# ANALYSIS OF TEMPERATURE DISTRIBUTION AND THERMAL STRESSES ON FGM COMPOSITE BEAM BY MATLAB TOOL

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## ABSTRACT

*Functionally graded materials (FGM) are most commonly used for barrier coating against large thermal gradient & simplification reduces modeling complexity and computation requirements but sacrifices the accuracy of through the thickness information. Now a day's FG materials are replacing the composite materials because in high temperature environment various discontinuities like cracks, debonding, delamination etc. are accounted in composite material. In FGM, variation of material properties are continues across the thickness. This investigation explores the effects of spatial temperature variation in the axial and through the thickness direction of the proposed 3-layer FGM composite. Then micromechanical modeling of functionally graded thermal barrier coating is considered to predict stresses under thermal and mechanical loading. In mechanical loading uniformly distributed load is subjected to FG cantilever beam and in thermal loading temperature difference is used for obtaining the axial stress results and then compared with the previous research work. Thermo-mechanical stress distribution for a three layered FGM composite beam having a middle layer of FGM is obtained by analytical method. The performance is evaluated by taking young's modulus as per power law (P-FGM), sigmoid law (S-FGM) and exponential law (E-FGM) across the thickness. The mathematical tool MATLAB is employed for generating the code.*

***Keywords: FGM temperature variation, Residual stresses & Thermal stress***

## I. THEORY

### 1.1 Motivation

In recent time composite material are changed in functionally graded materials (FGMs). Which are advanced multiphase composites and have a smooth spatial variation in material. Functionally graded materials (FGMs) are made from a chemical-alloy mixture of metals and ceramics. FGMs are useful for many engineering sectors such as the aerospace, aircraft, automobile, and defense industries, spring and most recently the electronics and biomedical sectors [1]. A functionally graded material (FGM) is made from metal & ceramic. Ceramic have mechanically brittle and good high-temperature behavior.

**1.2Drawbacks of Laminated Composites**

The laminated composite materials provide the design flexibility, stiffness and strength. The anisotropic constitution of laminated composite structures often result in stress concentrations near material and geometric discontinuities that can damage in the form of matrix cracking and adhesive bond separation. FGMs alleviate these problems because of a continuous variation of material properties from one surface to other.

**1.3 FGMs Applications**

A wide variety of applications exist for smart FGM structures. Aerospace, Engineering, Nuclear energy, Optics, Electronics, Bone, Biomaterials.

**1.4 Research Goal**

The material (FGM) properties are usually continuous variation in one direction. So the temperature distribution used in several applications such as nuclear reactors, ovens, space shuttles, aircrafts and combustion chambers.. The aim of this research is to determine the thermal and normal stresses generation and deflection in neutral axis of FGM materials which is substitute of traditional materials. The study will focus on the modeling and imitation of:

1. Functionally graded beam structures with material properties varying throughout the thickness of the beam.
2. Relationship & graph generation between according to variation of thickness with different property of the material. Example: Residual Stresses, thermal expansion, modulus of elasticity, modulus of rigidity, thermal conductivity, poison ratio etc.
3. Thermal gradient due to one-dimensional through-thickness steady heat conduction is considered.
4. The material properties are taken from literature which having a smooth temperature variation usually in one direction heat flow in FGM for Different material. Examples SIC- C, $Al_2O_3$ – Steel or $Al_2O_3$ – (W, Ti)C.
5. Analysis on Elastic thermo-mechanical stresses in FGM structures and thermal modeling with different temperature.

Studies are doing on the static and dynamic thermo-elastic behavior of FGM beams, cantilever or beam-like structures and Mathematical Analysis or Mat-Lab Formulation on thermal stress , thermo-mechanical loads behavior on different martial

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## II. CALCULATION

### 2.1 FGM Material Structure Composition

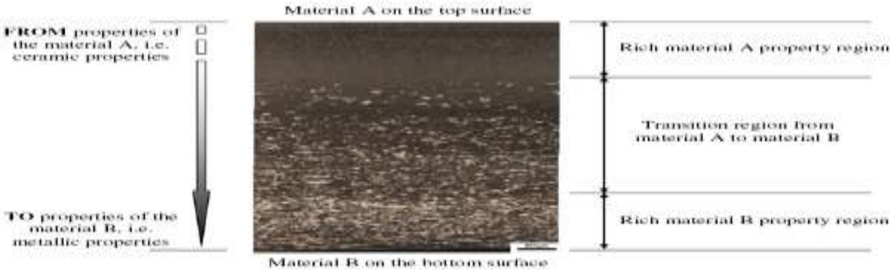


**Figure. 3.2a Illustration of the FGM concept by means of microphotography for FGM [1].**

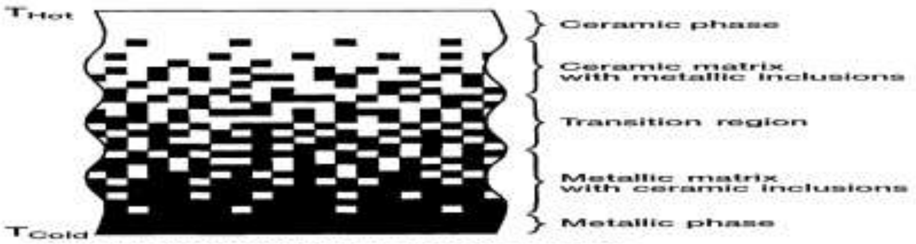


**Figure 3.2b Graphical FGM Representation of Gradual Transition in the Direction of the Temperature Gradient**

***3.Calculation*** **:-**

### 2.2 Volume fraction distribution law's of FGMs

In Power Law (P-FGM), a model is created that describes the function of composition throughout the material. In Figure 3.3b, the volume fraction $V_c$, describes the volume of ceramic at any point $z$ across, the thickness $h$ according to a parameter $n$ which controls the shape of the function [2].

$$V_c(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^n \qquad (3.1)$$

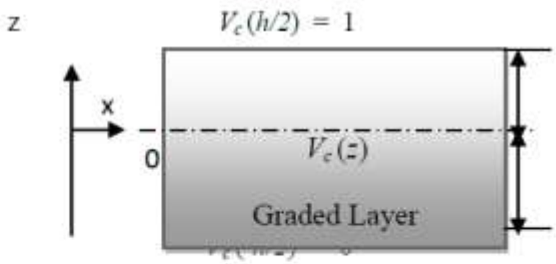


**Figure 3.3b Ceramic Volume Fractions Across the FGM Layer**

### 2.3 Effective Properties of FGM

Effective properties of FGM are obtained by basic three laws i.e. Power Law (P-FGM), Exponential Law (E-FGM) and Sigmoid Law (S-FGM).

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**Table 3.1 Effective property formulas of FGMs [57]**

| **Material property** | **Property related formula** |
|---|---|
| Thermal conductivity $(k)$ | $k(z) = k_t\left(1 + \dfrac{3(k_b - k_t)V_m(z)}{3k_tV_m(z) + (k_b + 2k_t)V_c(z)}\right)$ |
| Modulus of elasticity $(E)$ | $E(z) = E_t\left(\dfrac{E_t + (E_b - E_t)(V_c(z))^{\frac{2}{3}}}{E_t + (E_b - E_t)\left[(V_c(z))^{\frac{2}{3}} - V_c(z)\right]}\right)$ |
| Poission's ratio $(v)$ | $v(z) = (v_t - v_b)V_c(z) + v_b$ |
| Coefficient of thermal expansion $(\alpha)$ | $\alpha(z) = (\alpha_t - \alpha_b)V_c(z) + \alpha_b + \left(\dfrac{V_m(z)V_c(z)(\alpha_t - \alpha_b)(k_b - k_t)}{(k_b - k_t)V_c(z) + k_b + \left(\frac{3k_bk_t}{4G_m}\right)}\right)$ |
| Density ( $\rho$ ) | $\rho(z) = (\rho_t - \rho_b)V_c(z) + \rho_b$ |
| Yield strength $(\sigma_y)$ | $\sigma_y$ (z)=( $\sigma_{yt} - \sigma_{yb}$)"V" $_{"c"}$ ("z" ) $+ \sigma_{yb}$ |

In Table 3.1, $K$ and $G$ are the bulks modulus and modulus of rigidity, respectively. Also, the undefined parameters are given by

$$K_t = \frac{Et}{3(1 - 2Vt)} \;;\quad G_t = \frac{Et}{2(1 + Vt)} \qquad G_b = \frac{E_b}{2(1 + V_b)} \;;\quad K_b = \frac{E_b}{3(1 - 2V_b)}$$

The subscripts $t$ and $b$ stand for the material property at the top and bottom, respectively for the corresponding property. $t$ corresponds to the material property of the pure ceramic, and $b$ corresponds to the material property of the pure metal

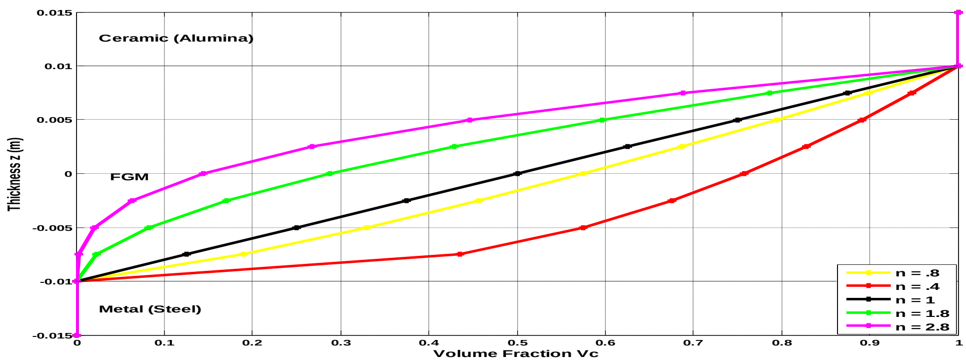


**Figure 3.4** (a) Effect of Power Law Index (n) on the Volume Fraction

One of most common methods to determine the effective properties of FGM is the rule of mixtures and is given by

$$P(z) = (P_t - P_b)V_c(z) + P_b \tag{3.2}$$

$$E(z) = (E_t - E_b)V_c(z) + E_b \tag{3.3}$$

## III. FORMULATION OF GOVERNING EQUATIONS

### 3.1.1 One-dimensional Heat Conduction Steady-State Exact Solution for 3-Layer FGM beam

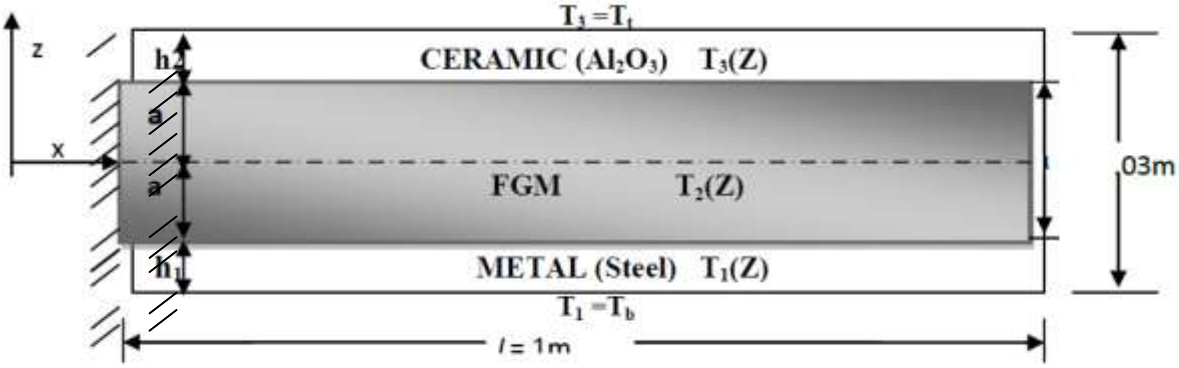


**Figure 4.1 Three layer beam with perfect thermal contact at the interface surface**

**The mathematical formulation of this problem is given with boundary condition as**

$$\frac{d}{dz}\left[\frac{k_1 dT_1(z)}{dz}\right]=0, \qquad -(h_1+a)< z < -a \tag{4.1}$$

$$\frac{d}{dz}\left[\frac{k_2 dT_2(z)}{dz}\right]=0, \qquad -a< z < a \tag{4.2}$$

$$\frac{d}{dz}\left[\frac{k_3 dT_3(z)}{dz}\right]=0, \qquad a< z < (a+h_2) \tag{4.3}$$

Subject to boundary and interface condition

$$T_1 = T_b \quad \text{at} \quad z = -(h_1 + a) \tag{4.4}$$

$$\left.\begin{aligned}\frac{k_1 dT_1(z)}{dz} &= \frac{k_2 dT_2(z)}{dz} \\ T_1 &= T_2\end{aligned}\right\} \tag{4.5}$$

$$\text{at} \quad z = -a \tag{4.6}$$

$$\frac{k_2 dT_2(z)}{dz} = \frac{k_3 dT_3(z)}{dz} \tag{4.7}$$

$$T_2 = T_3 \quad \text{at} \quad z = a \tag{4.8}$$

Where $k_1, k_2$ and $k_3$ are the thermal conductivity coefficient for metal (steel), graded layer, and ceramic (alumina).the solution to the equation (4.1-4.3) subjected to the boundary and interface condition given by Eqs.( 4.4-4.8) can be found the numerically. In Special cases can results in exact solution such as when $k_1 = k_b$ and $k_3 = k_t$ are constant throughout layers 1 and 3, while $k_2(z)$ is assumed to vary only in direction of the beam thickness

$$k_2(z) = k_t e^{-.5\ln\left(\frac{k_t}{k_b}\right)\left(1-\frac{z}{a}\right)} \tag{4.9}$$

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The solution of the ordinary differential equation (4.1-4.3) for each layer is given in form

$T_1\ (z)= C_1 z\ \ + C_2$ 4.10

$T_2\ (z)= C_3 \left(\frac{k_t}{k_b}\right)^{-\frac{.5z}{a}} + C_4$ 4.11

$T_3\ (z)= C_5 z\ \ + C_6$ 4.12

**3.2 Beam Theory for Stress Calculations**

$$V_m(z) = 1 - V_c(z) \qquad (4.13)$$

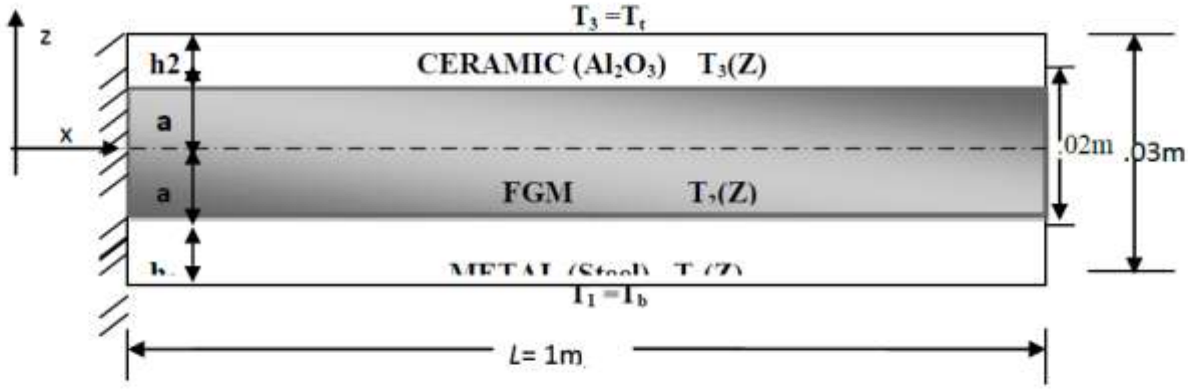


**Figure 4.2 Three Layer Composite FGM Beam under Distributed Load**

The mathematical modeling for evaluating the properties of functionally graded materials ( P(z)) or $P_b$ is the bottom layer property and $P_t$ is the top layer property which are chosen from any of the three laws expresses as per the Equations 3.2, 3.7, 3.9-3.11.

$$P(z) = P_b + (P_t - P_b)V_c(z) \qquad (4.14)$$

The basic assumptions are derived by that laws which is:

1. The beam is assumed to be in a state of plane strain, it is normal to the xz plane.
2. Euler-Bernoulli type beam theory is applied.
3. There is no variation in thickness along the length of beam.
4. Poisson's ratio is to be held constant along FG layer.
5. Material properties are independent of temperature gradient.

For a cantilever beam, the displacement field can be written as [51]:

$$w\ (x, z) = w(x)$$

$$u\ (x, z) = u_0(x) - z\ \frac{dW(x)}{dx}$$

In above equations, u and w are denoted as horizontal and vertical displacement of beam across the thickness. It may be noted that $u_0$ denotes displacement of points on the middle surface of the beam along the x direction. It is assumed that $\sigma_{zz}$ is negligible. Then the stress-strain relations take the form:

$$\sigma_x(z) = \breve{E}(z)\varepsilon_x\ , \qquad \tau_{xz}(z) = \breve{G}(z)\gamma_{xz} \qquad (4.15)$$

Where the plane strain Young modulus is given by:

$$\breve{E} = \frac{E}{1-\nu^2}$$

The expressions for axial strain and stress can be derived as:

$$\varepsilon_x = \frac{du(x,z)}{dx} = \frac{d}{dx}\left(\mathbf{u_0}(x) - z\frac{dW(x)}{dx}\right) = \varepsilon_{x0} + zk_x$$

$$\varepsilon_{x0} = \frac{du_0}{dx} \quad , \quad k_x = -\frac{d^2w(x)}{dx^2}$$

$$\sigma_x(z) = \breve{E}(z).\varepsilon_{x0} + z.\breve{E}(z).k_x \tag{4.16}$$

$$\begin{bmatrix}\sigma_x \\ \sigma_z \\ \tau_{xz}\end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{13} & 0 \\ Q_{13} & Q_{33} & 0 \\ 0 & 0 & Q_{55}\end{bmatrix}\begin{bmatrix}\varepsilon_x \\ \varepsilon_z \\ \gamma_{xz}\end{bmatrix} \tag{4.17}$$

$$[\bar{Q}_{ij}] = [Q_{ij}]$$

$$Q_{11} = \frac{E}{1-\nu^2} = Q_{33} \quad , \quad Q_{13} = \frac{\nu E}{1-\nu^2} \quad , \quad Q_{55} = \frac{E}{2(1+\nu)}$$

Here, $[\bar{Q}_{ij}]$ and $[Q_{ij}]$ both are stiffness matrices and $\varepsilon_{x0}$, k are axial strain in the middle surface and the beam curvature. According to Euler-Bernoulli beam theory, the axial force and bending moment, N and M, are defined

$$(N, M) = \int_{-h_1}^{h_1} [\breve{E}(z).\varepsilon_{x0} + z.\breve{E}(z).k](1, z)dz \tag{4.18}$$

$$C_0\varepsilon_{x0} + C_1k = 0$$

$$C_1\varepsilon_{x0} + C_2k = M_{maximum}$$

$C_0$, $C_1$, and $C_2$ are the coefficients of mid-plane strain and curvature. Using Equation 4.16, the axial stresses in ceramic, metal and FGM section across the thickness of proposed model are obtained.

**3.3 Temperature Profile modeling for thermal stress formulation**

When proposed FGM beam model is subjected to uniform temperature change (ΔT), the total strain under a small strain assumption, can be taken as made up of elastic and thermal part. For a beam under plane strain condition, the only non-zero stress component is $\sigma_x$ [4]:

$$\sigma_x = E(z)[\varepsilon_{x0}^T + z.k^T - \alpha(z)\Delta T] \tag{4.19}$$

Where $\varepsilon_{x0}^T$ is the strain at the mid-plane (z = 0) of the FGM layer and $k^T$ is the laminate curvature due to temperature gradient. Since only thermal loading is considered here:

$$\sum F_x = 0 \quad , \quad \sum M_x = 0$$

On the other hand

$$(N, M) = \int_{-h_1}^{h_1} \sigma_{xx}\,(1, z)dz = 0$$

The axial force and bending moment in thermal gradient can be obtained as given below:

$$N^T = (\Delta T)\sum_{k=1}^{m}[\overline{Q}]_k[\alpha]_k(h_k - h_{k-1}) \tag{4.20}$$

$$M^T = \frac{1}{2}(\Delta T)\sum_{k=1}^{m}[\overline{Q}]_k[\alpha]_k(h_k^2 - h_{k-1}^2) \tag{4.21}$$

Here, m is the number of lamina and in proposed model three laminas is considered. Further thermal strain, mid-plane strain and curvature, mechanical strain and thermal stresses are calculated by below formulas:

$$\{\varepsilon^T\} = (\Delta T)\{\alpha\}$$

$$\begin{bmatrix}\varepsilon_{x0}^T \\ k^T\end{bmatrix} = \begin{bmatrix}A & B \\ B & D\end{bmatrix}^{-1}\begin{bmatrix}N^T \\ M^T\end{bmatrix}$$

$$\{\varepsilon\} = \{\varepsilon_{x0}^T\} + z\{k^T\}$$

$$\{\varepsilon^M\} = \{\varepsilon\} - \{\varepsilon^T\}$$

$$\{\sigma^T\} = [\overline{Q}]\{\varepsilon^M\} \tag{4.22}$$

The coefficient of thermal expansion for FGM is obtained by rule of mixture

$$\alpha(z) = (\alpha_c - \alpha_m)(V_c) + \alpha_m \tag{4.23}$$

## IV. PERFORMANCE EVALUATION

### 4.1.1 Temperature Distribution

**Heat conduction analysis in 3 layer FGM beam from bottom (metal) layer to top (ceramic) layer**

For the different FGMs composite material have temperature distribution $T_1$ (z), $T_2$ (z) and $T_3$ (z) according to different value of the thickness of depth z in the 3 layer FGM beam from eq. (4.10-4.12).

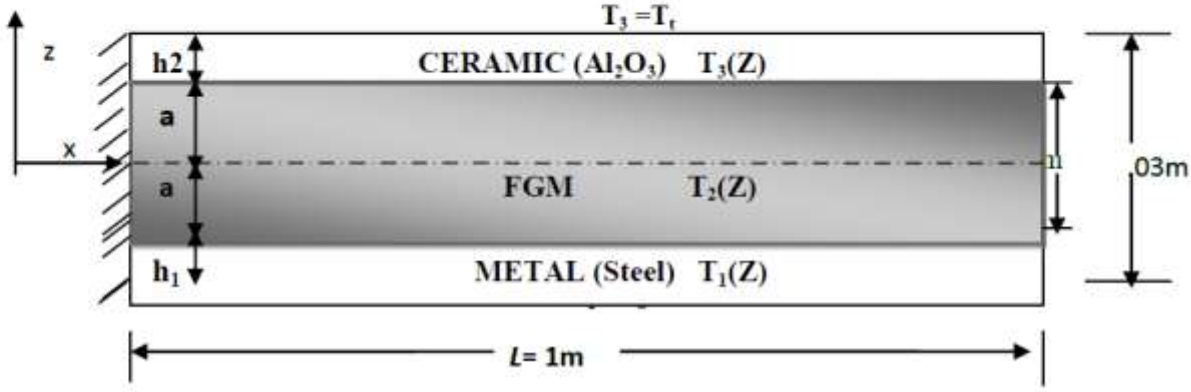


**Figure 5.1 Three layer beam with perfect thermal contact at the interface surface**

Take model figure 5.1 for Heat conduction analysis according temperature distribution in different composite material. The solution of two unknown constants for each layer; then, for a 3-layer problem, are detrmined them by 6 unknown constants. Substituing the solution given by eqs.(4.13-4.18) . then obtains 6 equation for 6 unknown constant the final solution of each layer is:

**Temperature distribution for Metallic (steel, C, (W,Ti)C) and Ceramic ($Al_2O_3$, Sic, $\alpha Al_2O_3$) in gernal equation from**

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$$T_1(z) = \frac{k_t \ln(k_t/k_b)(T_t - T_b)(z+a) + \ln(k_t/k_b)(k_t h_1 T_t + k_b h_2 T_b) + 2aT_b(k_t - k_b)}{(k_t h_1 + k_b h_2)\ln(k_t/k_b) + 2a(k_t - k_b)} \quad (5.1)$$

$$T_2(z) = \frac{2ak_b(k_t/k_b)^{.5\left(1-\frac{z}{a}\right)}(T_t - T_b) + \ln(k_t/k_b)(k_t h_1 T_t + k_b h_2 T_b) + 2a(T_t k_t - T_b k_b)}{(k_t h_1 + k_b h_2)\ln(k_t/k_b) + 2a(k_t - k_b)} \quad (5.2)$$

$$T_3(z) = \frac{k_t \ln(k_t/k_b)(T_t - T_b)(z+a) + \ln(k_t/k_b)(k_t h_1 T_t + k_b h_2 T_b) + 2aT_t(k_t - k_b)}{(k_t h_1 + k_b h_2)\ln(k_t/k_b) + 2a(k_t - k_b)} \quad (5.3)$$

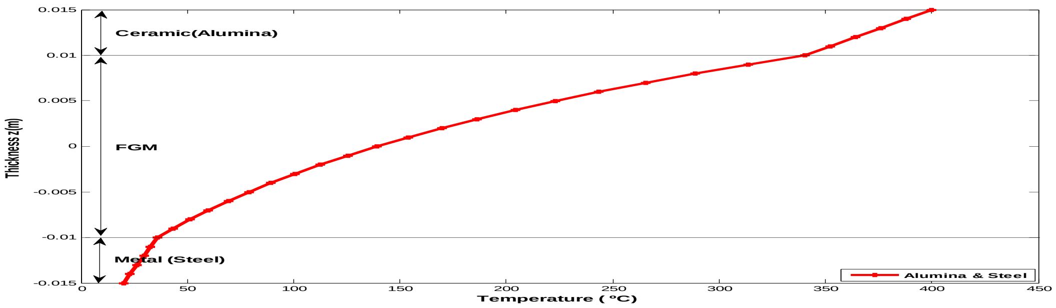



**Figure 5.2(a)** Temperature distribution graph between $Al_2O_3$ & Steel (present work)

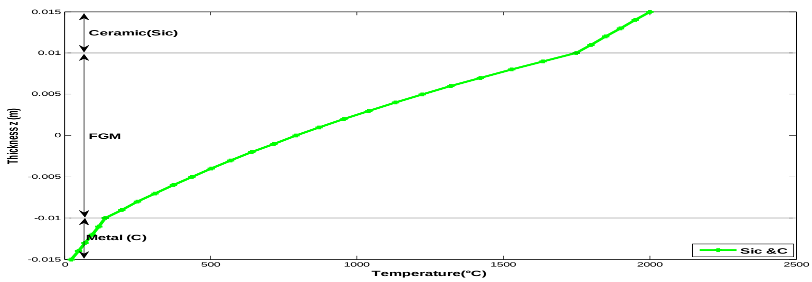
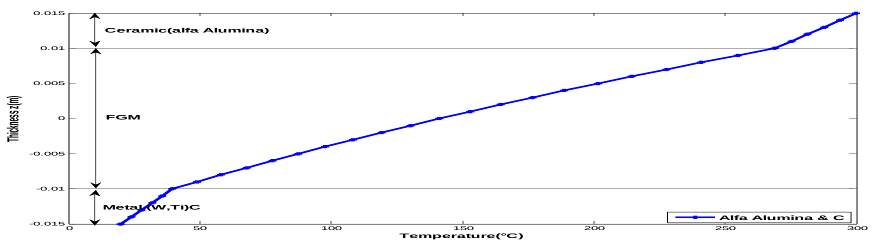



**Figure 5.2 (b)** Temperature distribution graphs between Sic & C (present work)




**5.2 (c)** Temperature distribution graph between $\alpha Al_2O_3$ & (W, Ti) C (present work)

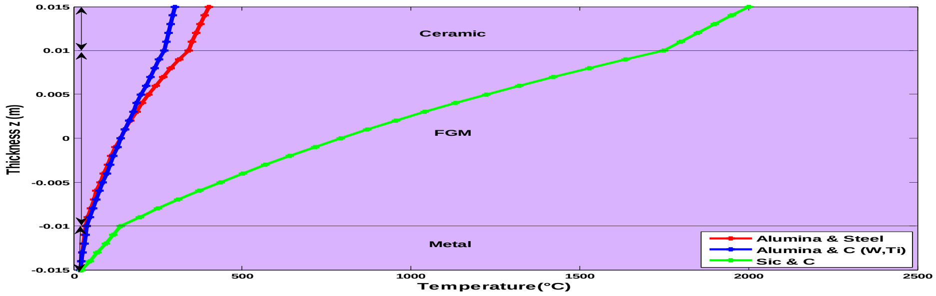



**Figure 5.3** Temperature distribution graph between $\alpha Al_2O_3$ & (W, Ti) C, Sic & C and $Al_2O_3$ & Steel (present work)

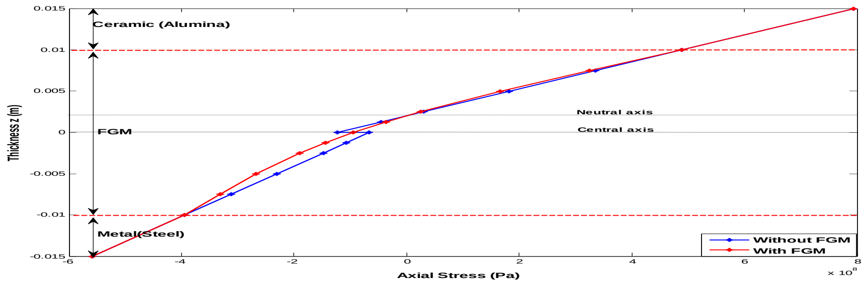



**Figure 5.7** (a) Axial stress with FGM & without FGM ( present Work)

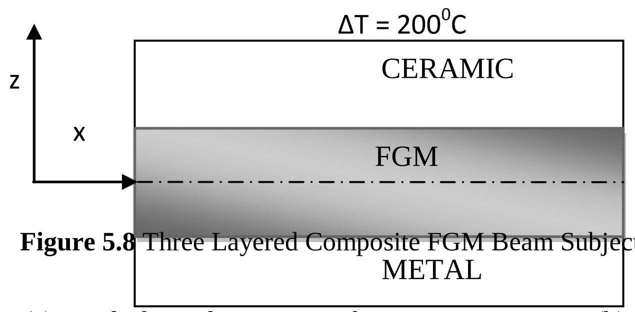



**Figure 5.8** Three Layered Composite FGM Beam Subjected to Thermal Loading

**Figure 5.9**(a) Axial Thermal Stress Distribution in FGM Beam, (b) Reprinted From ref [5].

## V. CONCLUSIONS

Functionally graded materials are good replacement of composite materials because they overcome the debonding type problems. These materials are commonly used in aerospace industries where the harsh temperature is major issue. The basic properties of FGM can be obtained by any of the three function laws, power law (P-FGM), sigmoid law (S-FGM) and exponential law (E-FGM).

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