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ANALYSIS RESIDUAL STRESSES AND VOLUME FRACTION ON FGM COMPOSITE BEAM BY MATLAB TOOL

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ABSTRACT

Functionally graded materials (FGM) are most commonly used for barrier coating against large thermal gradient & simplification reduces modeling complexity and computation requirements but sacrifices the accuracy of through the thickness information. Now a day's FG materials are replacing the composite materials because in high temperature environment various discontinuities like cracks, debonding, delamination etc. are accounted in composite material. In FGM, variation of material properties are continues across the thickness. This investigation explores the effects of spatial temperature variation in the axial and through the thickness direction of the proposed 3-layer FGM composite. Then micromechanical modeling of functionally graded thermal barrier coating is considered to predict stresses under thermal and mechanical loading. In mechanical loading uniformly distributed load is subjected to FG cantilever beam and in thermal loading temperature difference is used for obtaining the axial stress results and then compared with the previous research work

Keywords: FGM temperature variation, Residual stresses & Thermal stress

I. THEORY

1.1 Motivation

In recent time composite material are changed in functionally graded materials (FGMs). Which are advanced multiphase composites and have a smooth spatial variation in material. Functionally graded materials (FGMs) are made from a chemical-alloy mixture of metals and ceramics. FGMs are useful for many engineering sectors such as the aerospace, aircraft, automobile, and defense industries, spring and most recently the electronics and biomedical sectors [1]. A functionally graded material (FGM) is made from metal & ceramic. Ceramic have mechanically brittle and good high-temperature behavior. Another may be a metal which is exhibits better mechanical and heat-transfer properties but cannot withstand to high temperatures. But ceramic is the hotter region and metal is the cooler region. In the high-temperature condition, the strength of the metal is reduced.

1.2Drawbacks of Laminated Composites

The laminated composite materials provide the design flexibility, stiffness and strength. The anisotropic constitution of laminated composite structures often result in stress concentrations near material and geometric discontinuities that can damage in the form of matrix cracking and adhesive bond separation. FGMs alleviate these problems because of a continuous variation of material properties from one surface to other.

1.3 FGMs Applications

A wide variety of applications exist for smart FGM structures. Aerospace, Engineering, Nuclear energy, Optics, Electronics, Bone, Biomaterials.

1.4 Research Goal

The material (FGM) properties are usually continuous variation in one direction. So the temperature distribution used in several applications such as nuclear reactors, ovens, space shuttles, aircrafts and combustion chambers.. The aim of this research is to determine the thermal and normal stresses generation and deflection in neutral axis of FGM materials which is substitute of traditional materials. The study will focus on the modeling and imitation of:

1. Functionally graded beam structures with material properties varying throughout the thickness of the beam.
2. Relationship & graph generation between according to variation of thickness with different property of the material. Example: Residual Stresses, thermal expansion, modulus of elasticity, modulus of rigidity, thermal conductivity, poison ratio etc.
3. Thermal gradient due to one-dimensional through-thickness steady heat conduction is considered.
4. The material properties are taken from literature which having a smooth temperature variation usually in one direction heat flow in FGM for Different material. Examples SIC- C, Al_2O_3 – Steel or Al_2O_3 – (W, Ti)C.

II. LITERATURE REVIEW

YEAR	Progress of FGM layer
1984	Original concept by Dr. Niino and other material scientists in Sendai area, Japan
1986	Feasibility study on 'The basic technology for the development of functionally gradient materials for relaxation of thermal-stress' under the auspice of Science and Technology Agency (STA)
1987-89	National project 'Research on the basic technology for the development of Functionally gradient materials for relaxation of thermal-stress', FGM PART I--Phase 1
1988	Functionally Gradient Materials Forum (FGMF). 1st FGM Symposium organized by FGMF
1989	Germany-Japan FGM Seminar in K61n, Germany
1990	1st International Symposium on FGM in Sendai, Japan. International Advisory Committee of FGM (IACFGM)
1990-91	FGM PART I-Phase 2
1992	Feasibility study on R & D of FGMs as functional materials under the auspice of STA. 2nd International Symposium on FGM and Japan-Germany USA International Workshop in San Francisco, USA
1993	National project 'Research on energy conversion materials with functionally gradient structure' FGM PART II--Phase 1

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III. CALCULATION

3.1 FGM Material Structure Composition

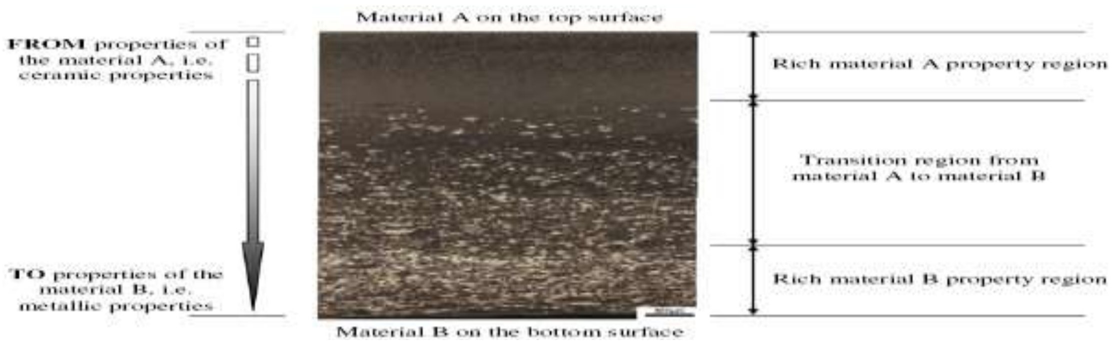

Figure. 3.2a Illustration of the FGM concept by means of microphotography for FGM [2].

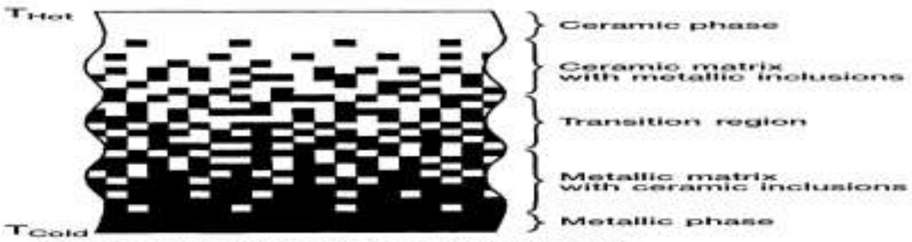

Figure 3.2b Graphical FGM Representation of Gradual Transition in the Direction of the Temperature Gradient

***<u>3.Calculation</u>* :-**

3.1 Volume fraction distribution law's of FGMs

In Power Law (P-FGM), a model is created that describes the function of composition throughout the material. In Figure 3.3b, the volume fraction V_c, describes the volume of ceramic at any point z across, the thickness h according to a parameter n which controls the shape of the function [3].

$$V_c(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^n \qquad (3.1)$$

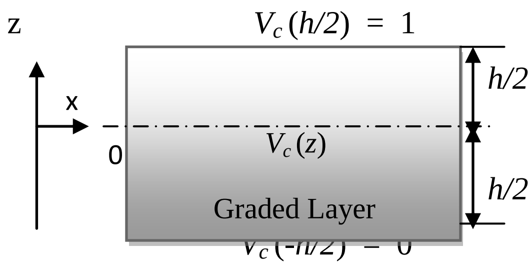

Figure 3.3b Ceramic Volume Fractions Across the FGM Layer

In the law of FGM denoted the volume fraction of metal, $V_m(z)$, in the FGM is $1\text{-}V_c(z)$. A graphical representation of volume fraction of ceramic for various values of the parameter n can be seen in Figure 3.4.

The area to the right of each line represents the amount of metal, and the area to the left represents the amount of ceramic in the material. It should be noted that $n \to 0$, the material approaches to a homogeneous ceramic, while as $n \to \infty$, the material becomes entirely metal. For $0 < n < \infty$, the metal will contain both metal and ceramic. When $n = 1$, the distribution of ceramic and metal is in equal portion. According to Nakamura and Sampath [4], the values of *n* should be taken in the range of **(1/3, 3),** as values outside this range will produce an FGM having too much of one phase.

3.2 Effective Properties of FGM

Effective properties of FGM are obtained by basic three laws i.e. Power Law (P-FGM), Exponential Law (E-FGM) and Sigmoid Law (S-FGM).

Table 3.1 Effective property formulas of FGMs [57]

Material property	Property related formula
Thermal conductivity (k)	$k(z) = k_t\left(1+\frac{3(k_b-k_t)V_m(z)}{3k_tV_m(z)+(k_b+2k_t)V_c(z)}\right)$
Modulus of elasticity (E)	$E(z) = E_t\left(\frac{E_t+(E_b-E_t)(V_c(z))^{2/3}}{E_t+(E_b-E_t)[(V_c(z))^{2/3}-V_c(z)]}\right)$
Poission's ratio (v)	$v(z) = (v_t - v_b)V_c(z) + v_b$
Coefficient of thermal expansion (α)	$\alpha(z) = (\alpha_t-\alpha_b)V_c(z)+\alpha_b+\left(\frac{V_m(z)V_c(z)(\alpha_t-\alpha_b)(k_b-k_t)}{(k_b-k_t)V_c(z)+k_b+(3k_bk_t/4G_m)}\right)$
Density (ρ)	$\rho(z) = (\rho_t-\rho_b)V_c(z)+\rho_b$
Yield strength (σ_y)	$\sigma_y(z)=(\sigma_{yt}-\sigma_{yb})V_c(z)+\sigma_{yb}$

In Table 3.1, *K* and *G* are the bulks modulus and modulus of rigidity, respectively. Also, the undefined parameters are given by

$$K_t = \frac{Et}{3(1-2Vt)} \quad ; \quad G_t = \frac{Et}{2(1+Vt)} \qquad G_b = \frac{E_b}{2(1+V_b)} \quad ; \quad K_b = \frac{E_b}{3(1-2V_b)}$$

The subscripts *t* and *b* stand for the material property at the top and bottom, respectively for the corresponding property. *t* corresponds to the material property of the pure ceramic, and *b* corresponds to the material property of the pure metal

Table 3.2 Effect of Power Law Index (n) on the Volume Fraction

Thickness z(m)	Power index (n=.4)	Power index (n=.8)	Power index (n=1)	Power index (n=1.8)	Power index (n=2.8)
.015	1	1	1	1	1
.01	1	1	1	1	1
.0075	.9480	.8987	.8750	.7863	.6881
.005	.8913	.7944	.7500	.5958	.4469
.0025	.8286	.6866	.6250	.4291	.2682
0	.7579	.5743	.500	.2892	.1436
-.0025	.6755	.4563	.3750	.1711	.0642
-.005	.5743	.3299	.2500	.0825	.0206
-.0075	.4353	.1895	.1250	.0237	.003
-.01	0	0	0	0	0
-.015	0	0	0	0	0

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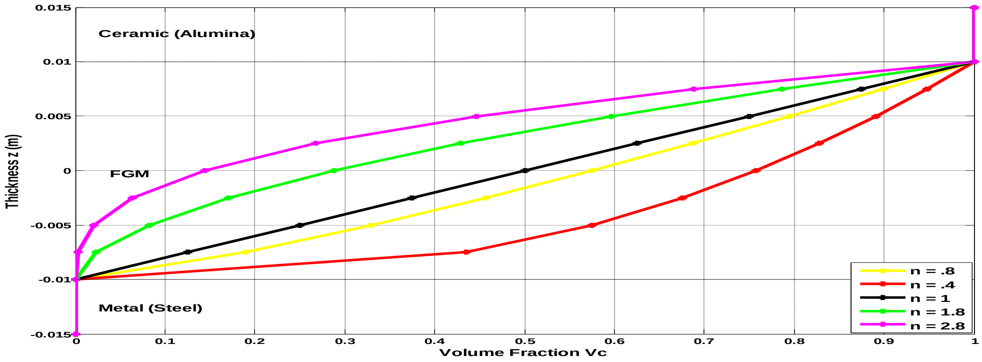

Figure 3.4 (a) Effect of Power Law Index (n) on the Volume Fraction

Table 3.3 Effect of Power Law Index (n) on the Young modulus E

Thickness z(m)	Power index (n=.4)	Power index (n=.8)	Power index (n=1)	Power index (n=1.8)	Power index (n=2.8)	Power index (n=3.8)
.015	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}
.01	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}
.0075	3.80×10^{11}	3.71×10^{11}	3.67×10^{11}	3.51×10^{11}	3.338×10^{11}	3.18×10^{11}
.005	3.70×10^{11}	3.53×10^{11}	3.45×10^{11}	3.17×10^{11}	2.904×10^{11}	2.70×10^{11}
.0025	3.59×10^{11}	3.33×10^{11}	3.22×10^{11}	2.87×10^{11}	2.582×10^{11}	2.40×10^{11}
0	3.46×10^{11}	3.13×10^{11}	3.00×10^{11}	2.61×10^{11}	2.358×10^{11}	2.22×10^{11}
-.0025	3.31×10^{11}	2.92×10^{11}	2.77×10^{11}	2.40×10^{11}	2.2155×10^{11}	2.14×10^{11}
-.005	3.13×10^{11}	2.69×10^{11}	2.55×10^{11}	2.24×10^{11}	2.137×10^{11}	2.109×10^{11}
-.0075	2.88×10^{11}	2.44×10^{11}	2.32×10^{11}	2.14×10^{11}	2.105×10^{11}	2.1007×10^{11}
-.01	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}
-.015	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}

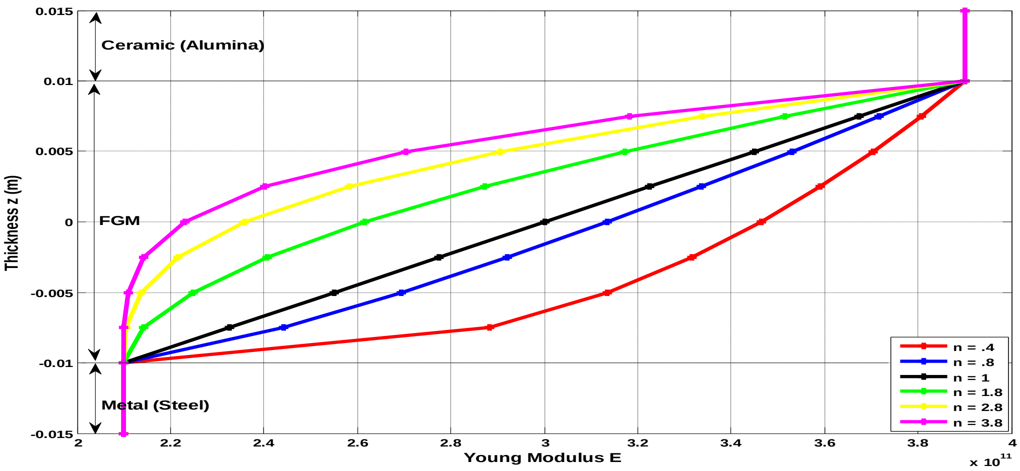

Figure 3.4 (b) Effect of Power Law Index (n) on the Young modulus E

One of most common methods to determine the effective properties of FGM is the rule of mixtures and is given by

$$P(z) = (P_t - P_b)V_c(z) + P_b \quad (3.2)$$

$$E(z) = (E_t - E_b)V_c(z) + E_b \quad (3.3)$$

Where P_t and P_b represent the material property of the top and bottom respectively. P_t corresponds to P_c or the material property of the pure ceramic and P_b corresponds to P_m or the material property of the pure metal. This equation holds true for the modulus of elasticity, density, thermal expansion, thermal conductivity and Poisson's ratio.. The material property for the center of the discrete layer within the beam should be determined using the above approaches and applied to the entire layer of the FGM. Using the power law index (n), total composition of ceramic percentage in a composite material can be determined with the help of Equation 3.4 as seen below. This composition helps to understand the basic characteristics of materials

$$V_{total,ceramic} = \frac{1}{n+1} \quad (3.4)$$

Another valuable parameter which can be determined from the value of *n* is the average material property across the thickness. This is derived from Equation 3.2 and Equation 3.4. This expression can be used for quick calculations treating the material as homogeneous such as solving for the volume fraction of a functionally graded structure. The average material property can be determined as [5].

$$P_{average} = \frac{P_t - P_b}{n+1} + P_b \quad (3.5)$$

Another quantity that might be interested in for modeling or judging the basic properties of the material is the point within the FGM at which the volume fraction transition from mostly metal to mostly ceramic is given as [6].

$$\frac{z_{transition}}{h} = 0.5^{1/n} - \frac{1}{2} \quad (3.6)$$

The value of n along with the materials used defines the characteristics of the material composition and can be tailored to produce desired result. Structural designers requiring significant thermal protection should consider low values of *n* which will yield a ceramic rich panel. Many researchers used Exponential Law (E-FGM) to describe the material properties of FGM. This function is more convenient than power law because there is no need to take power index and the properties of FGM are totally dependent on ceramic and material properties. It directly generates the young's modulus across the thickness and change according to exponential law as given below.

$$E(z) = Ae^{\beta(z+\frac{h}{2})} \quad (3.7)$$

$$A = E_m \quad \text{and} \quad \beta = \frac{1}{h}\ln\left(\frac{E_c}{E_m}\right)$$

$$E(z) = E_m\, e^{\frac{1}{h}\ln\left(\frac{E_c}{E_m}\right)(z+\frac{h}{2})}$$

Here, E_c and E_m represent the modulus of elasticity of top (ceramic) and bottom (metal). The material distribution in the thickness direction of the E-FGM beam is plotted in Figure 3.5.

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Table 3.4 Variation of Young's Modulus in an E-FGM Beam across the Thickness

Thickness z(m)	Young modulus Ez ($\times 10^{11}$)
.015	3.9
.01	3.9
.0075	3.609
.005	3.340
.0025	3.092
0	2.868
-.0025	2.648
-.005	2.451
-.0075	2.269
-.01	2.1
-.015	2.1

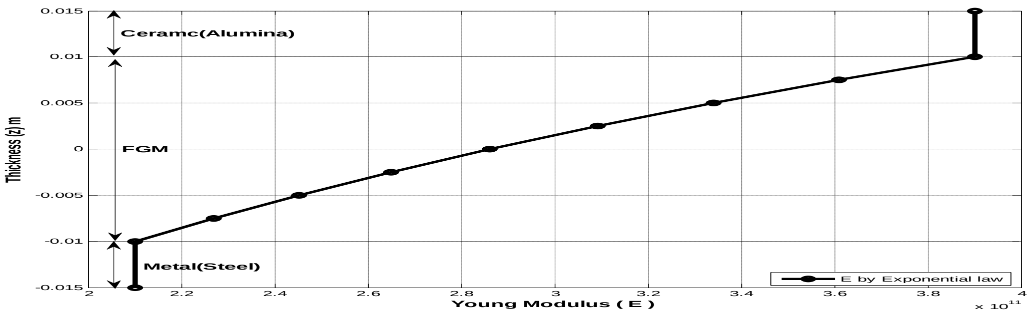

Figure 3.5 Variation of Young's Modulus in an E-FGM Beam across the Thickness

Now the use of Sigmoid Law (S-FGM) is discusses when a single power-law function is considered for the multi-layered composite, stress concentrations appear on one of the interfaces where the material is continuous but changes rapidly. Therefore, Chung and Chi (2001) defined the volume fraction using two power-law functions to ensure smooth distribution of stresses among all the interfaces. The two power law functions are defined by:

$$V_1(z) = 1 - \frac{1}{2}\left(\frac{h/2-z}{h/2}\right)^n \quad \text{for} \quad 0 \le z \le h/2 \tag{3.8}$$

$$V_2(z) = \frac{1}{2}\left(\frac{h/2+z}{h/2}\right)^n \quad \text{for} \quad -h/2 \le z \le 0 \tag{3.9}$$

By using rule of mixture, the Young's modulus of the S-FGM can be calculated by:

$$E(z) = V_1(z)E_c + [1 - V_1(z)]E_m \quad \text{for} \quad 0 \le z \le h/2 \tag{3.10}$$

$$E(z) = V_2(z)E_c + [1 - V_2(z)]E_m \quad \text{for} \quad -h/2 \le z \le 0 \tag{3.11}$$

Table 3.5 Variation of Young modulus in an S-FGM Beam across the Thickness.

Thickness z(m)	Power index (n=.4)	Power index (n=.8)	Power index (n=1)	Power index (n=1.8)	Power index (n=2.8)	Power index (n=3.8)
.015	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}
.01	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}	3.9×10^{11}
.0075	3.38×10^{11}	3.603×10^{11}	3.675×10^{11}	3.825×10^{11}	3.884×10^{11}	3.895×10^{11}
.005	3.21×10^{11}	3.383×10^{11}	3.45×10^{11}	3.641×10^{11}	2.770×10^{11}	2.835×10^{11}
.0025	3.09×10^{11}	3.185×10^{11}	3.225×10^{11}	3.363×10^{11}	2.497×10^{11}	2.598×10^{11}
0	3×10^{11}	3×10^{11}	3×10^{11}	3×10^{11}	3×10^{11}	3×10^{11}
-.0025	2.902×10^{11}	2.815×10^{11}	2.775×10^{11}	2.636×10^{11}	2.502×10^{11}	2.401×10^{11}
-.005	2.782×10^{11}	2.616×10^{11}	2.55×10^{11}	2.358×10^{11}	2.229×10^{11}	2.164×10^{11}
-.0075	2.616×10^{11}	2.396×10^{11}	2.325×10^{11}	2.174×10^{11}	2.118×10^{11}	2.104×10^{11}
-.01	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}
-.015	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}

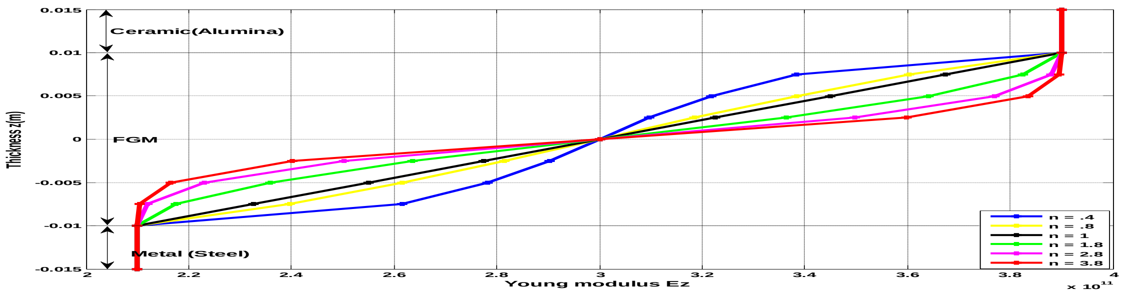

Figure 3.6 Variation of Young modulus in an S-FGM Beam across the Thickness.

3.6 FGM Applications

There is an extensive variety of applications in engineering practice which requires materials performance to vary with locations within the component. One of these applications is shown in Figure 3.7, where FGM is used to improve the thermo-mechanical performance by the help of volume fraction change according to young modulus variation along with the thickness.

Another application, is shown in Figure 3.8, where graded region (FGM) is defined the thermal conductivity variation in metal to ceramic tip & ceramic to metal tip with minimum to maximum density of mixture of metal and ceramic. There are many more current and future applications for FGM

IV. CONCLUSIONS

Functionally graded materials are good replacement of composite materials because they overcome the debonding type problems. These materials are commonly used in aerospace industries where the harsh temperature is major issue. The basic properties of FGM can be obtained by any of the three function laws, power law (P-FGM), sigmoid law (S-FGM) and exponential law (E-FGM).

In the present work the beam model simulation with temperature distribution is taken, distribution of temperature profile in one dimensional analysis in different FGM composite material for high heat insulation property & explore the effects of spatial temperature variation in the axial and through the thickness and compare three FGM composite and find out alumina and carbon (W,Ti) has low residual stresses than the other two. So for designing purpose this FGM composite is better than the other two.

The axial stresses in FGM beam under uniformly distributed load and the residual stresses of the FGM composite.These results are found to be compared with the different function law (P-FGM, S-FGM & E-FGM) & previous work.

V. FUTURE SCOPE OF WORK

The following recommendations and future work is suggested.

- We recommend further investigation of functionally graded beam structures with material properties varying in directions other than through the thickness
- A further investigation regarding the techniques for estimating effective material properties of functionally graded materials is desirable. In the graded layer of real FGMs, ceramic and metal particles of arbitrary shapes are mixed up in arbitrary dispersion structures. Hence, the prediction of the thermo-elastic properties is not a simple problem, but complicated due to the shape and orientation of particles, the dispersion structure, and the volume fraction.
- The thickness of the middle FGM can be optimized.

REFRENCES

[1.] Ichikawa K, editor. Functionally graded materials in the 21st century: a workshop on trends and forecasts, Japan. Kluwer Academic Publishers, 2000.

[2.] Shao, Z. S. and Wang, T. J. “Three-Dimensional Solutions for the Stress Fields in Functionally Graded Cylindrical Panel with Finite Length Subjected to Thermal/Mechanical Loads”. International Journal of Solids and Structures, 43:3856-3874,2006.

[3.] Reddy, J. N. “Mechanics of Laminated Composite Plates and Shells: Theory and analysis”. CRC Press, Boca Raton.

[4.] Nakamura, T., Wang, T., and Sampath, S., “Determination of Properties of Graded Materials By Inverse Analysis and Instrumented Indentation,” Acta Materialia, 48(17):4293-4306. 2000.

[5.] Cooley, W. G., and Palazotto, A. “IMECE2005-82776 Finite Element Analysis of Functionally Graded Shell Panels under Thermal Loading,” ASME-Publications-AD, 70:517-526, 2005.

[6.] Miyamoto, Y., Kaysser, W., Rabin, B., Kawasaki, A., and Ford, R. “Functionally Graded Materials: Design Processing and Applications,” Kluwer Academic Publication, Boston, 1999.

[7.] B.V. Sankar, An elasticity solution for functionally graded beams,Compos Sci Tech, 61 (2001), pp. 689–696.

[8.] H. Zhu, B.V. Sankar, A combined Fourier series-Galerkin method for the analysis of functionally graded beams, J Appl Mech Trans ASME, 71 (2004), pp. 421–424.

[9.] B.V. Sankar, J.T. Tzeng, Thermal stresses in functionally graded beams, AIAA J, 40 (2002), pp. 1228–1232.

[10.] R.E. Rossi, P.A.A. Laura, R.H. Gutierrez, A note on transverse vibration of a Timoshenko beam of non-uniform thickness clamped at one end and carrying a concentrated mass at the other, J Sound Vib, 143 (1990), pp. 491–502.