International Journal of Advance Research in Science and Engineering
Volume No.06, Special Issue No.(02), December 2017
www.ijarse.com
IJARSE
ISSN: 2319-8354

Stability Analysis of Job Shop Problem

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ABSTRACT

The job-shop problem (JSP) is an optimization technique, in which ideal jobs are assigned to resources at particular times. Practical view of deterministic scheduling process is not valid for every process in practice. In present study, stability analysis has been performed and to test the suitable techniques of optimization which will be applicable for job-shop problem also discussed how parametric relation are affected for two jobs. Comparative study of some existing techniques with present study is also discussed in this paper.

Keyword - ***Comparative study, Job-shop problem, Optimization, Parametric- relation, Stability analysis***

I INTRODUCTION

The problem under consideration is to minimize the value of the given desired function of completion times of n jobs J = {1, 2, …,n} processed on m machines N = {1, 2, …, m}. First, we assume that processing time $t_{j,k}$ of job j ∈ J on machine k ∈ N (i.e., processing time of operation $O_{j,k}$) is known before scheduling.

Operation preemptions are not allowed. This problem is denoted as J||φ where φ desired objective function. Let $C_{i,k}$ denote the completion time of the job in position i on machine k ∈ N. We assume that desired function φ ($C_{1,m,}$ $C_{2,m,}$ $_{\dots,}$ $C_{n,m}$) is non-decreasing function of job completion times. Such a criterion is called regular.

For the job-shop problem J |n=1|Cmax with two jobs and make span desired function C_{max} = max{C1,m, $C_{2,m,}$ …,$C_{n,m}$}, the geometric algorithm was proposed by Akers and Friedman [1] and developed by Brucker [2], Szwarc [7], Hardgrave and Nemhauser [4]. Sotskov [5] generalized the geometric algorithm for the problem J|n=1| φ with any given regular criterion. Sotskov [6] proven that both problems:

$$J\ |n=1|C_{max} \text{ and } J\ |n=2|\Sigma C_{i,m} \qquad (1)$$

are binary NP-hard. Hereafter, the criterion Σ $C_{i,m}$ means minimization of total completion time

$$\sum_1^n C_{i,m} \qquad 2.\ 2.$$

II METHODOLOGY

Describing geometric model for the case of a flow-shop problem $J|n=1|\varphi$, i.e., when all n jobs have the same technological through m machines, namely, (1, 2, …,m).

Let $TM_{j,k}$ denote the sum of the processing times of job $j \in J = \{1, 2\}$ on a subset of k machines $\{1, 2, ..., k\} \subseteq N$:

$$TM_{j,k} = \sum_{i}^{k} t_{j,i} \qquad 1 \le k \le m \qquad (3)$$

Assuming that $TM_{1,0} = TM_{2,0} = 0$. Introducing a coordinate system xy on the plane, and draw the rectangle with corners (0, 0), ($TM_{1,m}$, 0), (0, $TM_{2,m}$) and ($TM_{1,m}$, $TM_{2,m}$). In the rectangle , we draw m rectangles H_k, $k \in \{1, 2, …,m\}$, with corners ($TM_{1,k-1}$, $TM_{2,k-1}$), ($TM_{1,k}$, $TM_{2,k-1}$), ($TM_{1,k-1}$, $TM_{2,k}$), ($TM_{1,k}$, $TM_{2,k}$).

South-west corner ($TM_{1,k-1}$, $TM_{2,k-1}$) of the rectangle H_k as SW_k, north-west corner ($TM_{1,k-1}$, $TM_{2,k}$) as NW_k, south east corner ($TM_{1,k}$, $TM_{2,k-1}$) as SE_k, and north-east corner ($TM_{1,k}$, $TM_{2,k}$) as NE_k. Obviously, point (0, 0) is SW_1 and point ($TM_{1,m}$, $TM_{2,m}$) is NE_m.

Using Chebyshev's metric, i.e., the length d [(x, y), (x', y')] of a segment [(x, y), (x', y')] connecting points (x,y) and (x', y') in the rectangle H is calculated as follows:

$$D [(x, y), (x', y')] = \max \{|x - x'|, |y - y'|\}. \qquad (4)$$

The length $D[(x_1, y_1), (x_2, y_2), …, (x_r, y_r)]$ of a continuous polygonal line $[(x_1, y_1), (x_2, y_2), …, (x_r,y_r)]$ is equal to the sum of the lengths of its segments. Since φ ($C_{1,m}$, $C_{2,m}$) is a increasing function, the search for the optimal schedule can be restricted to set S of schedules in which at any time of the interval [0, max{$C_{1,m}$, $C_{2,m}$}] at least one job is processed. A schedule from set S can be suitably represented within the rectangle H on the plane xy as a trajectory (Continuous polygonal line) $\tau = [SW_1, (x_1, y_1), (x_2, y_2) , …, (x_r, y_r), NE_m]$ where either $x_r = TM_{1,m}$ or $y_r = TM_{2,m}$. Let a point (x, y) belong to the trajectory τ and let d be the length of the part of trajectory τ from the point SW1 to the point (x, y). The coordinate x (coordinate y) of point (x, y) defines the state of processing job 1 (job 2) as follows.

If $SWu \le x \le SEu$ and $SWv \le y \le NWv$, $u \in M$, $v \in M$, then job 1 (job 2) is completed on the machines 1, 2,…, u-1 (on the machines 1, 2,…, v-1) at time d. Moreover at time d, job 1 (job 2) has been processed on machine u (machine v) during x - SWu (during y - SWv) time units.

Since a machine cannot process more than one job at a time and operation preemptions are not allowed, each straight segment [(x, y), (x', y')] of a trajectory τ may be either

- Horizontal (when only job 1 is processed) or
- Vertical (when only job 2 is processed) or
- Diagonal with slope of 450 (when both jobs are processed simultaneously).

IJARSE
ISSN: 2319-8354

It is clear that a horizontal segment (vertical segment) can only pass along south boundary (west boundary) of the rectangle Hk, k ∈ M, or along north (east) boundary of the rectangle H. The diagonal segment of trajectory τ can only pass either outside rectangle Hk or through point NWk or point SEk. Sotskov [5] proven that problem J|n=1|Φ of finding the optimal schedule or, in other words, of finding the optimal trajectory, can be reduced to the shortest path problem in the digraph (V, A) constructed by the following Algorithm 1. Again for simplicity, we describe this algorithm for the case of a flow-shop problem F |n=1| φ, when all n jobs have the same technological route through m machines.

Vertex set V of the digraph (V, A) is a subset of set

$V_0 = \{SW_1, NE_m\} \cup \{NW_k, SE_k : k \in M\} \cup \{(x_k, TM_{2,m}), (TM_{1,m}, y_k) : k \in M\}$.

III ALGORITHM

1. Set $V = \{SW_1, SE_1, NW_1, NE_m\}$ and $A = \{(SW_1, SE_1), (SW_1, NW_1)\}$.
2. Take vertex $(x, y) \in V \setminus \{NE_m\}$ with zero out degree. If $(x, y) = SE_k$, go to step 3. If $(x, y) = NW_k$, go to step
3. If set $V \setminus \{NE_m\}$ has no vertex with zero out degree.
   STOP
4. Draw a diagonal line with slop 450 starting from vertex SE_k until either east boundary $[(TM_{1,m}, 0), NE_m]$ of the rectangle H is reached in some vertex $(TM_{1,m}, y_k)$ or open south boundary (SW_h, SE_h) of the rectangle H_h, $k+1 \leq h \leq m$, is reached. In the former case, set $V: =V \cup \{(TM_{1,m}, y_k)\}$ and $A: =A \cup \{(SE_k, (TM_{1,m}, y_k)), ((TM_{1,m}, y_k), NE_m)\}$. In the latter case, set $V: =V \cup \{SE_h, NW_h\}$ and $A: =A \cup \{(SE_k, SE_h), (SE_k, NW_h)\}$. Go to step 2.
5. Draw a diagonal line with slope 450 starting from vertex NW_k until either north boundary $[(0, TM_{2,m}), NE_m]$ of the rectangle H is reached in some vertex $(x_k, TM_{2,m})$ or open west boundary (SW_h, NW_h) of the rectangle Hh, $k+1 \leq h \leq m$, is reached. In the former case, set $V: = V \cup \{(x_k, TM_{2,m})\}$ and $A: = A \cup \{(NW_k, (x_k, TM_{2,m})), ((x_k, TM_{2,m}), NE_m)\}$. In the latter case, set $V: = V \cup \{SE_h, NW_h\}$ and $A: = A \cup \{(NW_k, SE_h), (NW_k, NW_h)\}$. Go to step 2.

In order to find the optimal path (i.e., optimal schedule) for the problem J|n=1|Φ we can use the following Algorithm, where the length of arc ((x, y), (x’, y’)) ∈ A is assumed to be equal to the length of the polygonal Line constructed by Algorithm with origin in the point (x, y) and with end in the point (x’, y’).

IV STABILITY ANALYSIS

In what follows, we consider stability of an optimal schedule with respect to possible variations of the given vector $t = (t_{1,1}, t_{1,2}, \dots, t_{1,m}, t_{2,1}, t_{2,2}, \dots, t_{2,m})$ of operation processing times.

Let (V_t, A_t) denote the digraph (V, A) constructed by Algorithm 1 for the problem $F|n=2|\varphi$ with vector t of operation processing times. Let P_t be set of all shortest paths from vertex SW1 to the border vertices in the digraph (V_t, A_t). As follows from Algorithm 1, the same path may belong to sets Pt constructed for different vectors t of operation processing times (since for any vector t we have Vt ⊆ V0). Notation su(t) is used for a schedule defined by path $\tau_u \in P_t$. The objective function value calculated for schedule su(t) is denoted as φ (su(t)). A schedule is called *active* if none of the operations can start earlier than in this schedule, provided that the remaining operations could start no later. It is known (see Giffler and Thompson [3]) that a set of active schedules is dominant (i.e., it contains at least one optimal schedule) for any regular criterion. The following claim may be proven by induction with respect to number of machines m.

To test whether optimality of the path τu ∈ Pt is stable takes O(m log m) time for problem F |n=1|Φ and $O(m_2 \log m)$ time for problem J $|n=1|\varphi$. Indeed, we can use Algorithm 2 for the vector t of the operation processing times and construct optimal paths with different border vertices. Number of the optimal paths which have to be tested due to theorem is restricted by the number of border vertices asymptotically restricted by O(m) for problem F $|n=1|\varphi$ and by $O(m_2)$ for problem J $|n=1|\varphi$.

It is easy to convince that for the above sufficiency proof of Theorem 2 we can replace increasing function φ by non-decreasing function φ. It should be noted that the most objective functions considered in classical scheduling theory are continuous non-decreasing functions of job completion times, e.g.,

- Make span C_{max},
- Total completion time $\sum_1^n C_{i,m}$
- Maximal lateness $L_{max} = \max\{ C_{i,m} - D_i : i \in J\}$ and
- Total tardiness = $\sum_1^n max\{0, C_{i,m} - D_i : i\epsilon J\}$ where Di denotes the given due date for a job i.

And so sufficiency of applicable theorem may be violated in the break points of such a function φ.

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