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Mathematical model of Blood flow in arteries in presence of applied Magnetics field and effect of velocity slip

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ABSTRACT

In this study, the flow of blood can be controlled by applying appropriate magnetic field on poiseuille flow of Bingham plastic fluid model for blood with velocity slip. The application of Magneto dynamics in physiological flow problem is of growing interest. Mathematical modeling for poiseuille flow of blood plastic fluid model with an axial velocity slip along an artery wall in presence of magnetic field, is considered. It is observed that when Hartmann number increases the fluid velocity is greatly affected. The present model includes the poiseuille flow models of slip at artery wall and one- layered Bingham plastic fluid model with zero-slip, as its special cases. Applications of this theoretical modelling to cardiovascular diseases and the role of slip in the better functioning of the diseased or occluded arteries are included in brief.

Keywords: Bingham plastic fluid model Reynolds Number, Hartmann number, Velocity profile, Non -Newtonian fluid.

I. INTRODUCTION

Mathematical analysis of arterial blood flow is currently receiving intensive study by both life scientist and engineers. Stenosis refers to localized narrowing of an artery and is a frequent result of arterial disease and is caused mainly due to intravascular atherosclerotic plaque which develops at the arterial wall and protrudes into the lumen of the vessel. Such constriction disturb normal blood flow through the artery [23] [26]. There is considerable evidence the hydrodynamic factor can play a significant role in the development and progression of the disease [20] [21] [22]. Blood flow through arteries can be complicated by the formation of atherosclerotic plaque the artery wall its subsequent advancement impedes the flow through and artery. This unwanted growth at vessel wall may ultimately affect the wall shear stress distribution [12] [13] [15]. The cardiovascular system of man and animals is characteristically a branch network of distensible tubes which carry blood from the heart to periphery and back again [4] [7] .The primary function of circulation is to transport nutrient to tissue and to remove metabolic product, Human blood is a suspension of red cells in a continuous and aqueous substance called plasma [8] [9]. The plasma behaves like a Newtonian fluid with a co-efficient of viscosity 1.2 centipoises whereas the whole blood is shear dependent that is the apparent viscosity of whole blood decreases with an increase rate of shear it has an infinite yield stress under certain flow condition and the viscosity of blood varies with hematocrit and also changes with temperature as well as dieses state .At high shear rates blood behaves as a Newtonian fluid with constant viscosity in larger arteries ,diameter nearly above 1 mm as shear stress decreases blood shows a Non Newtonian character [19] and other [11][18]have pointed out that viscosity of blood in general and interior viscosity of red cells in particular can be in significant factor in pathogenesis of ischemia and infraction and may play an important role in hypertension and cardiovascular disease[17][18].In most of the theoretical models on blood flow, usual no slip condition at vessel wall is considered[2][3][6][1] have suggested the likely presence of a red cell slip at vessel wall or in its immediate neighborhood and in view of a possible existence of slip at tube wall[14][10][25] and other have considered a velocity slip condition at blood vessel wall or at interface of fluid in their modeling, in the present modeling the blood flow through an artery a slip condition for velocity at tube wall of two different locations of CVS is employed.

Here we consider for one dimensional flow axial velocity $\hat{v}$ = (0, 0,$u_z(r)$) the equation for steady tube flow $0 \le r \le R$ of blood (a-Bingham fluid) in (r, θ, z)co-ordinate system reduce to obtain the following form in presence of transverse magnetic effect.

$$-\frac{\partial \hat{p}}{\partial \hat{r}} = 0 \tag{1}$$

$$-\frac{1}{\hat{r}}\frac{\partial \hat{p}}{\partial \hat{\theta}} = 0 \tag{2}$$

$$-\frac{1}{\rho}\frac{\partial \hat{p}}{\partial \hat{z}} + \frac{\mu}{\hat{r}}\frac{1}{\rho}\frac{\partial}{\partial \hat{r}}\left(\hat{r}\frac{\partial \hat{u}_z}{\partial \hat{r}}\right) + \frac{\sigma}{\rho}\hat{B}^2 u = 0 \tag{3}$$

From which we observed that pressure does not vary in the radial $\hat{r}$ circumferential $\hat{\theta}$ and axial $\hat{z}$ direction and that pressure remain constant across any cross-section of the tube and $\hat{p}$ is a function of only $\hat{z}$ that is $\hat{p}$=p($\hat{z}$) and so pressure gradient term in the last equation above becomes $\frac{d\hat{p}}{d\hat{z}}$

Then (3)

$$-\frac{1}{\rho}\frac{\partial \hat{p}}{\partial \hat{z}} + \frac{\mu}{\hat{r}}\frac{1}{\rho}\frac{\partial}{\partial \hat{r}}\left(\hat{r}\frac{\partial \hat{u}_z}{\partial \hat{r}}\right) + \frac{\sigma}{\rho}\hat{B}^2 u = 0$$

Non-dimensional form

$$r = \frac{\hat{r}}{R_0}, z = \frac{\hat{z}}{R_0}, R = \frac{\hat{R}}{R_0}, P = \frac{\hat{P}}{\rho {U_0}^2}, U = \frac{\hat{U}}{U_0}, B = \frac{\hat{B}}{U_0}$$

$$-\frac{dp}{dz} + \frac{1}{R_e r}\frac{d(r\tau_{rz})}{dr} + \frac{M^2}{R_e} = 0$$

$$C + \frac{1}{R_e r}\frac{d(r\tau_{rz})}{dr} + \frac{M^2}{R_e} = 0 \tag{4}$$

Let the solution of the above equation:

$$r^2\frac{d^2 u_Z}{dr^2} + \frac{du_Z}{dr} = Kr^2 \quad \text{Where } K = -\frac{C}{\mu}R_e r - \frac{M^2 r}{\mu}$$

$$U_z = \frac{1}{4}K.e^z$$

$$U_z = \frac{1}{4}\left[-\frac{C}{\mu}R_e r - \frac{M^2 r}{\mu}\right]r^2 \tag{5}$$

Again, shear stress component at any distance r from the tube axis is given by

$$\tau_{rz} = \mu\frac{du_z}{dr} = \mu e^0 \tag{6}$$

$$\tau_{rz} = \mu\frac{r}{2}\left[-\frac{C}{\mu}R_e - \frac{M^2}{\mu}\right] \tag{7}$$

Express for wall shear stress τ_w can be obtained from the formula

$$\tau_w = \tau_{rz}(r = R) \tag{8}$$

$$\tau_w = -\frac{R}{2}\left[CR_e + M^2\right] \tag{9}$$

Using Equation (7) and $\tau_Z(r_0) = \tau_0$ express for τ_0 will lead to the form

$$\tau_0 = -\frac{r_0}{2}\left[CR_e + M^2\right] \tag{10}$$

In between τ_0 and τ_w there may arises two cases wall shear stress is greater and that yield stress. In case $\tau_0 \rangle\ \tau_w$ that is if $r_0 \rangle R$ then there will occur no flow accordingly velocity function will become

$$U_Z = \frac{r^2}{4}\left[-\frac{C}{\mu}R_e - \frac{M^2}{\mu}\right]$$

Bingham equation defined in the form where

$$e^0 = f(\tau_{rz}) = \frac{1}{\mu}(\tau_{rz} - \tau_0)$$

$$\tau_{rz} \geq \tau_0 \tag{11}$$

$$e^0 = \frac{1}{\mu}\left[-\frac{r_0}{2}\left(CR_e + M^2\right) + \frac{r_0}{2}\left(CR_e + M^2\right)\right] \quad r = r_0$$

$$=0 \qquad\qquad \tau_{rz} \leq \tau_0 \tag{12}$$

In the above, vanishing of strain rate that is e^0

$$U_Z = \text{constant} = U_0 \quad \text{When } \tau = \tau_0 \tag{13}$$

Where U_0 is the core velocity at r=r_0 (core radius) .As such for blood flow when $r_0 < R$ there arises two region $0 \le r \le r_0$ and $r_0 \le r \le R$ it is clear for region between 0 and r_0 equation representing the flow is

$$\frac{du_Z}{dr} = 0 \quad 0 \le r \le r_0 \tag{14}$$

On integration $U_Z = U_0 \quad 0 \le r \le r_0$ indicating the velocity profile will become flat in the region and for $r_0 \le r \le R$ velocity U_Z will show deviation from flat profile and Bingham equation (11) has to be applied for this domain of blood flow the same equation it is easily seen that

$$\frac{du_Z}{dr} = \frac{d}{dr}\left[\frac{r^2}{4\mu}\left(-CR_e - M^2\right)\right]$$

$$\frac{du_Z}{dr} = \left[\frac{r_0 - r}{2\mu}\left(CR_e + M^2\right)\right] \qquad r_0 \le r \le R \tag{15}$$

The velocity slip condition at vessels wall is $u_Z = u_S \quad r = R$ (16)

Where U_S is the constant slip velocity at tube wall in axial distance.

$$\int_r^R \frac{du_Z}{dr} dr = \int_r^R \frac{r_0 - r}{2\mu}\left(CR_e + M^2\right)dr$$

$$u_z = u_s + \frac{\left(CR_e + M^2\right)(R - r)}{4\mu}\left[(R + r) - 2r_0\right] \qquad r_0 \le r \le R$$ (17)At

$r_0 = r$ expression for core velocity can be obtained from equation (17)

$$u_{0=}u_s + \frac{(CR_e + M^2)(R - r_0)^2}{4\mu} \tag{18}$$

And for all values of r between 0 and r_0 velocity function is

$$u_z = u_0 \quad 0 \le r \le r_0 \tag{19}$$

Thus from above expression and consideration velocity distribution u_z can be re-written in the following manners

$$u_z = \begin{cases} u_z(r) \;\; r_0 \le r \le R \\ u_0 \, 0 \le r \le r_0 \\ 0 r \rangle R \end{cases} \tag{20}$$

Where $u_z(r)$ and u_0 are given in equation (17) and (18) respectively

The rate of volume flow can be found from

$$Q = \int_{r=0}^{R} 2\pi r u_z dr$$

By integration after using the equation (16). (17) and (19)

$$Q = 2\pi \int_{0}^{r} r u_0 dr + 2\pi \int_{r_0}^{R} r u_z dr$$

$$Q = \pi R^2 u_s + \frac{\pi R^4}{8\mu} \left\{ \left(CR_e + M^2\right) - \frac{4}{3}\left(-\frac{2\tau_0}{R}\right) + \frac{1}{3}\left(\frac{-2\tau_0}{R}\right)^4 \left(CR_e + M^2\right)^{-3} \right\} \tag{21}$$

And expression for apparent viscosity μ_a can be found from the formula

$$\mu = \frac{\pi\left(CR_e + M^2\right)R^4}{8\left(\pi R^2 u_s + \frac{\pi R^4}{8\mu}\left\{\left(CR_e + M^2\right) - \frac{4}{3}\left(-\frac{2\tau_0}{R}\right) + \frac{1}{3}\left(\frac{-2\tau_0}{R}\right)^4\left(CR_e + M^2\right)^{-3}\right\}\right)} \tag{22}$$

And using equation (22), apparent viscosity take the following

$$\mu_a = \left[p(\alpha) \frac{8u_s \mu}{\left(CR_e + M^2\right)R^2} \right]^{-1}, \quad \mathrm{P}(\alpha) = \left[1 - \frac{4\left(-\frac{2\tau_0}{R}\right)\left(CR_e + M^2\right)^{-1}}{3} + \frac{1}{3}\left(\left(-\frac{2\tau_0}{R}\right)\left(CR_e + M^2\right)^{-1}\right)^4 \right]$$

The parabolic velocity profile for poiseuille flow the takes the form

$$u_z(r) = \frac{\left(CR_e + M^2\right)\left(R^2 - r^2\right)}{4\mu} \qquad 0 \le r \le R \tag{23}$$

Employing an axial velocity slip at the tube wall, instead of usual no slip in velocity along the wall the velocity function for poiseuille flow will takes the form

$$u_z(r) = u_s + \frac{\left(CR_e + M^2\right)\left(R^2 - r^2\right)}{4\mu} \qquad 0 \le r \le R \tag{24}$$

In the aforesaid cases velocity is maximum at the axis of the tube and expression for maximum velocity obtained from equation (23) and (24) are given by

$$u_{m1} = \frac{\left(CR_e + M^2\right)R^2}{4\mu} \tag{25}$$

$$u_{m1} = u_s + \frac{\left(CR_e + M^2\right)R^2}{4\mu} \tag{26}$$

Expression for rate of volume flow Q can be accordingly obtained for above two cases in the form

$$Q_1 = \frac{\left(CR_e + M^2\right)\pi R^4}{8\mu} \tag{27}$$

$$Q_2 = \pi R^4 u_s + Q_1 \tag{28}$$

Table-01:

Data for five different locations in Cardiovascular system(SUD AND SEKHON)(1985)

Sl. No.	**Name of an artery**	**Radium (R*) X 10^{-2}m**	**Pressure gradient (C*) X10 kg. $m^{-2}.s^{-2}$**	**r^0/R****		
				$\tau_0 = 0.00$	$\tau_0 = 0.04$	$\tau_0 = 0.10$
01	**Aorta**	**1.00**	**1.46**	**0.0000**	**0.0548**	**0.1370**
02	**Femoral**	**0.50**	**6.40**	**0.0000**	**0.0250**	**0.0625**
03	**Carotid**	**0.40**	**10.00**	**0.0000**	**0.0200**	**0.0500**
04	**Coronary**	**0.15**	**139.74**	**0.0000**	**0.0038**	**0.0095**
05	**Arteriole**	**0.008**	**400.00**	**0.0000**	**0.0250**	**0.0625**

Centre Line Velocity: Centre line velocity (u_0) is obtained from equation (18) and its variation with yield τ_0 and it is greater than equal to zero at all three location CVS and for both slip no-slip condition at artery wall is presented in the following table:

Table-02

Centre Line Velocity u_0 cm/sec				
Location	AORTA		FEMORAL	
Yield Stress	With no slip (u_s=0.0cm/sec)	With no slip (u_s=0.1cm/sec)	With no slip (u_s=0.0cm/sec)	With no slip (u_s=0.1cm/sec)
0.00	0.1830	0.2900	0.1999	0.3000
0.04	0.1700	0.2643	0.1880	0.2885
0.10	0.1355	0.2400	0.1700	0.2700

Conclusion:

Here we have attempted to study the behaviour of poiseuille flow of Bingham plastic fluid model for blood flow with velocity in presence of magnetic effect. A steady one-dimensional flow of blood (-a Bingham fluid) subject to the boundary conditions of velocity slip, suggested in the models of [15] [16] Chaturani and Biswas [6] and, Prahlad and Schultz [17], for five different locations of CVS, in presence of magnetic effect is investigated. Analytic expressions for velocity, flow rate, shear stress at wall, yield stress and apparent viscosity are presented. Axial velocity appears to be a function of pressure gradient C, radial coordinate r, tube semidiameter R, critical radius r_0 (or yield stress τ_0), Bingham fluid viscosity μ_a and u_s axial velocity slip at the boundary.

The following conclusion are observed (Table:01and 02) in this model they are :

i) If shear stress τ_{rz} at a distance is not higher than a finite yield stress, blood will not flow.

ii) If shear stress is not lower than its yield value, blood flow will be possible.

iii) Velocity profiles indicate a parabolic profile in all arteries and for slip and no-slip cases with the usual maximum magnitude at tube axis and a minimum velocity at the boundary in case of vanishing yield stress.

iv) These blunted for flat profiles in velocity $(\tau_0 > 0)$ clearly exposes the non-Newtonian nature of blood

(viii) Assumption that velocity variation in axial direction is negligible as compared to its variation in radial direction, may lead to the implication that the length of the artery is too large as compared to the radius.

(ix) Velocity profile increases when Hartmann number M increases in different fluid parameter viz., yield stress $\tau_0(\geq 0)$. The nature of velocity profile is also same in no slip.

(x) Velocity profile for a full scale of dimensionless radial co-ordinate $\frac{r}{R}$ from the tube axis to vessel will clearly state that.

(xi)Velocity u_0 increases due to an insertion of an axial velocity slip at tube wall that it is found higher with slip at wall than the velocity obtained with zero slip at an artery wall and it is true for two blood vessel.

(xii)As τ_0 increases velocity u_0 decreases from a higher value to a lower one in Aorta and femoral.

(xiii)Magnitude of u_0 is maximum at $\tau_0 = 0$ and minimum at $\tau_0 = 0.1$

(xiv)Flow rate Q increases due to velocity slip at the boundary ,as yield stress increases ,flow rate is increases in different arteries and Q is maximum seen at aorta the largest artery.

In present problem velocity is maximum at the axis and reduces to the magnitude of a slip velocity u_s .In the present analysis, A velocity slip condition at vessel wall is employed due to its physiological signifance .As a result this model clearly established the facts that a slip at an artery wall accelerates the flow and retards the resistance to flow.

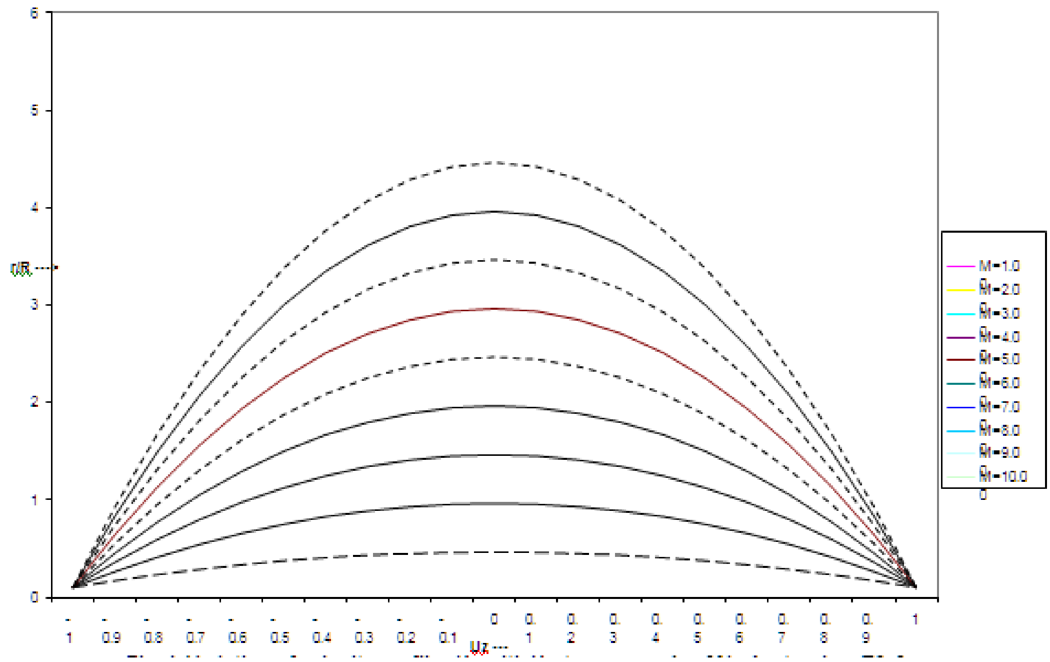

Fig. 1. Variation of velocity profiles Uz with Hartmann number M in Aorta when T0=0

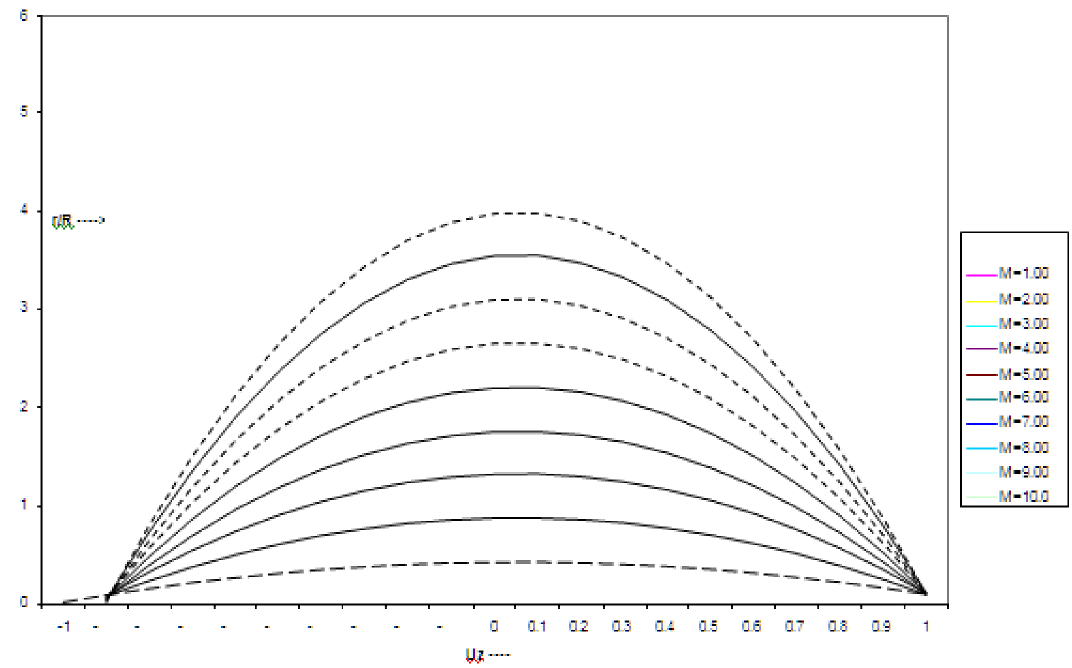

Fig-02 variation of velocity profile with Hartmann numberMin Aorta M T_0=0.04

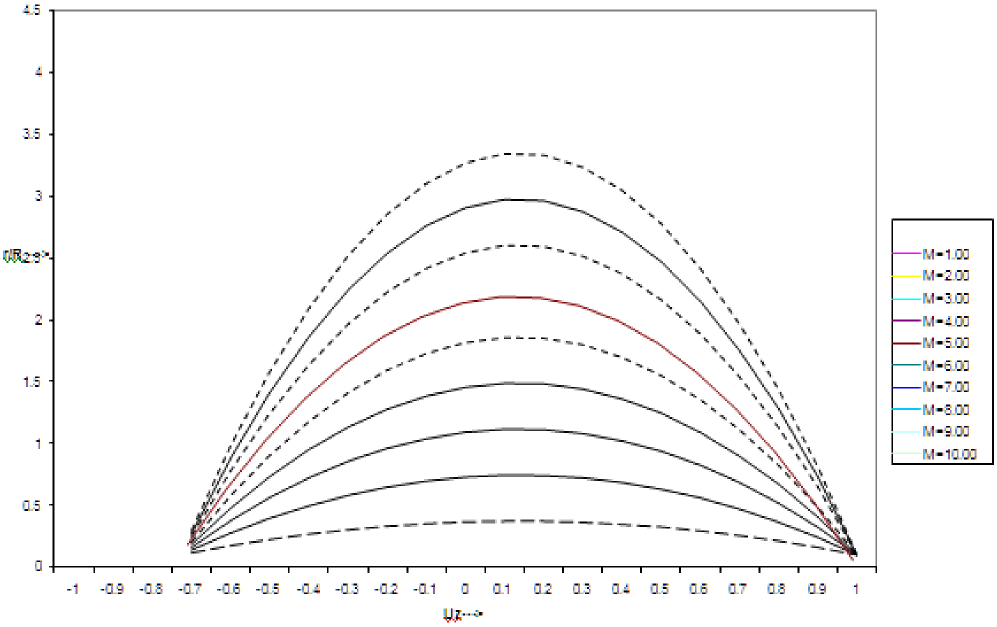

Fig. 3. Variation of velocity profiles with Hartmann number M when T0=0.10

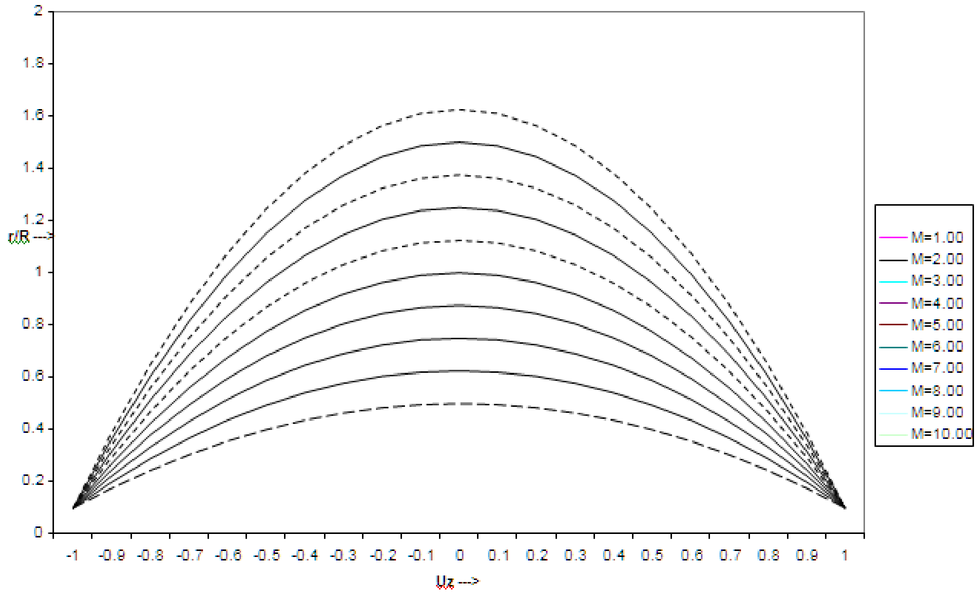

Fig. 4. Varation of velocity profiles Uz with Hartmann number M in femoral when T0=0

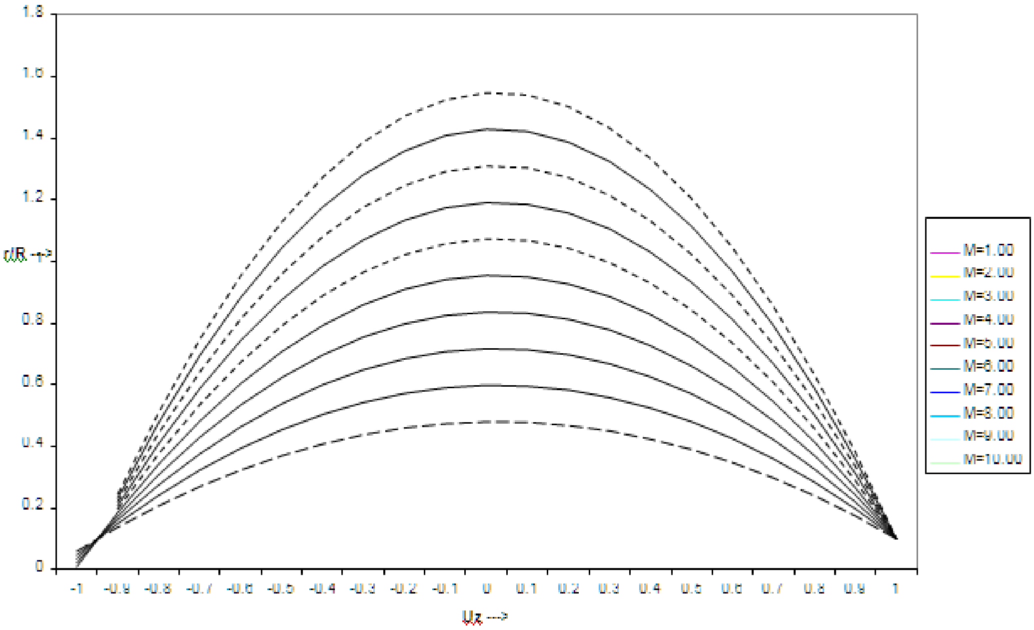

Fig. 5. Variation of Velocity profiles Uz with Hartmann number in Femoral when T0=0.04

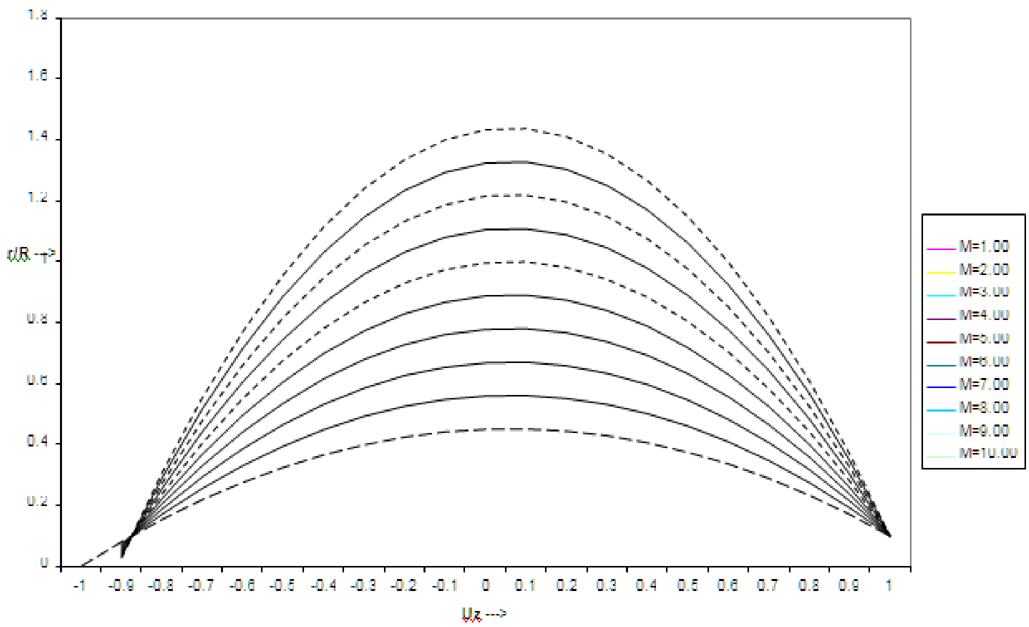

Fig. 6. Variation of velocity profiles Uz with Hartmann number M at femoral when T0=0.10

REFERENCES

[1.] Brunn,P.,The velocity slip of polar fluids,Rheol.Acta.,14,pp.1039-1054(1975)

[2.] Bennet,L.Red Cell Slip at Wall in Vitro,Science,15,pp.1554-1556(1967).

[3.] Bhuyan ,B.C and Hazrika,G.C., Bio-Science Research Bulletian,Vol.17(20,pp.105-112) (2001)

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ISSN: 2319-8354

[4.] Bijendra Singh, Padma Joshi and B.K. Joshi. Blood flow through an artery having radially non-symmetric mild stenosis. Applied Mathematical Sciences, 4, 1065-1072, 2010.

[5.] Caro.C.G.Transport of Material between blood and wall in arteries. In Atherogenesis:Initating Factors, Ciba foundation Symposium.vol,12.pp127-164(New Series),Elsevier, Excerpta Medica,North Holland, Amsterdam(1973).

[6.] Chaturani., P and Biswas,D., Effects of Slip in Flow Through Stenosed Tube, Physiological Fluid Dynamics I:Proc.of 1st International conf on Physiological Fluid Dynamics,September 5-7,pp.75-80.IIT-Madras(1983).

[7.] Deshpande, M. D. and Giddens, D. P., Computation of Turbulent Flow through Constrictions, J. Engng. Mech. 109, 2, 466-478 (1983)

[8.] D. S. Sankar, "Two-fluid nonlinear mathematical model for pulsatile blood flow through stenosed arteries," Bulletin of the Malaysian Mathematical Sciences Society, vol. 35, no. 2A, pp. 487–498, 2012. 1, no. 8, pp. 631–653, 2010.

[9.] Fry,D.I,Responses of the arterial wall to certain physical factors .In Atherogenesis:Initiating Factors,Ciba Foundation Symposium,Vol 12,pp.93-125(New Series),Elesiver, Excerpta Medica ,North Holland ,Amsterdam(1973).

[10.] Jan, K.M., Chien,S.,Bigger,J.T.Jr., Observation on Blood Viscosity changes After Acute Myocardial Infaraction, Circulation,51,pp.1079-1084(1975)

[11.] Light hill .M.J., Mathematical Biofluid Dynamics, SIAM Philadelphia (1975).

[12.] Mac Donald,D,A., On steady flow through Modelled Vascular Stenoses,J,Biomechanic ,12,pp13-20(1979).

[13.] Mukhopadhyay S, De P.R, Mandal M.S, Layek G.C, Applications and Applied Mathematics, 2011, 6(1), 304- 322 [7] Yadav S.S, Kumar, International Journal of Mathematics Trends & Technology, 2012, 3(2), 47-49

[14.] Nubar,Y.,BloodFlow,Slip and Viscometer,Biophys.,J.,11.pp.252-264(1971).

[15.] P.Chaturani, R.Ponnalagorsamy, Biorheology 22(1986) 521-531.

[16.] P.Chaturani and T.R. Ranganathan, proc.Int.conf of Mathematical Modeling in Science and Techonology, pp17 (1988)

[17.] Pralhad,R.N.and Schultz,D.H 1988 Biorheology,Vol-25,p.715

[18.] Roach .M.R., The effects of Bifurication and Stenoses on Arterial disease in cardiovascular flow Dynamics and Measurements,Eds.Hwang,N.H.C and Norman, N.A.University park press ,Baltimore(1977)

[19.] Schmid-Schonbein,H.Factor promoting and factor preventing the fluidity of blood in: Microcirculation, current Physiological, Medical and Surgical Concepts,eds.,Academic Press,Newyork ,pp.249-269(1981)

[20.] 20.Sellar,J.R., Journal of PP. Physics ,Vol.26,pp.489(1955)

[21.] Sapna Ratan Shah. Non-Newtonian flow of blood through an Atherosclerotic artery. Journal of Applied Sciences 6(1), 76-80, 2011

[22.] Sreenadh S, Pallavi A.R, Satyanarayana B.H, Advances in Applied Science Research, 2011, 2(5), 340-349

[23.] Shah, S. R, International Journal of Molecular Medicine and Advance Sciences, 2011, 7(4-6), 19-24

[24.] Sud, V.K. and Sekhon, G.S., *Phy. Med. Biol.*, Vol. 34, pp. 795-805, (1989).

[25.] Verma, S.R. and Anuj Srivastava. Study of Two-layered model of blood flow through narrow vessel with mild stenosis. Jour. PAS, Vol. 16, 38-49, 2010.

[26.] Verma, S.R. Mathematical model of unsteady blood flow through a narrow tapered vessel with stneosis. Jour. PAS, Vol. 17, 96-108, 2011.

[27.] Young ,D,F Fluid mechanic of Arterial stenos ,J.Biomechanical Eng Trans ASME,101,pp.157-175(1979).