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The lower hybrid suppression of drift waves in a magnetized dusty plasma cylinder

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ABSTRACT

The impact of positively charged dust grains is studied on lower hybrid suppression of drift waves in magnetized dusty plasma cylinder. The growth rate and mode frequencies were evaluated based on existing dusty plasma parameters. It is found that the unstable drift mode frequency increases more with $\delta_m (= n_{io} / n_{eo}$*, where* n_{i0} *is the ion plasma density and* n_{e0} *is the electron plasma density) in the presence of dust charge fluctuations (DCF). The growth rate of the unstable drift mode also decreases sharply for larger values of* δ_m *in the presence of DCF. The growth rate and unstable mode frequency also depends on pump wave amplitude.*

Keywords: Dust grains, Drift waves, growth rate & Lower hybrid waves.

I. INTRODUCTION

Drift waves are low frequency spontaneously excited waves which are produced due to density or temperature gradient in plasma. They are well known source of microinstabilities in plasma devices. The suppression of drift waves by the application of lower hybrid pump wave have been reported by many investigators [1-5]. Gore *et al*. [3] experimentally observed the suppression of the whole drift wave spectrum by lower hybrid waves of modest power (e.g., 1 watt) in a Q-Machine and experimental results have been explained by ponderomotive forces.

Recently, there has been a great deal of interest in studying electrostatic waves in dusty plasmas [6-11]. Barkan *et al*. [8] found experimentally that the presence of negatively charged dust grains enhanced the growth rate of the instability of current driven electrostatic ion cyclotron (EIC) wave in a dusty plasma. Sharma and Ajay [11] have studied the effect of dust charge fluctuations on the excitation of upper hybrid wave in a magnetized plasma cylinder. The dust has also been noted to influence a three-wave parametric process in unmagnetised plasmas [12-14] and magnetized plasma [15]. Praburam *et al*. [5] have studied lower hybrid suppression of drift waves in a plasma cylinder without dust environment. In this paper, we study the effect of positively charged dust on lower hybrid suppression of drift waves in a magnetized plasma cylinder.

II. INSTABILITY ANALYSIS

Consider a cylindrical dusty plasma column of radius r_0 immersed in a uniform axial magnetic field B_s in the z direction with equilibrium electron, ion and dust particle densities given as n_{e0}, n_{i0} and n_{d0}. The charge, mass and temperature of the three species are denoted by (-e, m_e, T_e), (e, m_i,T_i) and (-Q_{d0}, m_d,T_d), respectively. The density varies as $n_e(r) = n_{e0}\exp\left(\frac{-r^2}{r_0^2}\right)$ in the interior region of the plasma column and falls off rapidly near the edge. At equilibrium, the electrons acquire a diamagnetic drift velocity $\vec{v}_d = \frac{-\hat{\theta}2rv_{te}^2}{\omega_{ce}r_0^2}$,

where $v_{te} = \sqrt{\frac{2T_e}{m_e}}$ is the electron thermal velocity and $\omega_{ce} = \frac{eB_s}{m_e c}$ is the electron cyclotron frequency.

This equilibrium is perturbed by a low- frequency electrostatic perturbation (i.e., a drift wave)

$$\phi = \phi_{k,\omega}\exp[-i(\omega t - l\theta - k_z z)]. \tag{1}$$

The high amplitude lower hybrid pump wave $\phi_0 = \phi_0(k_{z0},\omega_0)\exp[-i(\omega_0 t - l_0\theta - k_{z0}z)]$ couples with a drift mode ϕ and two lower hybrid sidebands $\phi_{1,2}$ as

$$\phi_{1,2} = \phi_{1,2}(k_{z1,2},\omega_{1,2})\exp[-i(\omega_{1,2}t - l_{1,2}\theta - k_{z1,2}z)], \tag{2}$$

where $\omega_1 = \omega - \omega_0$, $\omega_2 = \omega + \omega_0$, $k_{z1} = k_z - k_{z0}$, $k_{z2} = k_z + k_{z0}$, $l_1 = l - l_0$, $l_2 = l + l_0$, i.e. , phase matching condition. Here we have considered only lowest order coupling. This is a four wave parametric interaction process.

The perturbed densities of electrons, ion and dust are given by

$$n_{e1} = \frac{n_{e0}e}{T_e}(\phi_p + \phi)(1 + i\alpha) , \tag{3}$$

where $\alpha = \frac{\sqrt{\pi}(\omega - \omega^*)}{k_z v_{te}}$, $\omega^* = k_\theta |v_d| = \frac{2lv_{te}^2}{r_0^2\omega_{ce}}$ is the adiabatic drift frequency and

$$\phi_p = \frac{e}{2im_e\omega_{ce}^2\omega_0}[(\nabla\phi_0 \times \omega_{ce}).\nabla\phi_1 - (\nabla\phi_0^* \times \omega_{ce}).\nabla\phi_2]\cdot$$

$$n_{i1} = \frac{n_{i0}e\nabla^2\phi}{m_i\omega_{ci}^2} + \frac{n_{i0}e\omega^*\phi}{T_e\omega} , \tag{4}$$

where $\omega_{ci}\left(= \frac{eB_s}{m_i c}\right)$ is the ion cyclotron frequency.

$$n_{d_1} = -\frac{n_{d0}Q_{d0}k^2\phi}{m_d\omega^2}. \tag{5}$$

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We obtain dust charge fluctuations by following Jana *et al*. [7] as

$$Q_{d1} = \frac{|I_{e0}|}{i(\omega + i\eta_m)}\left[\frac{e\nabla^2\phi}{m_i\omega_{ci}^2} + \frac{e\omega^*\phi}{T_e\omega} - \frac{e}{T_e}(\phi_p + \phi)(1+i\alpha)\right] \text{ where} \qquad (6)$$

$$\eta = 0.79a\left(\frac{\omega_{pi}}{\lambda_{Di}}\right)\left(\frac{1}{\delta_m}\right)\left(\frac{m_i}{m_e}\frac{T_i}{T_e}\right)^{\frac{1}{2}}$$ is the dust charging rate and $\delta_m = n_{i0}/n_{e0}$.

Substituting perturbed densities in the Poisson's equation $\nabla^2\phi = 4\pi(n_{e1}e - n_{i1}e + n_{d0}Q_{d1} + Q_{d0}n_{d1})$, we obtain

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \left(p_1^2 - \frac{l^2}{r^2}\right)\phi = \frac{(1+i\alpha)\phi_p}{L}\left[\frac{\omega_{pe}^2}{v_{te}^2} + \frac{i\beta}{(\omega+i\eta)}\frac{\omega_{pi}^2}{C_s^2}\frac{n_{e0}}{n_{i0}}\right], \qquad (7)$$

where $p_1^2 = p^2 - k_z^2$, $p^2 = \frac{M}{L}$,

$$M = \left[\frac{\omega_{pi}^2}{C_s^2}\frac{\omega^*}{\omega} - \frac{\omega_{pe}^2}{v_{te}^2}(1+i\alpha) + \frac{i\beta}{(\omega+i\eta)}\frac{\omega_{pi}^2}{C_s^2}\frac{\omega^*}{\omega}\frac{n_{e0}}{n_{i0}} + \frac{i\beta}{(\omega+i\eta)}\frac{\omega_{pi}^2}{C_s^2}\frac{n_{e0}}{n_{i0}}(1+i\alpha) + \frac{\omega_{pd}^2k^2}{\omega^2}\right],$$

$$L = 1 + \frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{i\beta}{(\omega+i\eta)}\frac{\omega_{pi}^2}{\omega_{ci}^2}\frac{n_{e0}}{n_{i0}},$$

$C_s^2 = \frac{T_e}{m_i}$, $\omega_{pe}\left(=\sqrt{4\pi n_{e0}e^2/m_e}\right)$, $\omega_{pi}\left(=\sqrt{4\pi n_{i0}e^2/m_i}\right)$ and $\omega_{pd}\left(=\sqrt{4\pi n_{d0}Q_{do}^2/m_d}\right)$ are the electron, ion and dust plasma frequencies, respectively and $\beta = \frac{|I_{e0}|}{e}\left(\frac{n_{d0}}{n_{e0}}\right) = 0.397\left(1-\frac{1}{\delta_m}\right)\left(\frac{a}{v_{te}}\right)\omega_{pi}^2\left(\frac{m_i}{m_e}\right)$ is the coupling parameter.

In the absence of non-linear coupling terms, Eq. (7) has a well known solution $\phi = AJ_l(p_1r), p_1 = p_{n1}$. At $r = r_0$, ϕ must vanish, hence $J_l(p_1r_0) = 0$, i.e., $p_1r_0 = x_n$ [where x_n are the zeros of the Bessel function $J_l(x)$, n=1, 2, 3,…] . In the presence of a finite R.H.S. of Eq. (7), we express ϕ in terms of a complete orthogonal sets of wave function:

$$\phi = \sum_m A_m J_l(p_{m1}r). \qquad (8)$$

Substituting the value of ϕ from Eq. (8) in Eq. (7), multiplying both sides of Eq. (7) by r J_l (p_{n1}r) and integrating over r from 0 to r_0 (where r_0 is the plasma radius), retaining only the dominant mode (m = n), we obtain

$$\{[\frac{\omega^*}{\omega}-(1+i\alpha)\frac{n_{e0}}{n_{i0}}]\frac{\omega_{ci}^2}{C_s^2}-k^2+\frac{i\beta}{(\omega+i\eta)}\frac{\omega_{ci}^2}{C_s^2}\frac{\omega^*}{\omega}\frac{n_{e0}}{n_{i0}}+\frac{i\beta}{(\omega+i\eta)}\frac{\omega_{ci}^2}{C_s^2}\frac{n_{e0}(1+i\alpha)}{n_{i0}}$$

$$-\frac{i\beta}{(\omega+i\eta)}k^2\frac{n_{e0}}{n_{i0}}+\frac{\omega_{pd}^2k^2}{\omega^2}\frac{\omega_{ci}^2}{\omega_{pi}^2}\}A_n=(1+i\alpha)\frac{n_{e0}}{n_{i0}}\frac{\omega_{ci}^2}{C_s^2}(1+\frac{i\beta}{(\omega+i\eta)})\frac{\int_0^{r_0}J_l(p_{n1}r)\phi_p rdr}{\int_0^{r_0}J_l^2(k_\perp r)rdr} \tag{9}$$

where $k^2=k_z^2+p_{n1}^2$.

Similarly following Praburam *et al*. [5], we obtain

$$(\frac{\omega_{p0}k_{z1}r_0}{(\omega_0-\omega)}-\lambda_{n_1,l_1})A_{n_1}=-\frac{e\omega^*2l}{i\omega_1\omega_{ce}^2\omega T_e}\frac{\int_0^{r_0}rdr\omega_p^2(r)\psi_{n_1,l_1}^{(1)*}(\nabla\phi_0^*\times\omega_{ce}).\nabla\phi}{\int_0^{r_0}J_l^2(k_\perp r)rdr}, \tag{10}$$

where A_{n_1} is the constant of wave function ϕ_1.

$$(\frac{\omega_{p0}k_{z2}r_0}{(\omega_0-\omega)}-\lambda_{n_2,l_2})A_{n_2}=-\frac{e\omega^*2l}{i\omega_2\omega_{ce}^2\omega T_e}\frac{\int_0^{r_0}rdr\omega_p^2(r)\psi_{n_2,l_2}^{(1)*}(\nabla\phi_0\times\omega_{ce}).\nabla\phi}{\int_0^{r_0}J_l^2(k_\perp r)rdr}, \tag{11}$$

where A_{n_2} is the constant of wave function ϕ_2.

Using the value of ϕ_p , assuming the pump to be azimuthally symmetric (l_0 =0), and considering the radial mode numbers of the two sidebands to be same, i.e., $n_1=n_2$, Eqs. (9), (10) and (11) gives a dispersion relation

$$[\frac{\omega^*}{\omega}-(1+i\alpha)\frac{n_{e0}}{n_{i0}}-k^2\rho_s^2+\frac{i\beta}{(\omega+i\eta)}\frac{\omega^*}{\omega}\frac{n_{e0}}{n_{i0}}-\frac{i\beta}{(\omega+i\eta)}k^2\rho_s^2\frac{n_{e0}}{n_{i0}}+\frac{i\beta}{(\omega+i\eta)}\frac{n_{e0}(1+i\alpha)}{n_{i0}}$$

$$+\frac{\omega_{pd}^2k^2}{\omega^2}\frac{C_s^2}{\omega_{pi}^2}\}]=\frac{n_{e0}}{n_{i0}}[1+\frac{i\beta}{(\omega+i\eta)}]\frac{\mu_1}{\omega}, \tag{12}$$

where

$$\mu_1 = \left(\frac{e^2 \omega^* \omega_{p0}^2 l^2}{2 m_e \omega_{ce}^2 \omega_0^2 T_e \int_0^{r_0} r dr J_l{}^2 (k_\perp r)} \right) \left| \int_0^{r_0} r dr \psi_{n_1, l_1} \frac{\partial \phi_0}{\partial r} J_l (p_{n1} r) \right|^2 \frac{2\left(\frac{\omega_{p0} k_z r_0}{\omega_0} - \lambda_1 \right)}{\left(\frac{\omega_{p0} k_z r_0}{\omega_0} - \lambda_1 \right)^2 - \left(\frac{\omega_{p0} k_{z0} r_0}{\omega_0} \right)^2},$$

$$\rho_s{}^2 = C_s{}^2 / \omega_{ci}{}^2 .$$

(13)

Equation (12) can be rewritten as

$$\varepsilon_r(\omega, k) + i\varepsilon_i(\omega, k) = 0,$$

(14)

where

$$\varepsilon_r(\omega,k) = \frac{\omega^*}{\omega} - \frac{n_{e0}}{n_{i0}} - k^2\rho_s^2 + \frac{\beta\eta}{(\omega^2+\eta^2)} \frac{\omega^*}{\omega} \frac{n_{e0}}{n_{i0}} - \frac{\beta\eta}{(\omega^2+\eta^2)} k^2 \rho_s^2 \frac{n_{e0}}{n_{i0}} - \frac{n_{e0}}{n_{i0}} \frac{\mu_1}{\omega}$$
$$- \frac{\beta\eta}{(\omega^2+\eta^2)} \frac{\mu_1}{\omega} \frac{n_{e0}}{n_{i0}} + \frac{\beta\eta}{(\omega^2+\eta^2)} \frac{n_{e0}}{n_{i0}} - \frac{\beta\alpha\omega}{(\omega^2+\eta^2)} \frac{n_{e0}}{n_{i0}} + \frac{\omega_{pd}^2 k^2}{\omega^2} \frac{C_s^2}{\omega_{pi}^2}, \quad (15)$$

$$\varepsilon_i(\omega,k) = -\alpha \frac{n_{e0}}{n_{i0}} + \frac{\beta\omega}{(\omega^2+\eta^2)} \frac{\omega^*}{\omega} \frac{n_{e0}}{n_{i0}} + \frac{\beta\omega}{(\omega^2+\eta^2)} \frac{n_{e0}}{n_{i0}} - \frac{\beta\omega}{(\omega^2+\eta^2)} k^2 \rho_s^2$$
$$- \frac{\beta\omega}{(\omega^2+\eta^2)} \frac{n_{e0}}{n_{i0}} \frac{\mu_1}{\omega} + \frac{\beta\alpha\eta}{(\omega^2+\eta^2)} \frac{n_{e0}}{n_{i0}}.$$

(16)

Let us write $\omega = \omega_r + i\gamma$ and assume that the wave is either weakly damped or growing (i.e., $|\gamma| << \omega_r$). Then we may set

$\varepsilon_r(\omega = \omega_r, k) = 0$ from Eq. (14).

(17)

Equation (14) yields

Growth rate:

$$\gamma = -\frac{\varepsilon_i(\omega_r, k)}{\partial \varepsilon_r(\omega_r, k)/\partial \omega_r}.$$

(18)

Now, we consider two cases of interest

Case I: In the presence of dust charge fluctuations, i.e., dust charging rate η is finite.

Case II: In the absence of dust charge fluctuations, i.e., $Q_{d_1} = 0$ when dust charging rate η→∞.

In the absence of dust grains, i.e., $\delta_m = n_{i0}/n_{e0}$ =1, $\beta \to 0$, we recover the expressions for the unstable drift mode frequency and growth rate of Ref. [5] (cf. page no. 3146). Dust grain is negatively charged for $\delta_m > 1$ and positively charged for $\delta_m < 1$.

III. RESULTS AND DISCUSSIONS

To estimate the numerical values of the real frequency and growth rate of the drift wave instability, we use typical dusty plasma parameters: $n_{i0}= 10^{10}$ cm^{-3}, Te=Ti=0.2eV , B_s=1.0x10^{3}G, ω^* =1.5x10^{5}rad./sec, r_0 =1.0cm, length of plasma column L = 70 cm, $m_i/m_e \approx 7.16\times10^4$ (Potassium), average dust grain size a=1μm, mode number n=1, i.e., the first zero of the Bessel function, $k_{\perp n} = 3.85$ cm^{-1}, $k_z = \pi/L$, pump wave amplitude $\phi_0 = 6.6\times10^{-4}$ esu. We vary δ_m from 0.35 to 0.9 for positively charged dust grains.

Using Eq. (17) we have plotted in Fig.1 the normalized real frequency ω_r/ω_{ci} of the unstable drift waves as a function of $\delta_m = n_{io}/n_{eo}$. In Fig.1 it is seen that the normalized wave frequency ω_r/ω_{ci} increases by a factor ~5.0 when δ changes from 0.4 to 0.8 if dust charge fluctuations are taken into account, and by a factor ~1.46 in the absence of dust charge fluctuations under the plasma parameters listed above. Barkan *et al* [8] have found that the wave frequency was about 10-20% larger than the ion-cyclotron frequency in the presence of negatively charged dust grains. Chow and Rosenberg [9] have shown, in their kinetic analysis on the effect of negatively charged dust grains on the collisionless electrostatic ion cyclotron instability, the wave frequency ω_r/ω_{ci} increases about 11% when δ_m is changed from 1 to 4 under similar conditions. Thus the increase is more in case of positively charged dust grains.

In Fig.2, we have plotted the normalized growth rate γ/ω_{ci} obtained from Eq. (18) as a function of δ_m for the same parameters as those used in Fig.1. From Fig.2 it can be seen that the normalized growth rate γ/ω_{ci} increases initially with δ_m in both cases but decreases for higher values of δ_m. However, the increase & decrease is more drastic when the dust charge fluctuations are taken into consideration. Thus the dust charge fluctuations can play a significant role in suppression of the drift mode for larger values of δ_m in case of positively charged dust grains.

IV. CONCLUSION

The present work investigates the role of positively charged dust grains in suppression of drift waves. The dust charge fluctuations can play a major role in influencing drift wave instability which plays a crucial role in international ITER and other fusion reactors.

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FIGURE CAPTIONS

FIG.1: Normalized real frequency ω_r/ω_{ci} of the unstable drift wave instability as a function $\delta_m\left(=n_{i0}/n_{e0}\right)$ [with and without dust charge fluctuations].

FIG. 2: Normalized growth rate γ/ω_{ci} of the unstable drift wave instability as a function δ_m [with and without

dust charge fluctuations].

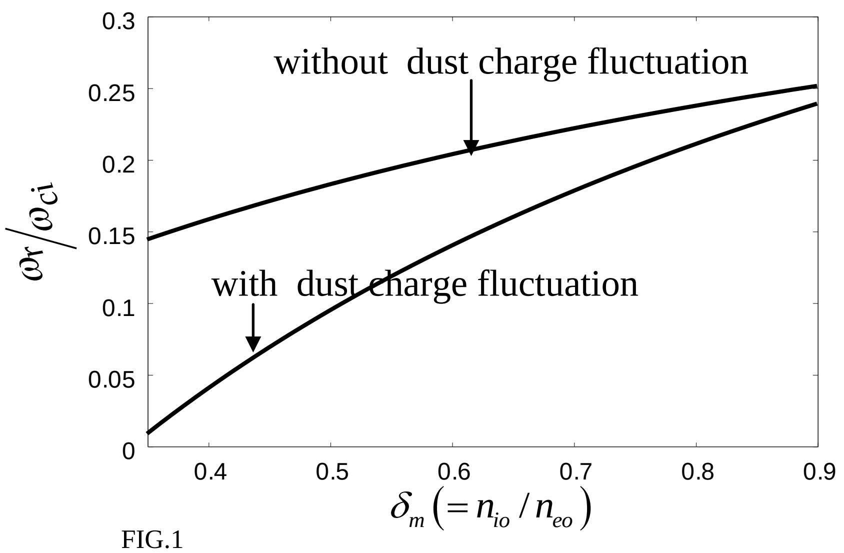

FIG.1

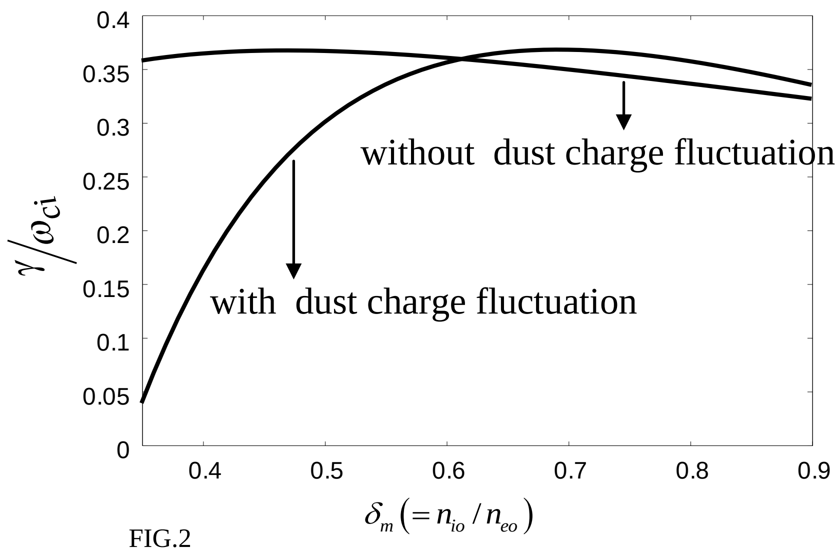

FIG.2