

IDENTIFICATION OF ISOMORPHISM USING DIFFERENT KINEMATIC PAIRS

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ABSTRACT

The establishment of the method for generalizing the kinematic chains having same number of links but different kinematic pairs. By using link-link form of the Kinematic Pairs [KP] and square kinematic pairs matrix or [SKPM] Matrix and elements of the matrix was chosen as one and zero depending on the absence or presence of a direct kinematic connection in between the joints, we found that kinematic chains having same number of link and different kinematic pair are isomorphic but in real practice it should be different. To overcome this problem we are trying to develop a different set of matrix called kinematic pair matrix.

KEYWORD: *Isomorphism, Kinematic Chain, Structural Invariants, Kinematic Pair Matrix*

I INTRODUCTION

Recently people have done several attempts to develop different method to find out isomorphism among two kinematic chains. Mohammad, A. and Agrawal, V. P [1], performed of identify the different mechanisms and they detect the isomorphism among the chain having different kinematic pairs. Ashok Kumar Sharma and Arvind Kumar Shukla [2], use kinematic chains which is represent by matrices and elements of matrices that is used for detection of isomorphism among the kinematic chains, they used chains of one degree of freedom ,six links and eight links. Uicker, J. J. and Raicu, A [3], described a method for the identification and recognition of equivalence of kinematic chains and used to testing the equivalence of kinematic chains using different kinematic pairs. Mruthyunjaya, T. S. and Raghavan, M. R.[4], presented the Computer aided analysis of the structural synthesis of kinematic chains, and they used the link link incidence matrix to represented the simple jointed kinematic chains ,procedures have produced to determine its structural characteristics such as degree of freedom, chains, different mechanisms. Nageswara Rao, C. and Rao, A. C. [5], preformed the Selection of best frame, input and output links for function generators modelled as probabilistic system, they identify to replace the 0 or 1 in the given known matrix by the distances motion have to flow in the chains from the one vertex to other. Rao, A. C. [6], performed the Kinematic chains, isomorphism, inversion and types of degree of freedom, using the concept of Hamming distances. Ambekar,

A. G. and Agrawal, V. P. [7], suggest the method to identify the mechanisms, path generators and the function generators through the set of identification numbers. Ambekar, A. G. and Agrawal, V. P. [8], performed the Identification and classifications of kinematic chains and their mechanisms used the identification codes. Shende, S. and Rao, A.C. [9], identify the problems in the detection of isomorphism which encountered in the structural synthesis of kinematic chains. Manoj Kumar Lohumi, Aas Mohammad and Irshad Ahmad Khan [10], performed the hierarchical clustering algorithm which developed for the identification of distinct mechanisms. J. N. YADAV and C. R. Pratap, V. P. Agrawal, [11], given the method for detecting the isomorphism among the kinematic chains with the help of simple joints used new invariants known as arranged sequence of the modified total distance ranks of all links, ASMTDRL. Ashok Dargar, Ali Hasan, Rasheed Ahmed Khan,[12], identify the distinct mechanisms derived from the given kinematic chain. these presented in the form of a flow matrix. Zichos, H. [13], proposed the tribology a system approach to science and technology of friction, lubrication and wear. Czichos, H. [14], identify the System approach to the analysis of wear problems. Gandhi, O. P. and Agrawal, V. P. [15], developed a diagraph approach to system wear evaluation and analysis. Hsiung, C. Y. and Mao, G. Y [16], used linear algebra concept for detecting the isomorphism. Mohammad, R. A. Khan and V. P. Agrawal, [17], performed the Identification of kinematic chains and distinct mechanisms using extended adjacency matrix. To identify the isomorphism among the kinematic chains and having different kinematic pairs and using graph theory [18] to [24]. In this paper we developed a new matrix called Kinematic pair Matrix to identify the isomorphism of kinematic chains and mechanism having different type of kinematic pairs.

During development of kinematic chain and mechanisms there is chances of duplication. To identify this duplication researchers have proposed various methods.

These methods were based on adjacency matrix and the distance matrix.

II PROPOSED METHODOLOGY

2.1. Structural Invariant of the mechanism

KP matrix is the unique representation of the mechanism having different types of kinematic pairs. All information related to the types of links and types of kinematic pairs existing in the mechanism is incorporated in kinematic pair matrix. Therefore its determinant of kinematic pair matrix may be taken as **structural invariant** of the kinematic pair matrix and in turns may be treated as structural invariants of the particular kinematic chain or mechanism. This invariant may be used to detect isomorphism among the planer kinematic chain and mechanism. Therefore structural invariant is written as determinant KP (Det. KP).

[KP] Matrix is the representation of kinematic chains and mechanisms, hence identification number is obtained from this matrix is also the representation or characteristics number of this chain.

Identification Number: The identification number is the number which is desired sum of the [KP] Matrix.

Determinant [KP] = Invariants of KC or mechanisms

The proposed method is used to detect the isomorphism using kinematic pair matrix and its invariants.

2.2 The Kinematic Pair [KP] Matrix

The KP matrix representation may be given as,

$$[KP] = \{P_{ij}\};$$

Where P_{ij} = Square of the type of KP elements between the i^{th} and j^{th} link those are directly connected.

Otherwise $P_{ij} = 0$; those are not connected directly

Designation of the kinematic pairs:

Lower pair : 1.0

Higher pair : 2.0

Further classification of kinematic pairs:

For Lower Pair:

Turning pair : 1.1;

Sliding pair : 1.2;

Screw pair : 1.3;

For Higher Pair:

Point Contact : 2.1;

Line Contact : 2.2;

Representation of [KP] Matrix:

$$KP = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \dots & P_{1n} \\ P_{21} & P_{22} & P_{23} & \dots & P_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ P_{n1} & P_{n2} & P_{n3} & \dots & P_{nn} \end{bmatrix}$$

2.3 Representation of SQUARE KINEMATIC PAIR

MATRIX [SKPM]

$$[SKPM] = \{P_{ij}\}$$

Where, P_{ij} = Square of the sum of the joint value in [SKPM] between i^{th} and j^{th} link those are connected directly (i.e., i is not equal to j).

Otherwise: $P_{ij} = 0$ i.e. those are not connected directly.

P_{ii} = Square of the degree of the i^{th} link ($i = j$)

Off course: $P_{ij} = 0$

The Square Joint Value Matrix [SJVM] matrix is represented as:

$$[SKPM] = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} & \cdot & \cdot & \cdot & P_{1n} \\ P_{21} & P_{22} & P_{23} & P_{24} & \cdot & \cdot & \cdot & P_{2n} \\ P_{31} & P_{32} & P_{33} & P_{34} & \cdot & \cdot & \cdot & P_{3n} \\ P_{41} & P_{42} & P_{43} & P_{44} & \cdot & \cdot & \cdot & P_{4n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ P_{n1} & P_{n2} & P_{n3} & P_{n4} & \cdot & \cdot & \cdot & P_{nn} \end{bmatrix}$$

EXAMPLE 1 (8 – bar Kinematic Chain Having Simple Joints)

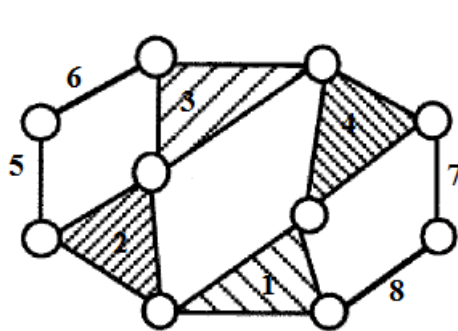


Figure 1 (a): Kinematic Chain

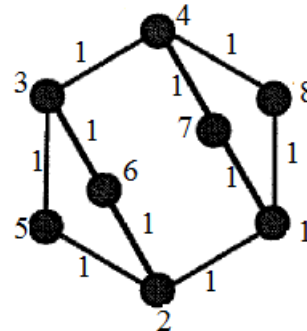


Figure 1 (b): Kinematic Graph

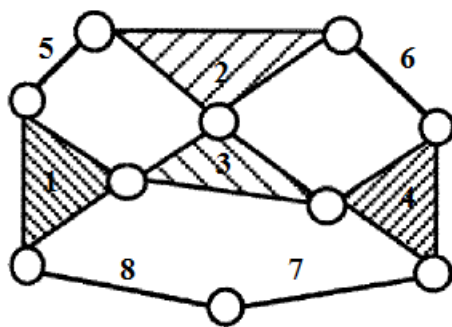


Figure 1 (c): Kinematic Chain

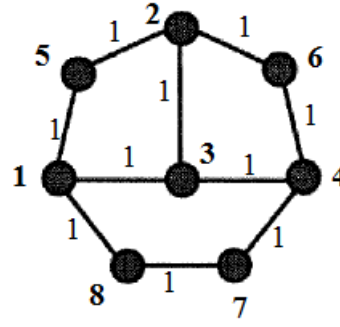


Figure 1 (d): Kinematic Graph

Figure 1 : 8 – links, 10 – joints, 1–F Kinematic Chains and their Kinematic Graphs

To identify the kinematic chain, let us consider a configuration of 8 – links, 10 – joints, 1 – F Kinematic chain and its kinematic graphs as shown in fig. 1 (a) and 1 (b) respectively. The all joints of the kinematic chain are simple. Therefore, each edge of the kinematic graph connecting i^{th} vertex to j^{th} vertex is assigned with the joint value 1.

The KP representing the kinematic graph (Fig. 1 b) using equation 1 is written as:

$$[\text{SKPM}] = \{P_{ij}\}$$

$$[\text{SKPM}] = [A_1] = \begin{bmatrix} 9 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 9 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 9 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 9 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 4 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

Structural Invariant of the Kinematic Chain

The structural invariant from [SKPM] matrix using software MATLAB for the kinematic chain shown in fig 1 (a) is:

$$\det (A_1) = 123500$$

The structural invariant is used as the identification number of the kinematic chain shown in fig.1(a).

EXAMPLE 1 (8 – bar Kinematic Chain Having Simple Joints)

To identify the kinematic chain, let us consider a configuration of 8 – links, 10 – joints, 1 – F Kinematic chain and its kinematic graphs as shown in fig. 1 (c) and 1(d) respectively. The all joints of the kinematic chain are simple.

Therefore, each edge of the kinematic graph connecting i^{th} vertex to j^{th} vertex is assigned with the joint value 1.

The Square Joint Value Matrix representing the kinematic graph (Fig. 1c) using equation 1 is written as:

$$[\text{SKPM}] = [A_4] = \begin{bmatrix} 9 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 9 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 9 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 9 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 4 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

Structural Invariant of the Kinematic Chain

The structural invariant from [SKPM] matrix using software MATLAB for the kinematic chain shown in fig 1 (c) is:

$$\det (A_4) = 1265300$$

The structural invariant is used as the identification number of the kinematic chain shown in fig.1(c).

The structural invariants of the above two kinematic graphs shown in fig. 1 (b) and 1(d) are different, therefore both the graphs and in turn the kinematic chains shown in figure 1 (a) and 1 (c) are **non – isomorphic**.

III RESULTS

This method is able to clearly identify the isomorphism among the kinematic chains or mechanisms having different kinematic pairs.

The proposed invariants are used as the identification number of the kinematic chains having simple joints. The identification numbers of all 1 – F, 8 – links . These invariants are also able to detect the isomorphism among the kinematic chains having simple joints, multiple joints and even kinematic chains with Co-Spectral graphs.

IV CONCLUSIONS

In the present work the usual work adjacency matrix is modified and is called KP in which the element P_{ij} are the square of the sum of the joint value of the i^{th} and j^{th} links those are directly connected, and maintain diagonally as 1, otherwise $P_{ij} = 0$. This [KP] matrix is able to distinguish the type of the joint between the two links. The structural invariants are able to detect the isomorphism among the kinematic chains having simple joints, multiple joints and even kinematic chains with Co-Spectral graphs. No counter example has been found to to detect the isomorphism in the kinematic chains up to 10 – links. It has been expected that these invariants are unique for the kinematic chains having more than 10 – links.

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