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Relativistic self focusing of two crossed intense laser beams and control of beat wave phase velocity in plasma beat wave accelerator

Prof. Shailesh Tiwari[1], Disha Kumawat[2]

[1] *Vice Principal, Vedant College of Engineering & Technology ,Bundi, Rajasthan (India)*

[2] *Asst.Professor, Department of Physics, Vedant college of Engineering and Technology, Bundi(India)*

ABSTRACT

Relativistic focusing allows two collinear short pulse radiation beams, provided they are of sufficiently high power, to propagate through plasma without diffracting. By further accounting for finite radial beam geometry, it is possible for the phase velocity of the radiation beat wave to equal to the speed of light. This removes one of the limiting factors, phase detuning between the accelerated electrons and the beat wave, in determining the maximum energy gain in the plasma beat wave accelerator.

Keywords: beat wave, dispersion, phase velocity, plasma accelerators, plasma waves

I. INTRODUCTION

We consider the propagation of two collinear intense laser beams of frequencies ω_1 and ω_2 are chosen in a uniform plasma . if ω_1 and ω_2 are chosen in such a way that $\omega_1 - \omega_2 = \omega_p$ and $\omega_p^2/\omega_1^2 << 1$, then large amplitude plasma waves are exited . the accelerating electric field associated with plasma waves can be estimated as –

$$E_m \approx \left(\frac{m_e c^2}{e}\right)\frac{\omega_p}{c} = \sqrt{n_p}\ \text{ev/cm}$$

Where n_p is the plasma density ($1/cm^3$).

In a plasma of density 10^{14} cm^{-3} the associated large acceleration field can give rise to an energy gain of 10^9 ev in accelerating distance of 1 meter . however to realize such an accelerating scheme it is very important to maintain that laser beams are able to propagate at high intensity without noticeable diffraction effects. This implies that laser beams retain their intensity over distance large compared to Rayleigh length $\sim\frac{\omega r_s^2}{2c}$. Where r_s is the radiation spot size. In addition to this it is also necessary to maintain the phase resonance between the accelerating electrons and the plasma wave over an equally large distance i.e. Rayleigh length .in order to counter the diffraction effects ,it is necessary to focus the laser beams .the focusing enhancement of laser beams can be achieved by the proper choice of the radius of laser spot sizes and their power . it can be demonstrated that the accelerating electrons can remain in tune with the beat wave if the two parameter of laser beams viz spot sizes and their respective power are properly chosen .

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Control of phase velocity of beat wave

In the plasma beat wave accelerator ,the phase velocity of plasma wave is equal to the phase velocity of the radiation beat wave . in the one dimensional limit ,the phase velocity is

$$\frac{v_p}{c} \approx \frac{\Delta\omega}{\Delta k}$$

The E.M wave propagating through plasma satisfy the dispersion relation :

$$\omega_1^2 = \omega_p^2 + c^2k_1^2$$

$$\text{and} \quad \omega_2^2 = \omega_p^2 + c^2k_2^2$$

$$(\omega_1 - \omega_2)(\omega_1 + \omega_2) = c\,(k_1 - k_2)\,(k_1 + k_2)$$

$$\frac{v_p}{c} = \frac{1}{c}\frac{(\omega_1-\omega_2)}{(k_1-k_2)} \approx \frac{c \cdot 2k_1}{c \cdot 2\omega_1} = \frac{(\omega_1^2-\omega_p^2)^{1/2}}{\omega_1}$$

$$\frac{v_p}{c} \simeq 1 - \frac{1}{2}\frac{\omega_p^2}{\omega_1^2} \qquad \left(\text{assuming } \frac{\omega_p^2}{\omega_1^2} << 1\right)$$

As the electrons are travelling with velocity ˜ C, the difference in the phase velocity of beat wave and accelerating electron is-

$$C - v_p \approx \frac{c \cdot \omega_p^2}{2\omega_1^2}$$

This means that the electron get detuned in a distance of the order of given by-

$$\frac{c \cdot \omega_p^2}{2\omega_1^2} \cdot \frac{L_d}{c} \sim \frac{\lambda_p}{2}$$

$$\sim \frac{1}{2}\frac{2\pi c}{\omega_p}$$

$$L_d \sim \frac{1}{2}\frac{2\pi c}{\omega} \cdot \frac{2c \cdot \omega_1^2}{c \cdot \omega_p^2}$$

$$L_d = \frac{\omega_1^2}{\omega_p^2} \cdot \lambda_p$$

This length for the case of $\frac{\omega_1}{\omega_2} = 20$ and $n_p=10^{14}cm^{-3}$ is –

$$\sim 400\, X\, \frac{3}{9} = 1.20 \text{ m}$$

<u>Control of beat wave phase velocity-</u>

As the radiation beam is of finite size i.e. the radiation spot is of radius r, this gives rise to finite wave number $k_\perp$. This wave number $k_\perp$ not only depends not only on the spot size but also on the power of radiation beam. The existence of finite $k_\perp$ given size to effective parallel wave number k_{11}.

$$\omega^2 = \omega_p^2 + c^2(k_\perp^2 + k_{11}^2)$$

$$C^2k_{11}^2 = \omega^2 - \omega_p^2 - C^2k_\perp^2$$

$$k_{11} = \frac{\omega}{c}\left[1 - \frac{\omega_p^2}{\omega^2} - \frac{k_\perp^2c^2}{\omega^2}\right]^{1/2}$$

$$\simeq \frac{\omega}{c}\left[1 - \frac{1}{2}\frac{\omega_p^2}{\omega^2} - \frac{k_\perp^2c^2}{2\omega^2}\right]$$

$$k_{11\,1} = \frac{\omega_1}{c}\left[1 - \frac{\omega_p^2}{2\omega_1^2} - \frac{k_1^2 c^2}{2\omega_1^2}\right]$$

$$k_{11\,2} = \frac{\omega_2}{c}\left[1 - \frac{\omega_p^2}{2\omega_2^2} - \frac{k_1^2 c^2}{2\omega_2^2}\right]$$

$$k_{11\,1} - k_{11\,2} = \frac{1}{c}(\omega_1 - \omega_2) - \frac{\omega_p^2}{2c}\left[\frac{1}{\omega_1} - \frac{1}{\omega_2}\right] - \frac{c^2}{2c}\left[\frac{k_{1\,1}^2}{\omega_1} - \frac{k_{1\,2}^2}{\omega_2}\right]$$

$$\frac{v_p}{c} = \frac{\omega_1 - \omega_2}{k_1 - k_2} = 1 - \frac{\omega_p^2}{2\omega_1\omega_2} + \frac{\left(k_{\perp 1}^2 - k_{\perp 2}^2\right)c^2}{2\omega\,\omega_p}$$

$$\frac{v_p}{c} = 1 \text{ if } \frac{k_{\perp 1}^2 - k_{\perp 2}^2}{2\omega\,\omega_p} = \frac{\omega_1^2}{2\omega^2}$$

That is by controlling the spot size and power of the two beams, it is possible to keep electron always in tune with the beat wave phase velocity. To see this explicitly, we consider the propagation of two radiation beams through the plasma. The wave equation for the vector potential is –

$$\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = \frac{4\pi}{c}\vec{J}_1 \qquad (1)$$

Defining the normalized vector potential

$\vec{a} = \frac{e\vec{A}}{mc^2}$, we can express equation (1) as

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{a} = \frac{4\pi}{c}\frac{e}{mc^2}\vec{J}_1 \qquad (2)$$

$$\vec{J}_1 = n_p(-e)\vec{v}_q$$

Where $\vec{V}_q$ is the quires velocity

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{a} = -\frac{\omega_p^2}{c^3}\vec{v}_q$$

For combined radiation fields:-

$$\vec{a}_\perp = \vec{a}_1 + \vec{a}_2$$

On substituting in equation (2) , one gets –

$$\vec{a}_1 = a_1 e^{i(k_1 z - \omega_1 t + \phi)} = a_1 e^{i\Phi_1}$$

$$\vec{a}_2 = a_2 e^{i\left(k_2 z - \omega_2 t + \phi_2\right)} = a_2 e^{i\Phi_2}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) a_\perp = -\frac{\omega_p^2}{c^3} a_\perp \left[1 + \left|\vec{a}_1\right|^2 + \left|\vec{a}_2\right|^2 + 2\left|\vec{a}_1\right|\left|\vec{a}_2\right|\cos\Delta\phi\right]^{1/2}$$

The wave equation gives –

$$\left(-k_z^2 + \frac{\omega^2}{c^2}\right) a_\perp = \frac{\omega_p^2}{c^2} a_\perp [1 + |a_1|^2 + |a_2|^2 + 2|a_1||a_2|\cos\Delta\phi]^{-1/2}$$

Where $\Delta\phi = \Phi_1 - \Phi_2$

$$\frac{\omega^2}{c^2}[1 - n^2] = \frac{\omega_p^2}{c^2}[1 + |a_1|^2 + |a_2|^2 + 2|a_1||a_2|\cos\Delta\phi]^{-1/2}$$

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$$n = \left[1 - \frac{\omega_p^2}{2\omega^2}\right]\left(1 - \frac{|a_1|^2}{2} - \frac{|a_2|^2}{2}\right)$$

$$n = 1 - \frac{\omega_p^2}{2\omega^2} + \frac{\omega_p^2}{2\omega^2}\left(\frac{|a_1|^2}{2} + \frac{|a_2|^2}{2}\right)$$

For focusing $\frac{\partial n}{\partial r} < 0$

<u>Cross focusing of two coaxial laser beams</u>:-

We consider the propagation of two cylindrically symmetric coaxial laser beams along the z - axis. Let the intensity distribution at the entrance (z =0), be given by

$$a_1^2 = E_{01}^2 \exp\left(-r^2/r_1^2\right)$$

$$a_2^2 = E_{02}^2 \exp\left(-r^2/r_2^2\right) \qquad (1)$$

Where a_1 and a_2 are the real amplitudes of the electric vector of two beams of angular frequencies $\omega_1$and ω_2 be expressed as –

$$\varepsilon_1(\omega_1) = \varepsilon_{10} + \phi_1(a_1, a_2)$$

$$\varepsilon_2(\omega_2) = \varepsilon_{20} + \phi_2(a_1, a_2)$$

Where $\phi_1$and ϕ_2 representing the contribution to the dielectric constant due to nonlinear effects.

We assume the solution for the combined electric field as

$$\vec{E} = a_1 e^{-ik_1 s_1} e^{i[\omega_1 t - k_1 \square]} + a_2 e^{-ik_2 s_2} e^{i[\omega_2 t - k_2 \square]} \qquad (2)$$

Substituting (2) in to wave equation and separating the real and imaginary parts, we get –

$$2\left(\frac{\partial s_j}{\partial \square}\right) + \left(\frac{\partial s_j}{\partial r}\right)^2 = \frac{\phi_j(a_1 a_2)}{\varepsilon_{j_0}} + \frac{1}{k_j^2 A_j}\left(\frac{\partial^2 a}{\partial r^2} + \frac{1}{r}\frac{\partial a_j}{\partial r}\right)$$

And

$$\frac{\partial a_j^2}{\partial \square} + \frac{\partial s_j}{\partial r}\frac{\partial a_j^2}{\partial r} + a_j^2\left(\frac{\partial^2 s_j}{\partial r^2} + \frac{1}{r}\frac{\partial s_j}{\partial r}\right) = 0$$

(A)

j = 1, 2

Assuming the eibonals to be given by –

$$s_j = \frac{r^2}{2}\beta_j(\square) + \phi_j \qquad , j = 1, 2$$

Then the general solution of equation (A) can be written as-

$$a_j = \frac{E_{0j}^2}{f_j^2} f\left(\frac{r^2}{r_0^2 f_j(\square)}\right) , \qquad j=1, 2$$

Where $\beta_j(\square) = \frac{1}{f_j(\square)}\frac{d f_j}{d \square}$ j=1, 2

If the intensity distribution is assumed to be Gaussian , thus

$$A_j^2 = \frac{E_{0j}^2}{f_j^2} exp\left[-\frac{r^2}{r_j^2 f_j^2}\right] \qquad j=1, 2$$

Then from equation (A), one gets-

$$2\left[\frac{r^2}{2}\frac{d\beta_j}{d\Box} + \frac{d\phi_j}{d\Box}\right] - \left[r\beta_j(\Box)\right]^2 = \frac{1}{2\varepsilon_{j_0}}\left[\varepsilon_{j_1}\frac{E_{01}^2}{f_1^2}\left(1-\frac{r^2}{r_1^2 f_1^2}\right) - \varepsilon_{j_2}\frac{E_{02}^2}{f_2^2}\left(1-\frac{r^2}{r_2^2 f_2^2}\right)\right] - \frac{2}{k_j^2 r_j^2 f_j^2} - \frac{r^2}{k_j^2 r_j^4 f_j^4} = 0$$

Equating the coefficient of r^2, we obtain

$$\frac{1}{f_1}\frac{d^2 f_1}{d\Box^2} = -\frac{R_{n_1}^{-2} - R_{d_1}^{-2}}{f_1^4} - \frac{R_{n_{12}}^{-2}}{f_2^4}$$

And

$$\frac{1}{f_2}\frac{d^2 f_2}{d\Box^2} = -\frac{R_{n_2}^{-2} - R_{d_2}^{-2}}{f_2^4} - \frac{R_{n_{21}}^{-2}}{f_1^4}$$

Where $R_{n_j} = \left[\frac{2\varepsilon_{j_0} r_j^2}{\varepsilon_{jj} E_{0j}^2}\right]^{1/2}$

$$R_{d_j} = \frac{1}{k_j r_j^2}$$

$$R_{n_{jj'}} = \left[\frac{2E_{j_0} r_j^2}{\varepsilon_{jj'} E_{0j'}^2}\right]^{1/2} \qquad j, j' = 1,2$$

$j \neq j'$

When cross focusing term are included , the beams get converged even when their powers are below critical powers ,

$$f_j = 1, \Box = 0$$

And also $\frac{df}{d\Box} = 0$ at $\Box = 0$

The condition for convergence of both the beams are-

$$R_{d_j}^{-2} - R_{n_j}^{-2} - R_{n_{jj'}}^{-2} < 0, \qquad j, j' = 1, 2 \text{ and } j \neq j'$$

And for self trapped propagation

$$R_{d_j}^{-2} - R_{n_j}^{-2} - R_{n_{jj'}}^{-2} = 0,$$

<u>Case I</u>:- if $E_{0_j}^2 >> E_{0_2}^2$

Then $f_1^2(\Box) = 1 - (R_{n_1}^{-2} - R_{d_1}^{-2}).\Box^2$

II. CONCLUSION

It has been shown that two collinear, short pulse radiation beams can propagate through a uniform plasma without diffracting due to relativistic focusing laser beams retain their intensity over large distance so it is also necessary to maintain the phase resonance between accelerating electrons and plasma wave over large distance

.the accelerating electrons can remain in tune with the beat wave depends on two parameters of laser beam spot size and their respective powers.

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