left[1-F(x)\right]^{\frac{1}{\beta}-1} \quad (4)$$

This paper is organised as follows. In section (1.1), we define the gamma-exponential distribution. Shannon entropy of the GED is given in section (2.1). Section (2.2),(2.3),(2.4) and (2.5) defines Renyi entropy, β-entropy, generalized entropy, Kullback-Leibler Divergence. In section (3.1),(3.2),(3.3),(3.4),(3.5),(3.6) we discuss some properties of the GED, including limit behaviour, moments,mean deviation, mode, moment generating function and characteristic function. Section 4 contains the method of maximum likelihood for estimating the parameters of gamma-exponential distribution. At last, we obtained Fisher'sinformation matrix for GED in section 5. Section 6 gives some brief concluding remarks.

1.1 The Gamma-Exponential Distribution (GED)

If X is a exponential random variable with density function

$$f(x) = \theta e^{-\theta x}; x > \theta \quad (5)$$

and cdf for exponential random variable is given by

$$F(x) = 1 - e^{-\theta x} \quad (6)$$

Using (5) and (6), then (4) results in

$$g(x) = \frac{\theta^{\alpha}}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1}(e^{-\theta x})^{\frac{1}{\beta}}; x > \theta, \alpha, \beta, \theta > 0 \quad (7)$$

Putting $\frac{\beta}{\theta} = c$, then (7) becomes

$$g(x) = \frac{1}{\Gamma(\alpha)c^{\alpha}} x^{\alpha-1} e^{-\frac{x}{c}}; \alpha, c > 0 \quad (8)$$

$g(x)$ follows the gamma-exponential distribution with parameters α and c. If $\alpha = 1$ in (8), the gamma-exponential distribution leads to the exponential distribution with parameters $\frac{1}{c}$ and if $c = 1$ it leads to the gamma distribution with parameter α.
The cdf of the gamma-exponential distribution results as

$$G(x) = \frac{1}{\Gamma(\alpha)}\gamma(\alpha, \theta x) \tag{9}$$

where $\gamma(\alpha, t) = \int_0^t u^{\alpha-1}e^{-u}du$ is the incomplete gamma function

The survival function of the GED is given by

$$R(x) = 1 - G(x) = 1 - \frac{\gamma(\alpha, \theta x)}{\Gamma(\alpha)}$$

and the hazard function of the GED can be obtained as

$$\begin{aligned} h(x) &= \frac{g(x)}{R(x)} \\ h(x) &= \frac{x^{\alpha-1}e^{-\frac{x}{c}}}{c^{\alpha}\Gamma(\alpha) - \gamma(\alpha, \theta x)} \end{aligned} \tag{10}$$

2 Information Measures

Information theory is a branch of probability with extensive potential applications to the communication systems. like several other branches of mathematics, information theory has physical origin. It was initiated by communication scientists C.E Shannon in 1948, who were studying the statistical structure of electrical communication equipments.

2.1 Shannon entropy

The Shannon entropy for the GED is given by the following theorem.
Theorem III: The Shannon entropy for GED is given by

$$\eta_x = \log c + \alpha + \log\Gamma(\alpha) + (1-\alpha)\psi(\alpha)$$

Proof: The Shannon entropy of the gamma$-X$ family of distributions is given by

$$\begin{aligned} \eta_x = & -E\left[\log f\left(F^{-1}\left(1-e^{-T}\right)\right)\right] + \alpha(1-\beta) + \log\beta + \\ & \log\Gamma(\alpha) + (1-\alpha)\psi(\alpha) \end{aligned} \tag{11}$$

where ψ is the digamma function and T is the gamma random variable with parameters α and β, using (5) and (6) in the expectation part of the above expression as

$$-E\left[\log f\left(F^{-1}\left(1-e^{-T}\right)\right)\right] = -\log\theta + E[T] \tag{12}$$

using $E(T) = \alpha\beta$ in (12), it reducse as

$$-E\left[\log f\left(F^{-1}\left(1-e^{-T}\right)\right)\right] = -\log\theta + \alpha\beta \tag{13}$$

substituting (13) in (12) and puttin $\frac{\beta}{\theta} = c$ the entropy for GED can be obtained as

$$\eta_x = \log c + \alpha + \log\Gamma(\alpha) + (1-\alpha)\psi(\alpha)$$

2.2 Generalized Entropy

Generalized entropy is often used in econometrics. It is indexed by a single parameter δ. The generalized entropy is defined as

$$I_\delta = \frac{v_\delta \mu^{-\delta} - 1}{\delta(\delta-1)}, \quad \text{where} \quad \delta \neq 0, 1 \tag{14}$$

$$v_\delta = E(x^\delta) = \int_0^\infty x^\delta \frac{1}{\Gamma(\alpha)c^\alpha} x^{\alpha-1} e^{-\frac{x}{c}} dx \tag{15}$$

Substituting $\frac{x}{c} = t$, (15), can be written as

$$v_\delta = \frac{c^\delta \Gamma(S+\alpha)}{\Gamma(\alpha)} \tag{16}$$

using (15) and (16), the generalized entropy in (14) reduces to

$$I_\delta = \frac{c^\delta \Gamma(\delta+\alpha) c^{-\delta^2} \Gamma^{-\delta}(1+\delta) - \Gamma(\alpha)}{\delta(\delta-1)\Gamma(\alpha)} \tag{17}$$

2.3 Renyi Entropy

The Renyi (1961) entropy for the random variable X with probability density function $g(x)$ is defined as

$$I_R(S) = \frac{1}{1-S} \log\left[\int g^S(x)dx\right], S > 0, S \neq 1 \tag{18}$$

By using the gamma-exponential probability density function in (8), we have

$$\int_0^\infty g^S(x)dx = \int_0^\infty \frac{1}{\Gamma^S(\alpha)c^{S\alpha}} x^{S(\alpha-1)} e^{-\frac{Sx}{c}} dx \tag{19}$$

Substituting $\frac{Sx}{c} = u$, (19) can be rewritten as

$$\int_0^\infty g^S(x)dx = \frac{c^{1-S}}{\Gamma^S(\alpha) S^{S(\alpha-1)+1}} \int_0^\infty e^{-u} u^{S(\alpha-1)} du$$

$$\int_0^\infty g^S(x)dx = \frac{c^{1-S}\Gamma S(\alpha-1)+1}{\Gamma^S(\alpha) S^{S(\alpha-1)+1}} \tag{20}$$

Using equation (20), the Renyi entropy in (18) can be written as

$$I_R(S) = \frac{1}{1-S}\log\left[\frac{c^{1-S}\Gamma S(\alpha-1)+1}{\Gamma^S(\alpha)S^{S(\alpha-1)+1}}\right]$$

$$\begin{aligned} I_R(S) &= \log c - \frac{1}{1-S}[S\log\Gamma(\alpha) + (S(\alpha-1)+1)\log S - \\ &\quad \log\Gamma(S(\alpha-1)+1)] \end{aligned} \tag{21}$$

Shannon entropy is a special case of Renyi entropy obtained by taking the limit of Renyi entropy as $s \longrightarrow 1$. The result in (14) follows by using the L'Hopital' rule for evaluating the limit equation (21) as $s \longrightarrow 1$.

2.4 β-Entropy

β-entropy is a one parameter generalization of the Shannon entropy. Applications of the β-entropy can be find in many physical systems. The β-entropy is defined by

$$H_\beta(g) = \frac{1}{\beta-1}\left[1 - \int g^\beta(x)dx\right], \quad \text{for} \quad \beta \neq 1 \tag{22}$$

$$\int_0^\infty g^\beta(x)dx = \int_0^\infty \frac{1}{\Gamma^\beta(\alpha)c^{\beta\alpha}} x^{\beta(\alpha-1)} e^{-\frac{Sx}{c}} dx$$

$$\int_0^\infty g^\beta(x)dx = \frac{c^{1-\beta}\Gamma(\beta(\alpha-1)+1)}{\Gamma^\beta(\alpha)\beta^{\beta(\alpha-1)+1}} \tag{23}$$

using (23) in (22), the β-entropy for GED can be written as

$$H_\beta(g) = \frac{1}{\beta-1}\left[1 - \frac{c^{1-\beta}\Gamma(\beta(\alpha-1)+1)}{\Gamma^\beta(\alpha)\beta^{\beta(\alpha-1)+1}}\right] \tag{24}$$

2.5 Kullback-Leibler Divergence

Kullback-Leibler divergence, between two probability distributions on a random variable is a measure of the distance between them. In case of discrete random variable the Kullback-Leibler divergence between two probability distributions is defined as

$$K(p||q) = \sum_{i=1}^n p_i \log\frac{p_i}{q_i}$$

and in continuous case

$$K(p||q) = \int p(x)\log\frac{p(x)}{q(x)}dx$$

Let $GED_o = GED(\alpha_o, c_o)$ be a given GED distribution, then the discrimination information function between GED and GED_o is given by

$$\begin{aligned} K(GED||GED_o) &= \int g(y)\log\frac{g(y)}{g_o(y)}dy \\ &= \int g(y)\log g(y)dy - \int g(y)\log g_o(y)dy \\ &= -H(g)+H(g_o) \end{aligned} \tag{25}$$

$$H(g) = -E\left[\log\left(\frac{1}{\Gamma(\alpha)c^{\alpha}}x^{\alpha-1}e^{-\frac{x}{c}}\right)\right]$$

$$H(g) = \log\Gamma(\alpha)+\alpha\log c-(\alpha-1)E(\log x)-\frac{1}{c}E(x) \tag{26}$$

$$\begin{aligned} E(\log x) &= \int_0^{\infty}\log x g(x)dx \\ &= \int_0^{\infty}\log x\frac{1}{\Gamma(\alpha)c^{\alpha}}x^{\alpha-1}e^{-\frac{x}{c}}dx \end{aligned}$$

$$E(\log x) = \frac{1}{c}(\log c+\psi(\alpha)), \quad \text{where} \quad \psi(\alpha) = \frac{\Gamma'(\alpha)}{\Gamma(\alpha)} \tag{27}$$

Using (27),(26) reduces to

$$H(g) = \log\Gamma(\alpha)+\alpha(\log c-1)-\frac{(\alpha-1)}{c}(\log c+\psi(\alpha)) \tag{28}$$

$$H(g_o) = \log\Gamma(\alpha_o)+\alpha_o(\log c_o-1)-\frac{(\alpha_o-1)}{c_o}(\log c_o+\psi(\alpha_o)) \tag{29}$$

Using (28) and (29) in (25), the following expression is given as

$$\begin{aligned} K(GED||GED_o) = \; & \log\frac{\Gamma(\alpha_o)}{\Gamma(\alpha)}+\alpha(\log c-1)+\alpha_o(\log c_o-1) \\ & -\frac{(\alpha-1)}{c}(\log c+\psi(\alpha)) \\ & -\frac{(\alpha_o-1)}{c_o}(\log c_o+\psi(\alpha_o)) \end{aligned} \tag{30}$$

3 Properties of the gamma-exponential distribution

Gamma-exponential distribution has some relations with other distributions. Using the transformation technique these relations can be obtained. Let Y be a random variable with parameters (α, c), then using the transformation $X = \theta e^{Y}$ probability density function of gamma exponential distribution is obtained.

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3.1 Limiting behaviour

The limiting behaviours of the gamma-exponential PDF is given in the following theorem.
Theorem I: The limit of the gamma-exponential density function is zero when x goes to infinity, and when $x \longrightarrow 0$, the limit is given by

$$\lim_{x\to 0} g(x) = \lim_{x\to 0} h(x) = \begin{cases} 0, & \alpha > 1 \\ \frac{1}{c}, & \alpha = 1 \\ \infty, & \alpha < 1 \end{cases}$$

Proof:

$$\lim_{x\to\infty} g(x) = \frac{1}{\Gamma(\alpha)c^{\alpha}} \lim_{x\to\infty} \frac{x^{\alpha-1}}{e^{\frac{x}{c}}}$$

Applying L, Hopital׳s rule, it reduces to

$$\lim_{x\to\infty} g(x) = \frac{\alpha-1}{\infty} = 0 \tag{31}$$

If $\alpha > 1$, then, (31) goes to zero, if $\alpha < 1$, it goes to infinity and if $\alpha = 1$, it reduces to $\frac{1}{c}$.
Theorem II: The limit of the hazard function for the gamma-exponential distribution when x goes to infinity is given by:

$$\lim_{x\to\infty} h(x) = \begin{cases} 0, & \alpha > 1 \\ \frac{1}{c}, & \alpha = 1 \\ \infty, & \alpha < 1 \end{cases}$$

Proof: we have

$$\begin{aligned} \lim_{x\to\infty} h(x) &= \lim_{x\to\infty} \frac{g(x)}{R(x)} \\ &= \lim_{x\to\infty} \frac{g(x)}{1-G(x)} \end{aligned}$$

Since $\lim_{x\to\infty} g(x) = 0$, it can be shown that $\lim_{x\to\infty} h(x) = 0$ by using L, Hopital׳s rule.

3.2 Moments of GED

The s^{th} non-central moments of GED are given by

$$E(X^s) = \int_0^{\infty} x^s \frac{1}{\Gamma(\alpha)c^{\alpha}} x^{s-1} e^{-\frac{x}{c}} dx$$

Letting $\frac{x}{c} = t$, then

$$E(X^s) = \frac{c^s \Gamma(s+\alpha)}{\Gamma(\alpha)} \tag{32}$$

$E(X) = \mu = \alpha c$ is the mean of the gamma-exponential distribution.

$$E(X-\mu)^s = \sum_{k=0}^{s} \binom{s}{k} (-1)^{s-k} \mu^{s-k} E(X^k) \tag{33}$$

The central moments shown by Alzaatreh et al (2012), expresses that for any random variable, it can be shown as Using (32) and (33), the central moments for the gamma-exponential random variable X can be obtained as

$$E(X-\mu)^s = c^s \sum_{k=0}^{s} \binom{s}{k} (-1)^{s-k} \alpha^{s-k} \frac{\Gamma(k+\alpha)}{\Gamma(\alpha)} \tag{34}$$

Using (34), the variance and the coefficient of variation (CV) of GED is given as

$$\sigma^2 = \alpha c^2$$

$$CV = \frac{1}{\alpha^{\frac{1}{2}}}$$

$$\gamma_1 = \frac{4\alpha^2 - \alpha - 1}{\alpha^{\frac{1}{2}}} \tag{35}$$

$$\begin{aligned} \gamma_2 \quad = \quad & \frac{1}{\alpha} \times [(\alpha+1)(\alpha+2)(\alpha+3) - 4(\alpha+1)(\alpha+2) + \\ & 6\alpha^2(\alpha+1) - 3\alpha^3] \end{aligned} \tag{36}$$

The above equations (35) and (36), are the skewness and kurtosis of the gamma-exponential distribution.

3.3 Mean Deviations of GED

By the definition

$$D(\mu) = \mu - 2\int_0^M xg(x)dx \tag{37}$$

$$D(M) = \mu - 2\int_0^M (M-x)g(x)dx \tag{38}$$

Now the integral

$$\int_0^m xg(x)dx = \int_0^m \frac{1}{c^\alpha \Gamma(\alpha)} x^\alpha e^{-\frac{x}{c}} dx \tag{39}$$

Putting $\frac{x}{c} = w$, the expression reduces to

$$\int_0^m xg(x)dx = \frac{\mu}{\Gamma(\alpha)} \gamma\left(\alpha+1, \frac{m}{c}\right) \tag{40}$$

$$\text{and} \quad \int_0^M xg(x)dx \quad = \quad \frac{\mu}{\Gamma(\alpha)} \gamma\left(\alpha+1, \frac{M}{c}\right) \tag{41}$$

Using (9), (40) and (41) in (37) and (38) the mean deviation from the mean and the mean deviation from the median for the gamma-exponential distribution can be written as

$$D(\mu) = \frac{2\mu\left[\gamma(\alpha, \frac{\mu}{c}) - \gamma\left(\alpha+1, \frac{\mu}{c}\right)\right]}{\Gamma(\alpha)}$$

$$D(M) = \mu\left[1 - \frac{2}{\Gamma(\alpha)}\gamma\left(\alpha+1, \frac{M}{c}\right)\right]$$

3.4 Mode of GED

The log of gamma-exponential distribution in (8) is as follows

$$\log g(x) = -\log\Gamma(\alpha) - \alpha\log c + (\alpha-1)\log x - \frac{x}{c} \tag{42}$$

Differentiating (42) with respect to x, the following expression is obtained

$$\frac{\partial}{\partial x}\log g(x) = \frac{c(\alpha-1)-x}{x}$$

Now equating $\frac{\partial}{\partial x}\log g(x) = 0$ implies

$$x = c(\alpha-1), \quad \text{where} \quad \alpha > 1 \tag{43}$$

This is the mode of GED when $\alpha > 1$.

3.5 Moment generating function of GED

$$\begin{aligned} M_x(t) &= E(e^{tx}) = \int_0^\infty e^{tx} g(x)dx \\ &= \frac{1}{c^\alpha\Gamma(\alpha)}\int_0^\infty x^{\alpha-1}\exp\left(-x\left(\frac{1}{c}-t\right)\right)dx \end{aligned} \tag{44}$$

Let $x\left(\frac{1}{c}-t\right) = z$, the integral in (44), can be written as

$$M_x(t) = \frac{\Gamma(z)}{\Gamma(\alpha)c^\alpha\left(\frac{1}{c}-t\right)^\alpha} \tag{45}$$

Equation (45), is moment generating function for gamma-exponential distribution.

3.6 Characteristic function of GED

$$\begin{aligned} \phi_x(t) &= E(e^{itx}) = \int_0^\infty e^{itx} g(x)dx \\ &= \frac{1}{c^\alpha\Gamma(\alpha)}\int_0^\infty x^{\alpha-1}\exp\left(-x\left(\frac{1}{c}-it\right)\right)dx \end{aligned} \tag{46}$$

Let $x\left(\frac{1}{c}-it\right)=z$, the integral in (46), can be written as

$$\phi_x(t) = \frac{\Gamma(z)}{\Gamma(\alpha)c^{\alpha}\left(\frac{1}{c}-it\right)^{\alpha}} \tag{47}$$

Equation (47), is characteristic function for gamma-exponential distribution.

4 Parameter estimation for GED

The likelihood function of (8) is

$$L(\alpha,c)=\left[\frac{1}{\Gamma(\alpha)c^{\alpha}}\right]^{n}\prod_{i=1}^{n}x_i^{\alpha-1}\exp\left(-\frac{\sum_{i=1}^{n}x_i}{c}\right) \tag{48}$$

and the log-likelihood function of (48) is given by

$$\log L(\alpha,c) = -n\log\Gamma(\alpha)-n\alpha\log c+(\alpha-1)\sum_{i=1}^{n}\log x_i - \frac{\sum_{i=1}^{n}x_i}{c} \tag{49}$$

Differentiating (49), with respect to α and c gives

$$\frac{\partial\log L(\alpha,c)}{\partial\alpha} = -n\psi(\alpha)-n\log c+\sum_{i=1}^{n}x_i \tag{50}$$

where $\frac{\Gamma'(\alpha)}{\Gamma(\alpha)}=\psi(\alpha)$ is the digamma function.

$$\frac{\partial\log L(\alpha,c)}{\partial c} = -\frac{n\alpha}{c}+\frac{\sum_{i=1}^{n}x_i}{c^2} \tag{51}$$

Setting (50) and (51) to zero, we obtain the following MLE of $\hat{\alpha}$ and $\hat{c}$

$$\psi(\hat{\alpha})=-\log\hat{c}+\frac{\sum_{i=1}^{n}x_i}{n}$$

$$\hat{c}=\frac{\bar{x}}{\hat{\alpha}}$$

5 Fishers information matrix of gamma-exponential distribution

Appling log on both sides in (8), we have

$$\log g(\alpha,c)=-\alpha\log c-\log\Gamma(\alpha)+(\alpha-1)\log x-\frac{x}{c} \tag{52}$$

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Differentiating (52), with respect to α and c, we get

$$\frac{\partial \log g(\alpha, c)}{\partial \alpha} = -\log c - \psi(\alpha) + \log x$$

where $\psi(\alpha) = \frac{\acute{\Gamma}(\alpha)}{\Gamma(\alpha)}$ is the digamma function

$$\frac{\partial^2 \log g(\alpha, c)}{\partial \alpha^2} = (\psi(\alpha))^2 - \psi(\acute{\alpha})$$

$$\frac{\partial}{\partial c} = -\frac{\alpha}{c} + \frac{x}{c^2}$$

$$\frac{\partial}{\partial \alpha \partial c} = -\frac{1}{c}$$

$$\frac{\partial^2}{\partial c^2} = \frac{\alpha}{c^2} - \frac{2x}{c^3}$$

Taking expectations on both sides of the equations, we get

$$I(1,1) = -E\left[\frac{\partial^2 \log g(\alpha, c)}{\partial \alpha^2}\right] = \psi(\acute{\alpha}) - (\psi(\alpha))^2$$

$$I(1,2) = -E\left[\frac{\partial^2 \log g(\alpha, c)}{\partial c \partial \alpha}\right] = \frac{1}{c}$$

$$I(2,1) = -E\left[\frac{\partial^2 \log g(\alpha, c)}{\partial \alpha \partial c}\right] = \frac{1}{c}$$

$$I(2,2) = -E\left[\frac{\partial^2 \log g(\alpha, c)}{\partial c^2}\right] = \frac{\alpha}{c^2}$$

Now, the Fishers information matrix of gamma-exponential distribution is given by

$$I(\alpha, c) = \begin{bmatrix} \psi(\acute{\alpha}) - (\psi(\alpha))^2 & \frac{1}{c} \\ \frac{1}{c} & \frac{\alpha}{c^2} \end{bmatrix}$$

6 Concluding remarks

In this paper, a special case of gamma-X family, the gamma exponential distribution is derived. The gamma exponential distribution is a generalization of exponential distribution. In general, the gamma exponential distribution is a generalization of the X distribution. The gamma exponential distribution can be over-dispersed, equi-dispersed or under-dispersed as well as left skewed, right skewed or symmetric. If the data is skewed, one should fit a gamma exponential distribution instead of an exponential distribution. We presented some information properties of gamma exponential distribution. It is hoped that the findings of the paper will be useful for the practitioners in various fields of theoretical and applied sciences.

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