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Whistler Waves Excited by Relativistic Electron Beam in Cold Plasma

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ABSTRACT

This paper presents the excitation of whistler waves in cold plasma by a relativistic electron beam. Using fluid model and linear first order perturbation theory, a dispersion relation of the whistler wave is obtained. When the phase velocity of the wave is comparable to the velocity of the beam, energy transfer between the beam and the wave takes place. Expressions of growth rate are provided for Cerenkov and cyclotron interactions and their dependence on beam density, plasma electron density and magnetic field is studied. The growth rate of the wave increases with beam density and is proportional to square root of the beam density.

Keywords: Whistler Wave, Frequency, Dispersion Relation, Growth Rate, Cerenkov, Cyclotron

I INTRODUCTION

Whistler waves are circularly polarized waves in the audio frequency range and are often observed in the outer earth's radiation belt and in the auroral kilometric radiation zone. Low frequency whistlers have low velocity, suffer dispersion in ionosphere and magnetosphere and therefore they can be excited by electron beam. The space experiments with injection of electron beams from satellites into the ionospheric plasma stimulate theoretical studies for electromagnetic emissions recorded by Very Low Frequency receivers in the electron whistler frequency range. A large number of papers [1-7] have appeared regarding the theory of Whistler excitation in beam-plasma system. Krafft and Lundin [1] have discussed the compatibility of the spectral characteristics of VLF receivers with the electromagnetic emissions radiated by electron beams artificially injected into the ionospheric plasma during active space experiments. Baranets *et al.* [2] have investigated the wave-particle interaction during simultaneous injection of electron and xenon ion beams from the satellite IK-25. Borcia *et al.*[3] have examined the parallel and oblique propagation of whistler waves generated by relativistic electron beams. In this paper, we present the theory of low frequency whistler wave excitation by an electron beam in homogenous plasma. In Sec. 2, we study the response of beam and plasma electrons to whistler wave perturbation. In Sec. 3, we derive the dispersion relation and growth rate of excited whistlers. Sec. 5 gives the discussion of our results.

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II. INSTABILITY ANALYSIS

Consider a plasma immersed in a magnetic field $B_s \| \hat{z}$, with electron density n_{eo}, electron mass m_e and electron charge e. A uniform relativistic electron beam propagates inside the plasma with density n_{b0}, mass m_e, and equilibrium beam velocity $\mathbf{v_{b0}} = v_{bx}\hat{x} + v_{bz}\hat{z}$. The quasineutrality condition at equilibrium is given by $en_{eo}+en_{bo}=en_{io}$.

An electromagnetic whistler wave propagates through it, with electric field

$$\mathbf{E} = A(x,y)e^{-i(\omega t - k_z z)},$$

The magnetic field of the wave is $\mathbf{B} = c\mathbf{k}\times\mathbf{E}/\omega$, and $\gamma = \gamma_o + \left(\gamma_o{}^3/c^2\right)\left(\mathbf{v_{b0}}\cdot\mathbf{v_{b1}}\right)$ is the relativistic gamma factor.

The equation of motion, governing the perturbed velocity of plasma and beam electrons is

$$m_e\left[\frac{\partial(\gamma\mathbf{v_e})}{\partial t} + \mathbf{v_e}\cdot\nabla(\gamma\mathbf{v_e})\right] = -e\,\mathbf{E} - \frac{e}{c}\mathbf{v_e}\times(\mathbf{B_S}+\mathbf{B}) \tag{1}$$

On linearizing Eq. (1), we obtain the perturbed beam velocities as

$$v_{b1x} = \frac{e}{i\,m_e}\frac{\gamma_o\bar{\omega}^2}{\omega\left(\bar{\omega}^2\gamma_o{}^2 - \omega_{ce}^2\right)}E_x - \frac{e}{m_e}\frac{\gamma_o\bar{\omega}\,\omega_{ce}}{\omega\left(\bar{\omega}^2\gamma_o{}^2 - \omega_{ce}^2\right)}E_y \tag{2}$$

$$v_{b1y} = \frac{e}{m_e}\frac{\gamma_o\bar{\omega}\,\omega_{ce}}{\omega\left(\bar{\omega}^2\gamma_o{}^2 - \omega_{ce}^2\right)}E_x - \frac{i\,e}{m_e}\frac{\gamma_o\bar{\omega}^2}{\omega\left(\bar{\omega}^2\gamma_o{}^2 - \omega_{ce}^2\right)}E_y \tag{3}$$

$$v_{b1z} = -\frac{i\,e}{m_e}\frac{k_z v_{bx}}{\omega\,(\gamma_o+\gamma_1)\bar{\omega}}E_x \tag{4}$$

where $\bar{\omega} = \omega - k_z v_{bz}$, $\omega_{ce} = \frac{e\mathbf{B_S}}{m_e c}$ and

$$\gamma_1 = \left(\gamma_o{}^3/c^2\right)v_{b0}{}^2.$$

Similarly, the perturbed plasma electron velocities obtained from linearization of Eq. (1) are

$$v_{e1x} = -\frac{e\left(i\omega\,E_x + \omega_{ce}\,E_y\right)}{m_e\left(\omega^2 - \omega_{ce}^2\right)} \tag{5}$$

$$v_{e1y} = \frac{e\left(\omega_{ce}E_x - i\omega\,E_y\right)}{m_e\left(\omega^2 - \omega_{ce}^2\right)} \tag{6}$$

Substituting the expressions of perturbed beam velocities from Eqs. (2)-(4) in the equation of continuity, we obtain the perturbed beam number density as

$$n_{b1} = -\frac{i\, n_{bo} e\, k_z}{m_e \omega\ (\gamma_o + \gamma_1)\bar{\omega}^2}\left[k_z v_{bx} E_x\right] \tag{7}$$

The perturbed beam current density is given as

$$J_{b1} = -e\, n_{b0} \mathbf{v_{b1}} - e\, n_{b1} v_{bx} \hat{x} - e\, n_{b1} v_{bz} \hat{z} \tag{8}$$

Writing the x,y and z components of Eq. (8) and using Eqs. (2)-(4) and Eq. (7), we obtain

$$J_{b1x} = \frac{i\, e^2\, n_{bo}}{m_e \omega}\left[\left(\frac{\gamma_o \bar{\omega}^2}{\left(\gamma_o{}^2 \bar{\omega}^2 - \omega_{ce}^2\right)} + \frac{k_z^2\, v_{bx}^2}{(\gamma_o + \gamma_1)\bar{\omega}^2}\right)E_x + \frac{\gamma_o \bar{\omega}\ \omega_{ce}}{i\left(\gamma_o{}^2 \bar{\omega}^2 - \omega_{ce}^2\right)} E_y\right] \tag{9}$$

$$J_{b1y} = \frac{-e^2\ \gamma_o n_{bo}}{m_e \omega\left(\gamma_o{}^2 \bar{\omega}^2 - \omega_{ce}^2\right)}\left[\bar{\omega}\ \omega_{ce} E_x - i\, \bar{\omega}^2\ E_y\right] \tag{10}$$

$$J_{b1z} = \frac{i\, e^2\ n_{bo}}{m_e\,(\gamma_o + \gamma_1)\bar{\omega}^2}\left[k_z v_{bx} E_x\right] \tag{11}$$

The perturbed electron current density is given as $\mathbf{J_{e1}} = -\, n_{e0} e\ \mathbf{v_{e1}}$ (12)

Substituting Eqs. (5) and (6) in Eq. (12), we get

$$J_{e1x} = n_{eo} \frac{e^2}{m_e} \frac{\left(i\omega\ E_x + \omega_{ce}\ E_y\right)}{\left(\omega^2 - \omega_{ce}^2\right)} \tag{13}$$

$$J_{e1y} = -n_{eo} \frac{e^2}{m_e} \frac{\left(\omega_{ce} E_x - i\omega\ E_y\right)}{\left(\omega^2 - \omega_{ce}^2\right)} \tag{14}$$

The low frequency electromagnetic whistler waves are governed by the wave equation given as

$$\nabla^2 \mathbf{E} - \nabla(\nabla \cdot \mathbf{E}) + \left(\frac{\omega^2}{c^2}\right)\mathbf{E} = -\frac{4\pi i \omega}{c^2} \mathbf{J_1} \tag{15}$$

Using Eqs. (9), (10), (11), (13) and (14) in Eq. (15), we get the dielectric tensor:

$$\in = \begin{bmatrix} \in_1 & \in_2 & \in_3 \\ -\in_1 & \in_4 & 0 \\ \in_3 & 0 & \in_5 \end{bmatrix}$$

$$\in_1 = -k_z^2 c^2 + \omega^2 - \frac{\omega_{pe}^2 \omega^2}{\left(\omega^2 - \omega_{ce}^2\right)} - \frac{\gamma_o \omega_{pb}^2 \bar{\omega}^2}{\left(\gamma_o{}^2 \bar{\omega}^2 - \omega_{ce}^2\right)} - \frac{\omega_{pb}^2 k_z^2\ v_{bx}^2}{(\gamma_o + \gamma_1)\bar{\omega}^2}$$

$$\in_2 = \frac{i\, \omega\ \omega_{ce} \omega_{pe}^2}{\left(\omega^2 - \omega_{ce}^2\right)} + \frac{i\, \gamma_o \bar{\omega}\ \omega_{ce} \omega_{pb}^2}{\left(\gamma_o{}^2 \bar{\omega}^2 - \omega_{ce}^2\right)}$$

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$$\epsilon_3 = -\frac{\omega_{pb}^2 \omega k_z \, v_{bx}}{(\gamma_o + \gamma_1)\bar{\omega}^2}$$

$$\epsilon_4 = -k_z^2 c^2 + \omega^2 - \frac{\omega_{pe}^2 \omega^2}{\left(\omega^2 - \omega_{ce}^2\right)} - \frac{\gamma_o \omega_{pb}^2 \bar{\omega}^2}{\left(\gamma_o{}^2 \bar{\omega}^2 - \omega_{ce}^2\right)}$$

$$\epsilon_5 = \omega^2 - \omega_{pe}^2 - \frac{\omega_{pb}^2 \omega^2}{(\gamma_o + \gamma_1)\bar{\omega}^2}$$

where $\omega_{pb}^2 = -\frac{4\pi n_{b0} e^2}{m_e}$, $\omega_{pe}^2 = \frac{4\pi n_{e0} e^2}{m_e}$

The dispersion relation obtained by putting the determinant $|\epsilon| = 0$ i.e. $\epsilon_1 \epsilon_4 + \epsilon_2^2 = 0$ is

$$k_z^4 c^4 - 2k_z^2 c^2 \frac{\omega_{pe}^2 \omega^2}{\omega_{ce}^2} - \frac{\omega_{pe}^4 \omega^2}{\omega_{ce}^2} = \frac{-2\gamma_o \omega_{pb}^2 \bar{\omega}^2}{\left(\gamma_o{}^2 \bar{\omega}^2 - \omega_{ce}^2\right)} \left(k_z^2 c^2 + \omega_{pe}^2 \frac{\omega}{\bar{\omega}}\right) + \left(\frac{\omega_{pe}^2 \omega^2}{\omega_{ce}^2} - k_z^2 c^2\right) k_z^2 v_{bx}^2 \frac{\omega_{pb}^2}{(\gamma_o + \gamma_1)\bar{\omega}^2}$$

In Cerenkov interaction, the growth rate is obtained as

$$\gamma = \mathrm{Im}(\delta) = \frac{\sqrt{3}}{2} \left[\frac{\left(\omega_{pe}^2 + k_z^2 c^2\right)}{2(\gamma_o + \gamma_1)\left(\omega_{pe}^2 + 2k_z^2 c^2\right)} \frac{v_{bx}^2}{c^2} \omega_2 \omega_{pb}^2 \right]^{1/3}. \qquad (16)$$

In slow cyclotron interaction, the growth rate is obtained as

$$\gamma = \left[\frac{\omega_{pb}^2 \omega_2 \gamma_o \omega_{ce}}{2k_z^2 c^2} \right]^{1/2} \qquad (17)$$

III. CONCLUSION

A relativistic electron beam can interact with the whistler waves via Cerenkov and slow cyclotron interaction in infinite geometry. The interaction between the plasma mode and the beam mode is found to excite the whistler waves, and hence the wave grows. In cyclotron interaction, only slow cyclotron interaction is found to excite the whistler waves and provide them a growth. The velocity of the beam for fast cyclotron interaction is such that it cannot interact with the whistler waves. Mathematically also, the growth rate in fast cyclotron interaction is found to be zero. The oblique beam results in growth of whistler mode via slow cyclotron interaction (Ref. Eq. (17)) and via Cerenkov interaction (Ref. Eq. (16)). From Eq. (16), it is observed that as the transverse component of beam velocity increases, the growth rate increases. Also the growth rate increases with an increase in the strength of magnetic field in Cerenkov as well as cyclotron interactions. The growth rate is sensitive to beam velocity in the case of Cerenkov interaction but is quite insensitive in cyclotron interaction.

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