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COMPUTATION OF THE RADIATION RESISTANCE OF ONE AND A HALF WAVE DIPOLE ANTENNA

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ABSTRACT

This paper presents a formal derivation of the radiation resistance of a one and a half wave dipole antenna. The one and a half wave dipole antenna is a thin linear antenna that finds use in wireless communications domain. It is so called because the wire that is used practically is considered to be thin such that tangential currents are neglected and the current can be considered as only linear. For the derivation, it is also assumed that the current distribution is sinusoidal. The boundary conditions of the current are that the currents are zero at the two ends and maximum at the center. A detailed mathematical analysis is presented that computes the value for the radiation resistance. The same approach can be followed for deriving the expression for the radiation resistance of any other thin linear antenna.

Keywords—antenna, integral, magnetic field, resistance, vector

I.INTRODUCTION

Thin linear antennas are antennas which are small in length and size when compared to other antennas that are used in practice. They come in handy for wireless applications. They have certain advantages like they are inexpensive and their installation is also not extortionate. Hence, these versatile antennas are the workhorses of wireless mobile applications. It is vital to understand the mathematical analysis behind these antennas in order to appreciate their working practically [1]. Half – wave ( ), Full – wave ( ), and, three – half wave ( ) dipole antennas are examples of thin - linear antennas. The radiation resistance of the dipole antenna is presented in the next section.

II.COMPUTATION OF THE RADIATION RESISTANCE

The geometry of symmetrical, thin, linear, center – fed antenna of length is considered in Fig. 1. It is assumed that the antennas are symmetrically fed at the center by a balanced two – wire transmission line [2]. The current distribution is sinusoidal in nature. We intend to compute the expressions for the far field of the thin linear antenna. The antenna is oriented along the axis and the plane waves are incident on the antenna in the negative direction of the axis [3]. The antenna field measurements are done w.r.t a distant point and it is assumed that the antenna is operating in the far field or the Fraunhofer diffraction zone [9]. Origin is taken to be the reference. Consider an infinitesimal dipole placed on the axis. The distant point is at a distance from the origin. Similarly, it is at a distance from the infinitesimal dipole . The center of the

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dipole makes an angle of with the line of length that connects the origin and the distant point . Here, retarded current is considered [4].

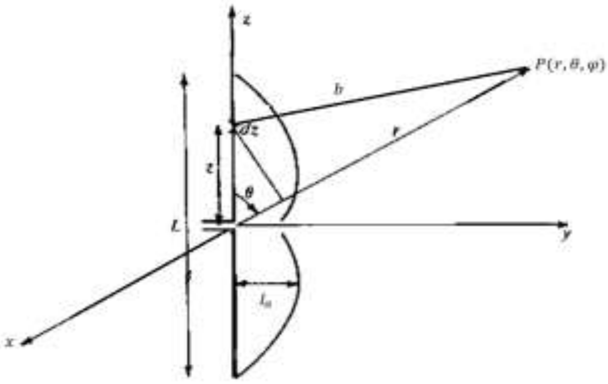

Fig. 1. Geometry of thin, linear antenna of length .

Hence, the retarded value of the current at any point on the antenna corresponding to the point at a distance is given by:

(1)

Since phase constant, , (1) can be written as:

(2)

The terms and indicate retardation time, and form factor for the current on the antenna respectively [5]. For the case of , the term is used. Similarly, for the case of , the term is used. The thin linear antenna can be treated as being consisting of a series of infinitesimal dipoles of length . The expressions for the fields of a thin linear antenna are well established and found in the literature and they are written directly here [3]. The magnetic field intensity (component of the thin, linear antenna is given by:

$$H_{\varphi} = \quad (3)$$

The electric field intensity is related to the magnetic field intensity by the well-known relation [10]:

(4)

Substituting (15) in (16), we obtain:

$$E_{\theta} = \frac{j60}{} \quad (5)$$

Equation (5) gives the electric field intensity (component of the thin, linear antenna in or . Thus, the equations (3) and (4) give the expressions for the far field corresponding to a thin, linear antenna [11].

The average Poynting vector is given by the expression:

$$\boldsymbol{W_{av}} = \frac{1}{2} Re[\boldsymbol{E} \text{x} \boldsymbol{H}^{*}] = \frac{1}{2} Re\left[120\pi H_{\varphi} \boldsymbol{a_{\theta}} \text{x} H_{\varphi}^{*} \boldsymbol{a_{\varphi}}\right] \quad (6)$$

It is to be noted that the symbols or letters written in block indicate that they are vector quantities. Since we are interested in computing the radiation resistance of dipole, we substitute in (3) and (5) to obtain:

(7)

and

(8)

The magnitudes of the electric field and magnetic field intensities of the dipole are given by:

(9)

and

(10)

Substituting (10) in (6), and simplifying, we get:

(11)

The total power radiated is obtained by integrating the average Poynting vector over a sphere of radius . Thus, we get:

$$P_{rad} = \oiint_S \boldsymbol{W_{av}} . \boldsymbol{dS} = \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} W_{av} \boldsymbol{a_r} . r^2 \sin\theta \, d\theta \, d\varphi \boldsymbol{a_r}$$

Substituting of from (11) into the above equation and simplification leads to:

$$P_{rad} = \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} W_{av} r^2 \sin\theta \, d\theta \, d\varphi$$

$$P_{rad} = \frac{15 I_o^2}{\pi} \int_{\varphi=}^{2\pi} \quad (12)$$

But, the radiated power can also be written as [12]:

(13)

Here, is the radiation resistance of the dipole antenna. Thus, from (12) and (13), it follows that:

$$R_{rad} = \frac{30}{\pi} \left(\int_{\varphi=0}^{2\pi} d\varphi \right) \left(\int_{\theta=0}^{\pi} cos^2 \left(\frac{3\pi}{2} \cos\theta \right) cosec\, \theta \, d\theta \right)$$

Simplifying the first integral, we get:

$$R_{1} \quad (14)$$

Substituting and . The new limits of integration are from to . With this new change of variables and further simplification, (14) reduces to:

(15)

The term can be resolved into partial fractions yielding:

(16)

Substituting (16) in (15) and simplifying, we get:

(17)

where,

(18)

and

(19)

Now, we consider the evaluation of the first integral.

(20)

Substituting , we get, and . The new limits of integration are to . Putting these in (20) and simplification of the same gives:

$$I_3 = \int_{\alpha=0}^{6\pi} \frac{cos^2\left(\frac{3\pi-\alpha}{2}\right)}{\alpha}\, dv$$

(21)

Equation (21) follows from the elementary property of the definite integral.

Next, we consider the evaluation of the second integral.

(22)

Substituting , we get, and . The new limits of integration are to . Putting these in (22) and simplification of the same gives:

$$I_4 = \int_{\alpha'=0}^{6\pi} \frac{cos^2\left(\frac{\alpha'-3\pi}{2}\right)}{\alpha'}\, d\alpha'$$

(23)

Equation (23) follows from the elementary property of the definite integral.

Substituting (21) and (23) in (17), we obtain:

(24)

Consider the integral defined by

(25)

is a well-known integral that is related to the cosine integral and is well tabulated in standard references [13]. The cosine integral and the sine integral are defined in the mathematical and technical literatures as:

(26)

and

(27)

is the integral of , which is zero for .

The is related to by this relation [6]:

$Cin(x) = \ln(\gamma x$(28)

Here, and is called the Euler's constant [7].In view of the above expressions, it is also possible to write the Cosine Integral in the form of a series as:

$$Ci(x) = \ln(\gamma x) + \sum_{m=1}^{\infty} \frac{(-1)^m x^{2m}}{(2m)(2m)!}$$

(29)

$\Rightarrow Ci(x) =$ (30)

When is small ( ), we have [14]:

(31)

When is large ( ), we have [1]:

(32)

is the integral of , which vanishes at [8].From (32), it is evident that for larger values of , converges around 0 [2]. The graphs of the and are depicted in Figs.2 and 3 respectively.

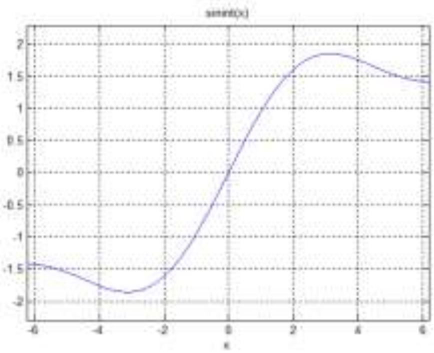

Fig. 2. Graph of

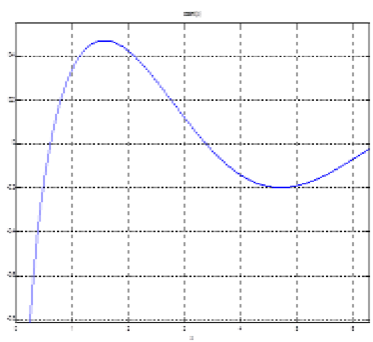

Fig. 3. Graph of

Thus, by the virtue of the above equations, it is possible to write as:

(33)

(34)

When is small ( ), the Sine Integral becomes [3]:

(35)

When is large ( ), we have [4]:

(36)

It is evident that for larger values of , converges to around . Also, from the graphs, we see that the sine integral is symmetric, i.e., and . The same argument holds good for the cosine integral, i.e., .

Finally, (24) reduces to:

$$R_{rad} = 30 \int_{v=0}^{6\pi} \frac{1-\cos v}{v}\, dv = 30\, Cin(6\pi)$$

$$R_{rad} = 30\, Cin[8.84955592]$$

$$R_{rad} = 30 \qquad (37)$$

This can also be written as:S

(38)

Thus, the radiation resistance of a dipole is given by . Thus, we see that the radiation resistances of the half – wave ( )and the full – wave ( ) dipole antennas are 73 and 200 respectively. When compared to these, the radiation resistance of the dipole antenna comes to . This is the final step in the derivation of the radiation resistance of the dipole antenna.

III.CONCLUSIONS

This paper gives the derivation of the one and a half dipole antenna or dipole antenna which is important for use in wireless applications. The computation of the radiation resistance plays a pivotal role in the performance analysis of any antenna and this is done for dipole antenna. This can be further used for the computation of the mutual impedance of the above said antenna, for which the reactance component needs to be calculated based on the induced emf method developed by Carter.

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