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BUCKLING AND POST-BUCKLING BEHAVIOUR OF FGM PLATE – A REVIEW

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ABSTRACT

This paper presents a comprehensive review of the buckling and post-buckling behavior of FGM plates. Both analytical and numerical methods are considered. The review is carried out on recent work published. In this paper, only buckling and post-buckling characteristics of FGM plates predicted using different theories along with different methods by several researchers are presented. The main objective of this paper is to serve the interest of researchers and engineers already involved in the buckling and post-buckling analysis and design of FGM structures.

Keywords: *FGM plate; Buckling; Post-buckling*

I. INTRODUCTION

Functionally graded materials (FGM) are the advanced composite materials widely used in various areas of engineering applications. FGM have wide range of adaptability to change its constituents phase with a continuously varying composition. The volume fraction distribution of FGM is of basically three types i.e. P-FGM; S-FGM and E-FGM. FGM are designed with varying properties that include various changes such as changing chemical properties, changing mechanical, magnetic, thermal and electrical properties. FGM are graded as stepwise-gradient structure or some of them are continuous graded structures depends on the application areas.

There are different types of processes utilized to manufacture FGM thin layer, such as

i. Powder Metallurgy (PM)
ii. Vapour Deposition Technique
iii. Other Methods for Fabrications of FGM
    - Electro-deposition method
    - Plasma spraying and etc., [1] [2].

FGM have wide range of applications in various scientific and engineering fields, such as, FGM's extensively used in dental and orthopedic applications [1]. It is also utilized in defensive covering on turbine edges in gas turbine motor [1]. FGM are appropriate for rocket engine components, structure and space plane body [1]. FGM are also used for optoelectronics applications and materials of audio-video disc having magnetic capacity as storage media. In defense industry, FGM are used in safeguarding and bullet proof vest and vehicles, also utilized in manufacturing of traditional Japanese sword [1] [2].

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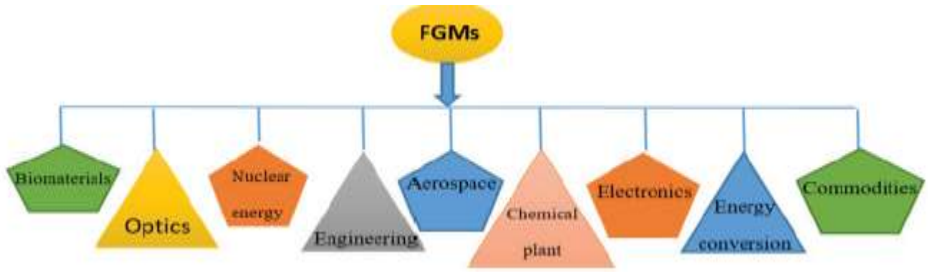

Figure 1: Schematic field of FGMs application [1]

1.2. MATERIAL PROPERTY IDEALIZATION

FGM often possess variety of micro-structure by gradually changing the volume fraction of constituents phases called material gradation [3].

A review of several micromechanical models and its accuracy in estimation of mechanical properties for a single layered and multi-layered FGM dealt in various papers some of them are, [4] to [5]. The methods for evaluating the effective material properties across the plates thickness are discussed below.

1. POWER LAW FUNCTION (P-FGM)

The methods of estimation are based on linear rule of mixture and has been extensively used in the literature to investigate the response of FGM [6] to [7].

The properties of material of P-FGM can be explained by the rule of mixture:

$$P(z) = (P_t - P_b)V_f + P_b \quad \text{.......................................} \; 1$$

Properties of material are dependent on the volume fraction V_f of P-FGM which obeys power law,

$$V_f = (z/h + ½)^n \quad \text{.......................................} \; 2$$

where n is a parameter that symbolize the material variation profile through the thickness known as is the volume fraction exponent. A bottom face, (z/h) = -1/2 and $V_f = 0$, hence $P(z) = P_b$ and at top face, (z/h) = ½ and so $V_f = 1$, hence $P(z) = P_t$ where P denotes a generic material property like modulus, P_t and P_b denotes the property of the top and bottom faces of the plate.

At n = 0 the plate is a fully ceramic plate while at n = ∞ the plate is fully metal. The variation of Young's modulus in the thickness direction of the P-FGM plate is shown in figure 3., which shows the Young's modulus changes rapidly near the lowest surface for n > 1, and increases quickly near the top surface for n < 1.

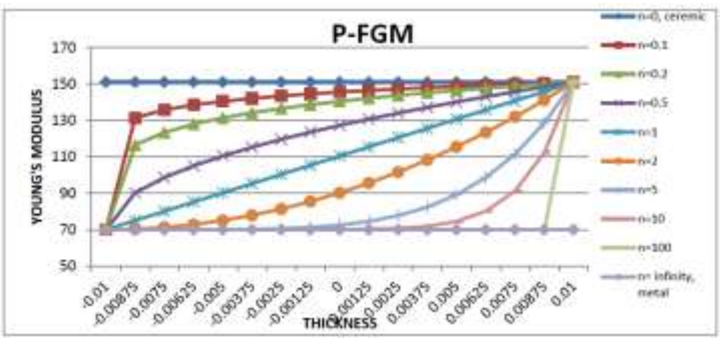

Figure 2: Variation of Young's Modulus in a P-FGM with 'n' [8].

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2. SIGMOID LAW:

In the case of adding an FGM of a single power-law function to the multi-layered composites, stress concentrations appear on one of the interfaces where the material is continuous but changes rapidly. Therefore, [9] defined that the volume fraction using two power-law functions to ensure smooth distribution of stresses among all the interfaces. The two power-law functions are defined by:

$G_1(z) = 1 - \frac{1}{2}\left(\frac{\frac{h}{2}-z}{\frac{h}{2}}\right)p$ for $\leq z \leq h/2$, $G_2(z) = \frac{1}{2}\left(\frac{\frac{h}{2}+z}{\frac{h}{2}}\right)p$ for $-h/2 \leq z \leq 0$....................................... 3

By using the rule of mixture, the Young's modulus of the S-FGM can be calculated by:

$E(z) = G_1(z)E_1 + [1 - G_1(z)]E_2$ for $0 \leq z \leq h/2$ and $E(z) = G_2(z)E_1 + [1 - G_2(z)]E_2$ for $-h/2 \leq z \leq 0$

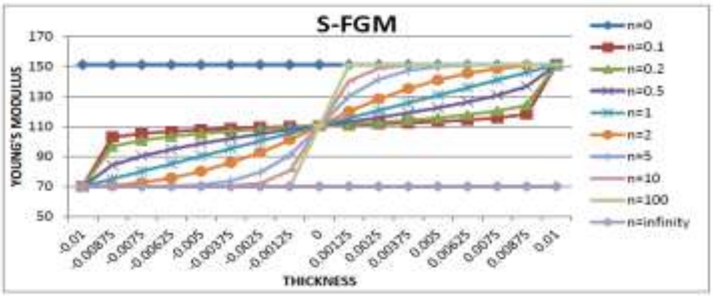

Figure 3: Variation of Young's modulus in S-FGM with 'n' [8].

The variation of Young's modulus in the thickness direction of the S-FGM plate is shown in figure 4., which describe that the Young's modulus changes are gradual because of using two power law functions together as shown in figure 4.

3. EXPONENTIAL LAW (E-FGM)

Many researchers used the exponential function to describe the material properties of FGMs as follows:

$$E(z) = E_2 e^{\frac{1}{h}\ln\left(\frac{E_1}{E_2}\right)\left(z+h/2\right)} \text{.. 4}$$

The material distribution in the E-FGM plates is plotted in figure 5.

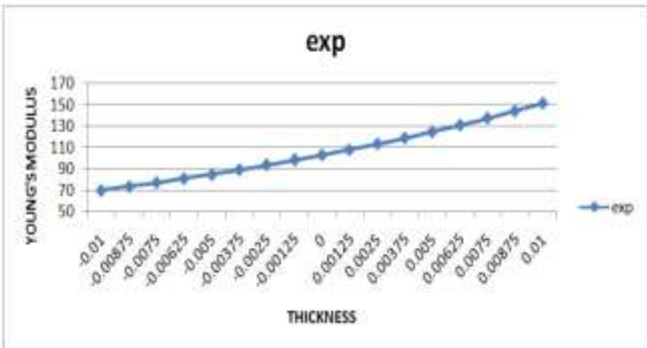

Figure 4: Variation of Young's modulus in a E-FGM plates [8].

II. BUCKLING AND POST-BUCKLING BEHAVIOUR

Buckling is the state of instability, when a structure is subjected to compressive stress, buckling may occur. Buckling is characterized by a sudden sideways deflection of a structural member. This may occur even though the stresses that develop in the structure are well below those needed to cause failure of the material of which the structure is composed. As an applied load is increased on a member, such as a column, it will ultimately become large enough to cause the member to become unstable and it is said to have buckled. There are three ways in which a compression member can buckle, or becomes unstable. These are as listed below.

1) Flexural buckling

   In this type, buckling can occur in any compression member that experience a deflection caused by bending or flexure.

2) Torsional buckling

   This type of buckling occurs in compression members that are doubly-symmetric and have very large slender cross-sectional elements.

3) Flexural-Torsional buckling

   In this case, buckling only occurs in compression members that have unsymmetrical cross-section with one axis of symmetry. Flexural-torsional buckling is the simultaneous bending and twisting of a member.

POST-BUCKLING

This stage is the continuation of the buckling stage. After the load reaches its critical value the load value may not change or it may start decreasing, while deformation continues to increase. In some cases, the structure continues to take more load after certain amount of deformation, to continue increasing deformation which eventually results in a second buckling cycle. Post-buckling analysis being non-linear due to large deformation.

III. PLATE THEORIES

In order to analyze the plate, various plate deformation theories developed. Plate deformation theories can be divided in to two domains on the basis of assumed fields: Stress based theories and displacement based theories. In stress based theories, the stresses are treated as primary variables. In displacement based theories, displacements are treated as primary variables.

Displacement based theories are divided into two parts:

I. Classical Plate Theory

II. Shear Deformation Plate Theories.

3.1. Classical Plate Theory

Kirchhoff developed the well-known classical plate theory (CPT). It is based on the Kirchhoff's hypothesis that straight lines normal to the un-deformed mid-plane remain straight and normal to the deformed mid-plane. In accordance with the kinematic assumptions made in the CPT all the transverse shear and transverse normal strains are zero. The CPT is widely used for static bending, vibrations and stability of the thin plates in the area of solid structural mechanics. The classical plate theory can be used for thin plates since it gives improper results for the thick plates.

3.2 Shear Deformation Plate Theories

These can be classified as following

3.2.1 First Order Shear Deformation Theory

First-order shear deformation theory (FSDT) can be considered as improvement over the CPT. It is based on the hypothesis that the normal to the un-deformed mid-plane remain straight but not necessarily normal to the mid-plane after deformation, this is known as FSDT because the thickness wise displacement field for the in-plane displacement is linear or of the first order. Reissner had developed a stress based FSDT which incorporates the effect of shear and Mindlin employed displacement based approach. In Mindlin's theory, transverse shear stress is assumed to be constant through the thickness of the plate, but this assumption violates the shear stress free surface conditions on the top and bottom surface of the plate. Mindlin's theory satisfies constitutive relations for transverse shear stresses and shear strains by using shear correction factor.

1.2.1. Higher Order Shear Deformation Theory

This limitation of CPT and FSDTs forced the development of higher order shear deformation theories (HSDTs) to avoid the use of shear correction factors, to include correct cross-sectional warping and to get the realistic variation of the transverse shear strains and stresses through the thickness of plate. The higher order theory is developed by Reddy to get the parabolic shear stress distribution through the thickness of plate and to satisfy the shear stress free surface conditions on the top and bottom surfaces of the plate to avoid the need of shear correction factors.

IV. LITERATURE SURVEY

The minimum in-plane edge compressive stress required just to start such instability is called critical buckling load. In this paper, discussion is done on the both analytical and numerical methods employing two-dimensional elasticity theory are included in following discussions.

4.1. Analytical Methods

The extensive investigations are done in the buckling analysis of FGM plates and FGM sandwich plates to anticipate the critical buckling loads under different boundary conditions. A review of various analytical and numerical methods used for buckling analysis are presented in the following sections.

Two-dimensional plate theory

To obtain the critical buckling load eigenvalue problems has to be solved. Unfortunately, the three-dimensionally elasticity theory is very suitable method of buckling analysis but not reported yet in the literature. Various two-dimensional plate theories are applied in the analysis of buckling of FGM plates under various boundary conditions, for two types of loading.

- Mechanical load case
- Thermo-mechanical load case

4.1.1. Mechanical load case

Birman [10] was the first person who tried to solve the buckling problem to FGM hybrid composite plates. The problem was formulated based on the multi-cell model proposed by **Chamis** [11] and **Hopkins and Chamis** [12]**.** Results were presented for the critical buckling load of FGM hybrid composite plates made of three different types of fibers i.e. Silicon Carbide, Boron and Nicalon fibers. The elastic bifurcation buckling of FGM plates with nonuniformly distributed fibers under uniaxial compressive loading was analyzed by **Feldman and Aboudi** [13] who combined micromechanical and structural approach. **Neves et.al.** [27] presented the various sinusoidal shear deformation theories are used for the buckling analysis of functionally graded sandwich plates, in this the governing equations and boundary conditions are derived using the principle of virtual work under a generalization of Carrera's unified formulation and further interpolated by collocation with radial basis function. **Heydari et.al.** [28] presented a buckling analysis of FGM plates resting on two-parameter Pasternak elastic foundation.

Song [29] examined the static bending and buckling analysis of FG multi layered nano platelets (GPL)/polymer composite plates within the framework of FSDT analytical solutions are obtained for the static deflection and critical buckling load of the simply supported functionally graded GPL/polymer plates by using the Navier's solution technique.

4.1.2. Thermo-mechanical load case

Lanhe [30] employed the Navier's solution technique to analyze the thermal buckling of simply-supported moderately thick rectangular FGM plates based on the FSDT and subjected to two types of temperature fields; uniform temperature rise and gradient across the thickness of the plate. **Najafizadeh and Heydari** [31] practiced the closed form solutions for the critical buckling temperature of clamped FGM circular plates based on the Reddy's TSDT. **Morimoto et.at** [32] solved the plane thermos-elastic problem using Galerkin's method to understand the buckling behavior of FGM rectangular plates subjected to partial heating in a plane and uniform temperature rise through the thickness. It was observed that there was no stretching-bending coupling in constitutive equations if the reference surface is selected properly.

. **Zenkour and Sobhy** [37] submitted an analytical solution to find the critical buckling temperature differences of various symmetric FGM sandwich plates based on the SSDT and von-Karman non-linearity. Here, three types of loading conditions were assumed namely, uniform, linear and non-linear distribution through the thickness. The results were compared with CPT, TSDT, and FSDT. It was observed that the results obtained are higher than that of rest of shear deformation theories. **Bodaghi and Saidi** [38] utilized the levy-type solution method to investigate the thermo-elastic buckling behavior of thick FGM rectangular plates based on the Reddy's TSDT. **Chen et.al.** [39] used average stress method to study the thermally induced buckling of initially stressed FGM hybrid composite plates based on the FSDT. **Kettaf et. al.** [40] utilized the new hyperbolic displacement model with four unknowns to study the thermal buckling behavior of sandwich plates consisting of FGM face sheets and homogeneous core made of isotropic ceramic material.

Zhang et. al. [41] utilized thermal buckling behaviors of ceramic–metal functionally grade plates (FGPs) were studied by using a local Kriging meshless method. The local meshless method was developed based on the local Petrov–Galerkin weak-form formulation combined with shape functions having the Kronecker delta function property, constructed by the Kriging interpolation. **Shariyat et. al.** [42] presented thermal buckling of general quadrilateral plates fabricated from heterogeneous, orthotropic and auxetic materials resting on the elastic Winkler-Pasternak elastic media. **Van Do et.al.** [43] presented the variational phase field model is adopted to analyze thermal buckling behavior of cracked FGM plates using FSDT where, crack is simulated by variational phase-field theory.

4.2 Numerical Methods

As it is already known that the analytical solution is not easy to attain for all plate geometry and boundary conditions. Solutions becomes more complex and material properties are varied non-linearly through the thickness. Therefore, numerical methods such as finite element methods and meshless methods are being extensively used for complex engineering problems.

4.2.1 Finite elements method

Different types of finite element techniques are used for linear and non-linear buckling analysis of FGM plates subjected to mechanical load and thermo-mechanical load.

4.2.1.1 Mechanical load case

Naei et. al. [44] utilized the energy method based on Love-Kirchhoff hypothesis along with the Sander's non-linear strain-displacement relation to analyze the buckling analysis of a radially-loaded circular FGM plates with variable thickness. **Lee and Kim** [45] investigated the post-buckling behavior of FGM plates in hygro-thermal environments based on the FSDT and von-Karman non-linearity. Properties of materials were graded in the thickness direction using power-law function. It was concluded that the effect of moisture on the post-buckling behavior significantly increases with the increase in the value of power-law parameter. **Song** [29] presented Numerical results shows that GPL distribution pattern, weight fraction, and geometry and size have significant influences on the bending and buckling behaviors of the functionally graded GPL reinforced nanocomposite plate.

4.2.1.2 Thermo-mechanical load case

Na and Kim [46] [47] [48] employed 18-node solid element and assumed the strain mixed formulation for 3D thermal buckling and post-buckling analysis of FGM plates with temperature dependent properties. **Jaberzadeh et. al.** [49] used EFG method and approximation to study the thermal buckling of FGM skew and trapezoidal plates based on CPT. **Jalali et. al.** [50] presented the finite element method which involves pseudo-spectral method and collocation method to examine the thermal stability of laminated FGM circular plates based on FSDT and subjected to uniform temperature rise. **Zandekarimi** [51] investigated the size-dependent thermal buckling and post-buckling behavior of FG circular micro-plate under uniform temperature rise field and in clamped boundary conditions.

4.2.2 Meshless method

Different meshless methods are utilized for linear and non-linear buckling analyses of FGM plates and FGM sandwich plates.

4.2.2.1 Mechanical load case

A 2D elastic plane stress problem of FGM plates on the Mindlin's plate assumptions are subjected to non-linearity distributed in-plane edge loads was investigated by **Chen and Liew** [52]**. Mahdavian** [53] applied the Airy stress field approach and Galerkin's approach for the stability analysis of FGM plates subjected to non-uniform in-plane compressive loads based on CPT and von-Karman non-linearity. Here, four types of loading were considered i.e. concentrated load, triangular load, uniform load, reverse rectangular load and sinusoidal load. The critical buckling load coefficient was found to be maximum in sinusoidal load and minimum in concentrated load. **Zhang** [54] practiced stability and local bifurcation behavior for a simply-supported FG rectangular plate subjected to the transverse and in-plane excitations in the uniform thermal environment. **Tran** [55] presented study intends to analyze nonlinear buckling behavior of functionally graded (FG) plates under thermal loading by a mesh-free method. Buckling formulations is derived by HSDT.

4.2.2.2 Thermo-mechanical load case

Lanhe et. al. [56] practiced the dynamic stability of thick FGM plates based on the FSDT and subjected to aero-thermo-mechanical loads. **Lee et. al.** [57] extended the post-buckling analysis of FGM plates based on FSDT under edge compression temperature field condition. **Malekzadeh** [58] presented 3D thermal buckling analysis using differential quadrature technique, and obtained results were validated by comparing them with HSDT. **Ghannadpour** [59] used the finite strip method for buckling analysis of FGM plates under thermal loading based on CPT and three types of loading were considered i.e. uniform temperature rise, linear temperature change across the thickness and nonlinear temperature change across the thickness. **Zhang et. al.** [60] analyzed the mechanical and thermal buckling behavior of FGM plates based on FSDT using MLPG approach along with moving Kriging interpolation technique which has Kronecker delta function property.

Tran [55] presented study intends to analyze nonlinear buckling behavior of functionally graded (FG) plates under thermal loading by a mesh-free method. Buckling formulations is derived by HSDT.

V. CONCLUSIONS

A review is presented on recent advances in analytical and numerical methods for the buckling and post-buckling FGM plates. Mostly, approaches for the analysis of FGM plates are the expansion of the similar approaches for the analysis used for the composite/isotropic plates. Analytical solutions using 3D elasticity equations are restricted only for FGM plates with exponential and some other simple types of material gradient. Use of the direct displacement method for 3D analysis is not allowed only for the FGM plates under uniform load. In numerical analysis, 3D analysis requires more computational effort and large computer core memory than 2D analysis. Hence, attention is given to the 2D analysis.

Various 2D plates theory are utilized such as CPT, FSDT, TSDT, SSDT, Soldato plate theory, Levinson plate theory, UTST, FOST etc., for FGM plate analysis but, FSDT and TSDT are extensively used. FSDT provide reasonably accurate results at very less computational cost, but it requires a shear correction factor to satisfy unrealistic variation of shear/strain through thickness. Various researchers proved that meshless method is reliable alternative for the finite element methods, regardless of the variation of plate thickness and ceramic/metal contents. This method also suffers because of high computational cost and difficulty in treating boundary conditions

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