

Type-2 Fuzzy System and Observer Based Controller for Non-linear system

Mr. Rohit B. Mane¹, Ms. Bhagyashri C. Chavan², Mr. Aniket Chavan³

^{1,2,3}Electrical, NMCE (India), ² Electrical, NMCE (India), ³ Electrical Mtech WCE,(India)

ABSTRACT

This paper presents the nonlinear system with time delay. For such nonlinear system, robust control problem is designed. Takagi-Sugeno (T-S) fuzzy models can give a proper analysis of nonlinear systems with time delay. The Type-2 T-S fuzzy model approach is extended to achieve temperature control in continuous time nonlinear systems with time delay. Also disturbance tracking response is checked by using Type-2 Fuzzy DOB technique under internal as well as external disturbances.

Keywords- Type-2 Fuzzy system approach, DOB, Fuzzy DOB

INTRODUCTION

Time delay is present in many systems such as chemical processes, pneumatic processes and hydraulic systems etc. Because of Time delay source has a problem of instability. Stability issue of time delay systems have been studied in past three decades [1-2]. During last few years, fuzzy logic technique has been successfully used for nonlinear system modeling, especially with incomplete plant knowledge for systems [3]. Fuzzy logic system is universal approximators. Takagi-Sugeno (T-S) fuzzy model is used for such approximations [4]. This model used the fuzzy rules to describe a nonlinear system in terms of a set of linear models which is based on fuzzy membership functions. For controller design of complex nonlinear system, T-S fuzzy model based control is used.

Mostly physical systems are nonlinear in nature. Thus, all control systems are nonlinear. Consider if the operating range of a control system is small and the exited nonlinearities are smooth, then the control system may be considered by a linearized system, whose dynamics is expanded by a set of linear differential equations. Time delays have been considered unsuitable in control system, because of its tendency to instability of the system. However, nonlinear systems with time-delay could be represented by a T-S fuzzy time-delay model [2]. Here in this paper, with plant rules we considered a nonlinear time-delay system this could be expanded by the following T-S fuzzy time-delay model. IF $\theta_1(t)$ is μ_{i1} and ... and $\theta_p(t)$ is μ_{ip} THEN

$$\dot{x}(t) = A_{1i}x(t) + A_{2i}x(t - \tau(t)) + B_i u(t)$$

$$y(t) = C_{1i}x(t) + C_{2i}x(t - r(t))$$

$x(t)$

Where μ_i is the fuzzy set, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, where $y(t) \in \mathbb{R}^p$ is the output vector, and $A_{1i}, A_{2i}, B_i, C_{1i}$, and C_{2i} are constant matrices, r is the number of IF-THEN rules, where $\theta(t) = [\theta_1(t) \theta_2(t) \dots \theta_p(t)]$ are the premise variables. It is considered that there is no dependency of premise variables on the input variables. $r(t)$ is the bounded time-varying delay in the state and it is assumed that

\square .

i.e. the time-varying delay functions with derivative, which is a natural supplementary condition. $\psi(t) \in \mathbb{C}$ is a vector-valued initial continuous function. Given a pair of $(x(t), u(t))$, which are the final outputs of the fuzzy systems are inferred as follow

$$\dot{x}(t) = \frac{\sum_{i=1}^r w_i(\theta(t)) [A_{1i}x(t) + A_{2i}x(t-r(t)) + B_i u(t)]}{\sum_{i=1}^r w_i(\theta(t))} \quad (3)$$

$y(t)$

I. FUZZY WITH CONSIDERATION TIME DELAY MODEL OF UNCERTAINTY

Consider the following nonlinear time-delay system:

$$\dot{x}(t) = F(x(t), x(t-d(t))) + \Delta F(x(t), x(t-d(t))) + G(x(t), u(t)) \quad (5)$$

\square .

Where, $x(t) \in \mathbb{R}^n$ which is a state vector, $u(t) \in \mathbb{R}^m$ is the input vector and d is the delay time of the system state with $d(t)$; $F(\cdot), G$ are nonlinear functions, and ΔF is an uncertain nonlinear function. $\varphi(t) \in \mathbb{C}$ is a vector-valued initial continuous function. A fuzzy model is developed [9] to understand input/output relations of nonlinear systems. A TS fuzzy time-delay model is expanded by r plant rules that can be represented as follows [8]. IF θ_1 is μ_{i1} and ... and θ_p is

$$THEN \dot{x}(t) = A_i x(t) + A_{di} x(t-d(t)) + B_i u(t) + \Delta f_i(x(t), x(t-d(t))) \quad (7)$$

Where the fuzzy is set, and A_i, A_{di} are some constant matrices, is the number of IF-THEN rules, and $\theta_j(t) = [\theta_1(t) \theta_2(t) \dots \theta_p]$ are the premise variables. The premise variables that do not depend on the input variables. Considering fuzzy model which is achieved by fuzzy blending of each individual rule as follows:

$$\dot{x}(t) = \sum_{i=1}^r h_i(\theta) \left(A_i x(t) + A_{di} x(t - d(t)) + B_i u(t) + \Delta f_i(x(t), x(t - d(t))) \right) \quad (8)$$

Where $h_i(\theta) = \mu_i(\theta) / \sum_{i=1}^r \mu_i$

(9)

With $\|\delta_i\|$ and

$$v_i(x(t), x(t - d(t))) = a \sin(t) \|x(t)\|^2 + b \cos(t) \|x(t) - d(t)\| \quad (10)$$

II. CSTR

Experimental Setup: CSTR plays important role in various chemical processes. In this first order exothermic reaction $A \rightarrow B$ takes place in which a fluid stream is continuously fed to the reactor [4]. Since the fluid is perfectly mixed, the temperature and concentration of the exit stream is same as the reactor fluid. The jacket which is assumed to have a uniform lower temperature than the reactor. From the reactor into the jacket then energy then passes through the reactor walls removing the heat generated by reaction and tries to maintain the temperature at desired value. When we consider the industrial reactors which are typically more complicated kinetics, but the characteristic behaviour is similar.

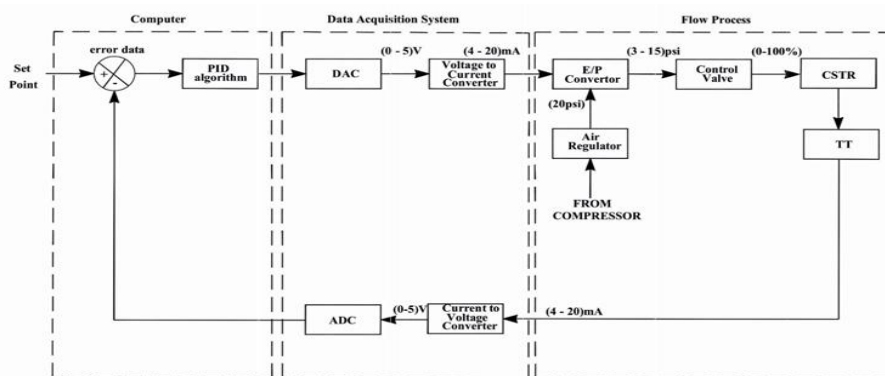


Fig.1:Block diagram of CSTR

By using pade's approximation the second order transfer function is calculated as

$$G(s) = \frac{-0.12s+0.12}{3s^2+4s+1} \quad (11)$$

The state space matrices are given as

$$A = \begin{bmatrix} -1.33 & -0.667 \\ 0.5 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0.25 \\ 0 \end{bmatrix}$$

$$C = [-0.16 \quad 0.32] \quad D = [0] \quad (12)$$

III. ROBUSTNESS OF TYPE-2 FUZZY CONTROLLER

1. INTERVAL TYPE-2 FUZZY CONTROLLER:

During the past decades, various control methods are used in a process control industry [1]. Type-1 (T1) Fuzzy Logic controllers (FLCs) have been widely used as alternatives to conventional controllers. Recently, the main research focus is on Interval Type-2 (IT2) FLCs. In a Type-2 fuzzy set we also have uncertainty about the membership function. Type-1 fuzzy sets has crisp membership function and hence not able to directly model the uncertainties. Whereas type-2 fuzzy sets (T2FS) which are able to modeling the uncertainties because their membership function is themselves fuzzy [7,8]. The respective two dimensional fuzzy sets membership function of type-1 (T1FS) and membership function of type-2 fuzzy sets are three dimensional. The robustness is provided by extra third dimension of type-2 fuzzy sets.

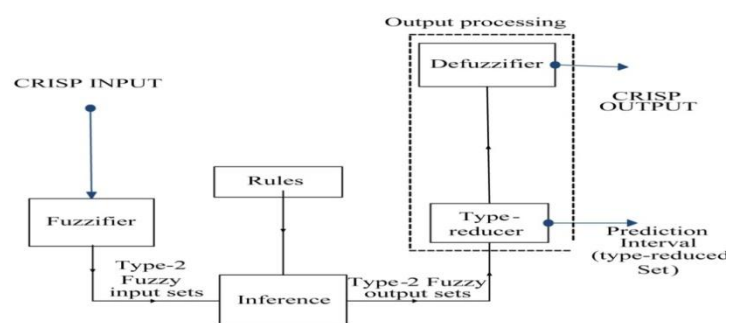


Fig.2: Block diagram of Type-2 Fuzzy

The IT2-FLC with internal structure which is similar to that of Type-1 Fuzzy logic system. The type-2 Fuzzy Yet, IT2-FLCs designed based on an evolutionary algorithms such that to generate a desired FM (i.e. control surface). The main problem with the use of this approach is due to the improper understanding of how the Foot print Of Uncertainty parameters affect the system performance. Type-2 fuzzy logic controllers consist of following 5 parts: rules, fuzzifier, inferences engine, the type reducer, and defuzzifier. These are connected as shown in Fig. as well.

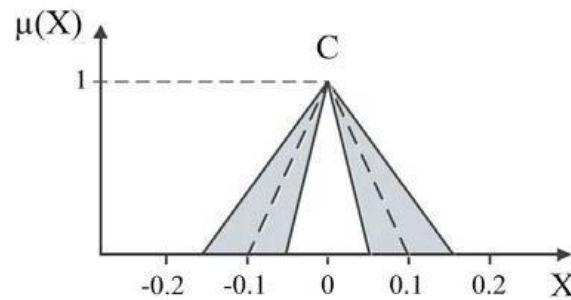


Fig.3: Fuzzy membership function

A higher control gain usually generates a faster tracking response and a less offset in the presence of disturbances. However, it can be observed from figure that rather higher control energy is generally required so as to achieve a faster tracking and disturbance rejection performance. The results of this simulation scenario demonstrate that there exists a contradiction between disturbance rejection and reasonable control energy for the high gain control method.

In many applications a DOB has been used which was proposed by ohnishi. The estimation of system uncertainties as well as external disturbances, and also robustness related to system is achieved by estimating the disturbances in feedback loop [12]. Developing disturbance estimation techniques would be good preferences alleviate the restrictions which are always faced by feed forward control. By nominal plant model achievement of performance goals, an outer-loop controller is designed nonlinear disturbance observer [13]. Disturbance observer-based controls (DOBC) approach for nonlinear systems under disturbances, that is, nonlinear DOBC or NDOBC. Within the NDOBC framework, instead of considering the control problem for a nonlinear system with disturbances as a single one, it is divided into two sub problems, each with its own design objectives.

The first sub problem is the same as the control problem for a nonlinear system without disturbances and its objective is to stabilize the nonlinear plant and obtain desired tracking and/or regulation performance specifications. The second sub problem is disturbance attenuation. A nonlinear disturbance observer is designed to estimate external disturbances and then the estimated value is employed to compensate for the influence of disturbances. DOBC for linear systems has been developed and employed in engineering over three decades. Ohishi et al. pioneered the development of DOBC for motion control systems. After that, DOBC has been employed in many mechatronic systems including disk drivers, machining centres, dc/ac motors, and manipulators. Most of the work on DOBC is engineering-oriented and lacks sound theoretic justification. When an attempt is made to extend DOBC from linear systems to nonlinear systems, this results in a composite controller consisting of a nonlinear controller and a nonlinear disturbance observer.

Developed nonlinear disturbance observer for unknown constant disturbance using Lyapunov stability theory and applied it to a two-link manipulator. The aim is aims to develop a more general framework for

NDOBC and also establish rigorous basis for NDOBC development. First, a general procedure for the design of DOBC for nonlinear systems is presented. This procedure is then applied to the control problem for a nonlinear system subject to disturbances generated by an exogenous system. This kind of disturbances widely exist in engineering systems including unknown load and harmonics, and has been investigated in several linear/nonlinear control approaches, most notably, nonlinear output regulation theory.

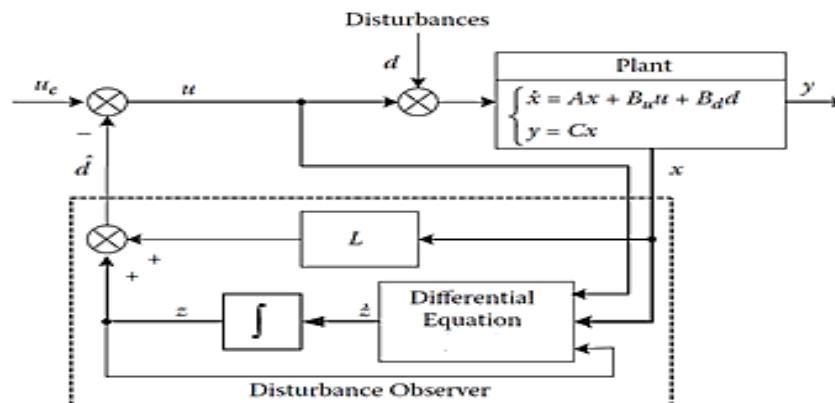


Fig.4: DOB block diagram

The high-order time varying disturbance case, this implies that the disturbance can be expressed in time series expansion. The output design of the nonlinear disturbance observer, which implies that only the output but not the state information, is employed. The method is inspired by the ideas of extended state observer and high-gain observer methods [12]. Time-domain disturbance observer with the above designed parameters has achieved quite fine disturbance estimation performances. Note that the time-domain DO here can be used for both minimum and nonminimum phase MIMO linear systems. However, it requires all the state information for observer design, while the frequency-domain DO only use the output and input information.

$$\hat{d} = z + L * x \tag{13}$$

$$\dot{e}_d = \dot{\hat{d}} - \dot{d} \tag{14}$$

$$\dot{e}_d = -L * B_u * u + B_d * \dot{d} \tag{15}$$

$$\dot{x} = A * x + B_u * u + B_d * \dot{d} \tag{16}$$

$$y = C * x$$

(17)

Where,

L=Disturbance compensation gain

z =internal parameter vector

\hat{d} =Estimated disturbance

\hat{e}_d =Disturbance estimation error

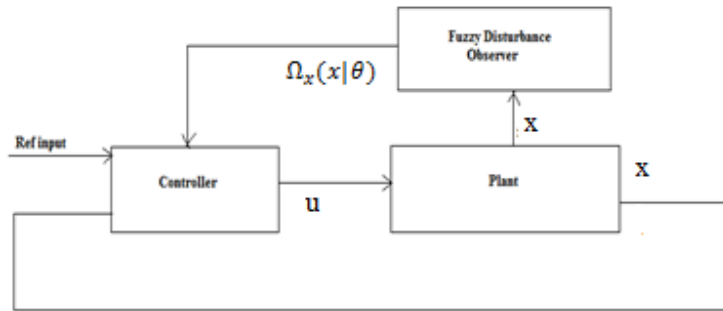


Fig.5:Block diagram of Fuzzy Disturbance Observer

The fuzzy logic system uses the configuration such as fuzzy inference engine, some IF-THEN rules with fuzzy and defuzzifier as well. Here the fuzzy IF-THEN rules are performing in the inference engine to mapping input to output. The output of fuzzy logic system expressed as,

$$y(x) = \frac{\sum_{i=1}^r y^i (\prod_{j=1}^n \mu_{A_j^i}(x_j))}{\sum_{i=1}^r (\prod_{j=1}^n \mu_{A_j^i}(x_j))} \quad (18)$$

$$= \hat{\theta}^T * \xi(x)$$

Where,

$\mu_{A_j^i}(x_j)$ =Fuzzy membership function

$\hat{\theta}^T = (y^1, y^2, \dots, y^r)$ =Adjustable parameter vector

r =Number of fuzzy rules

$x = (x_1, x_2, \dots, x_n)^T$ =Input linguistic vector (19)

$\xi^T = (\xi^1, \xi^2, \dots, \xi^r)$ (20)

$\xi^i = (\prod_{j=1}^n \mu_{A_j^i}(x_j)) / \sum_{i=1}^r (\prod_{j=1}^n \mu_{A_j^i}(x_j))$, are the fuzzy basis functions

IV.SIMULINK RESULTS

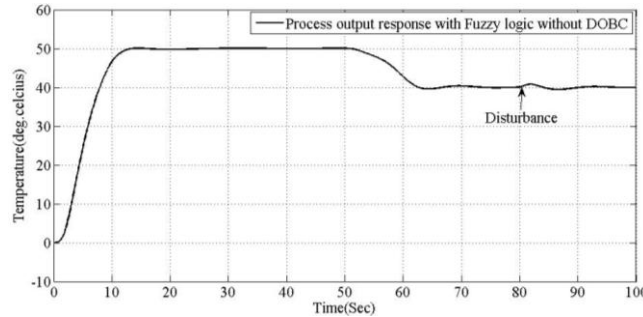


Fig.6: Response of CSTR with IT2 fuzzy controller

Here, the initial temperature is given to CSTR as 50 degree and set point is given as 40 degree. The control signal via such Type- 2 Fuzzy logic systems given to plant at 30 second. Such Simulink response of control action to achieve set point is shown in fig.6. With PI, Fuzzy and TYPE -2 Fuzzy Simulink results are also shown in fig.7.as well.

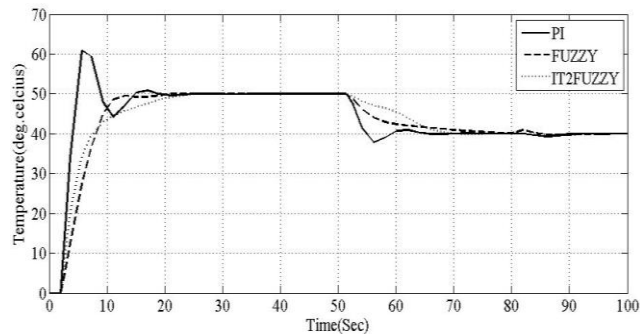


Fig.7: Output process response of CSTR with only Fuzzy logic, Output process response of CSTR with Fuzzy logic and DOBC logic, Output process response of CSTR with only Type-2 Fuzzy logic, Output process response of CSTR with Type-2 Fuzzy logic and DOBC logic

The structure of the type 2 fuzzy logic controller is same as that of structure of the type 1 fuzzy logic controller. The differences between types-2 and 1 are associated only with the nature of the membership function [10].The rules are a main component of the fuzzy logic system. These rules may be defined by a man (expert) or calculated analytically.

$$\bar{A} = \{((x, u), \mu_A(x, u))\} \quad \text{which is also defined as}$$

$$\bar{A} = \int_{x \in X} \int_{u \in U} \mu_A(x, u) / (x, u) \quad \text{with } x \in X$$

$\mu_A(x, u) \in [0,1]$ is a type-2 fuzzy membership function and secondary grad.

J_x is called as primary membership function of x whereas secondary membership function takes as $[0, 1]$ in generalized T2FLSs. The fuzzification which has structure of Type-2 which coincides with type-1 fuzzy rule. By considering M rules, the first rule of rule base has form as following.

The fuzzy rules of the fuzzy dynamic model have the form

$$IF \quad R^1: IF z_1 \text{ is } F_1^1 \text{ and } \dots z_v \text{ is } F_v^1$$

$$THEN \quad x(t + 1) = A_1 x(t) + B_1 u(t) + a_1$$

$$y(t) = C_1 x(t) \tag{20}$$

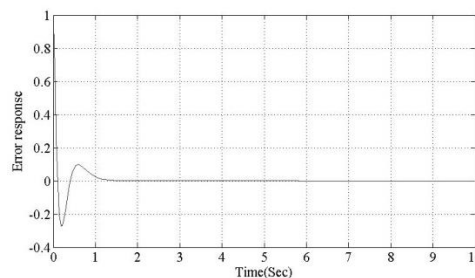


Fig.8: Response of disturbance estimation error

The assumption is considered as disturbance observation error should converge to zero. The objective is to drive x to zero in the presence of uncertainty $\Delta\alpha(x)$ and $\Delta\beta(\cdot)$. Also to track the disturbance which is coming in the system d [10]. The uncertainty and disturbance are added together and given as following,

$$\dot{x}_n = [\alpha(x) + \Delta\beta(x)]u + \pi_x(x, u) \tag{21}$$

Consider nonlinear system which is described as,

$$\dot{x}_n = [\alpha(x) + \Delta\alpha(x)] \dot{+} [\beta(x) + \Delta\beta(x)]u + d \tag{22}$$

$$\begin{aligned} \dot{x}_1 &= x_2 \\ &\vdots \\ \dot{x}_{n-1} &= x_n \end{aligned} \tag{23}$$

$\alpha(\cdot)$ and $\beta(\cdot)$ are the nominal functions

$\Delta\alpha(x)$ and $\Delta\beta(\cdot)$ are the uncertainty, and d is the disturbance

The dynamic system is expressed as,

$$\dot{\mu} = \alpha(x) + \beta(x)u + \hat{\Omega}x(x, u|\hat{\theta}) + \sigma(x_n - \mu) \quad (24)$$

The disturbance estimation error which shown in fig.8, is given as,

$$\xi = x_n - \mu \quad (25)$$

The disturbance estimation error should goes to zero and perfect matching condition occurs when there is no disturbance which may be external or internal. The condition for perfect matching is given as,

$$\hat{\theta} = 0.$$

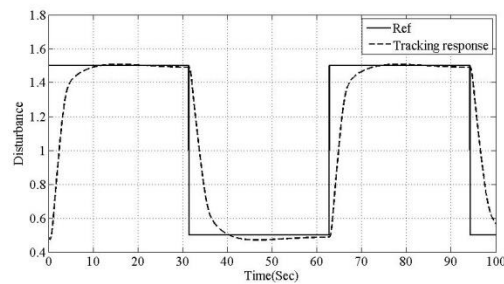


Fig.9: Type-2 Fuzzy DOB disturbance tracking response

Using parameter error $\tilde{\theta} = \theta - \hat{\theta}$ yields,

$$\dot{\xi} = -\sigma\xi + \tilde{\theta}\xi^T(x, u) + \varepsilon(x, u) \quad (26)$$

Fuzzy tuning rules are selected as,

$$\dot{\tilde{\theta}} = \gamma\xi\xi^T(x, u) \quad (27)$$

Assuming Lyapunov candidate function with equation (28) which gives,

$$\dot{V} = \xi\dot{\xi} + \frac{1}{\gamma}\tilde{\theta}^T \left\{ \xi\xi^T(x, u) + \frac{1}{\gamma}\dot{\tilde{\theta}} \right\} + \zeta\varepsilon(x, u) \quad (28)$$

$$\dot{V} = -\sigma\zeta^2 + \zeta\varepsilon(x, u)$$

$$= \frac{\sigma}{2}\zeta^2 + \frac{1}{2\sigma}\varepsilon^2(x, u) - \left(\sqrt{\frac{\sigma}{2}}\zeta - \sqrt{\frac{1}{2\sigma}}\varepsilon(x, u) \right)^2 \leq -\sqrt{\frac{\sigma}{2}}\xi^2 + \frac{1}{2\sigma}\varepsilon^2(x, u) \quad (29)$$

Under consideration of Type-2 Fuzzy DOB gives arbitrary closeness to nonlinear function as to monitor unexpectedly occurring disturbances. As V is negative for $|\xi| \geq \frac{\epsilon}{\sigma}$ then assumption considered as $\hat{\theta}$ is bounded, that is disturbance observer error is uniformly ultimately bounded [10, 13].

The designed FDO $\hat{\Omega}(x, u|\hat{\theta})$ is tuned in such way that it approaches unknown disturbance in the system. Such tuning parameter θ is adjusted with consideration of fuzzy basis function with respect to disturbance estimation error which is shown in equation (29) and such tracking response and for random data tracking response is shown in fig.9 and fig.10 respectively in above diagrams.

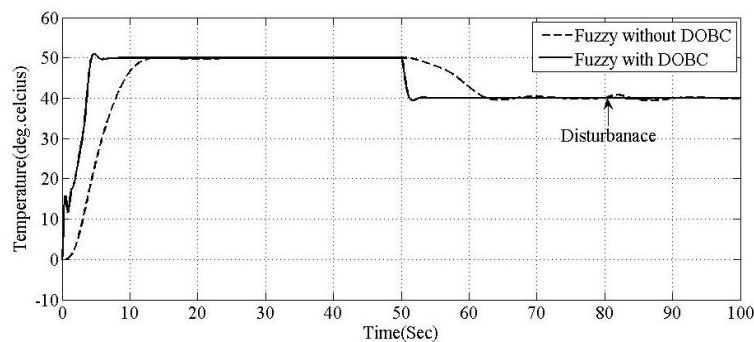


Fig.10 Tracking response (With Fuzzy+ DOBC)

All the simulation results shows the effectiveness of type-2 FLC in achieving a very high control performance and allowing a faster and more precise control of the process, both for set point tracking and disturbance rejection with less amount overshoot compared to type-1 controller. The rise time and settling time in both simulation and real time study reduces in interval type-2 fuzzy logic controller and considering DOB techniques as well.

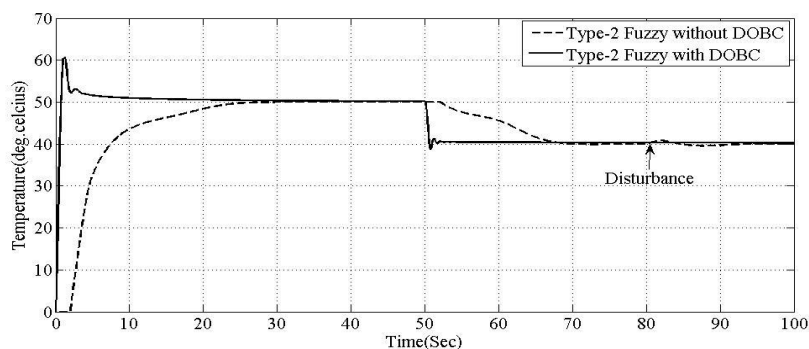


Fig.11 Tracking response (With Fuzzy+ DOBC)

This observer shows a much faster convergence rate than other types of disturbance observers. The observer gain is selected such that $-L * B_d$ is Hurwitz. The eigenvalues of matrix $-L * B_d$ are calculated as $\lambda_1 = -101.3836$ and $\lambda_2 = -30.6164$, which implies that matrix, $-L * B_d$ is Hurwitz and satisfies the design. In this system the step disturbance is added to the system and tracking response is observed, which is shown in fig.9.

$$\dot{z} = -L * B_d(z + L * x) - L(A * x + B_u * u) \tag{30}$$

Table 1. Different Fuzzy Controller with Tr & Ts

Sr no.	Controller	Tr (Rise time)	Ts (Settling Time)
1	With only Fuzzy controller	8	18
2	With Fuzzy +DOBC controller	2	8
3	With Type-2 fuzzy controller	5	12
4	With Type-2 Fuzzy+DOBC controller	1.5	5

V.CONCLUSION

In this paper, T-S Fuzzy technique is used to study the control problem of nonlinear time-delay system. In the presence of parameter perturbations and external disturbances stabilization problem for uncertain T-S fuzzy time-delay systems are studied. Various control methods are used to achieve the disturbance tracking responses. In this paper two different methods are studied ,where Type -2 Fuzzy system applied for achieving the temperature control in CSTR and disturbance tracking in the system is achieved by using DOB,Type-2 Fuzzy DOB as well and compared their results. Also an internal and external disturbance which determinates the system performance can also be brought nearer to perfect matching condition with disturbance observer control scheme as well. For this, CSTR example is used to validate the results and effectiveness with the help of MATLAB Simulink model.

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