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An Efficient Method For Solving Assignment Problems

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ABSTRACT

Assignment problems have various applications in the real world because of their wide applicability in industry, commerce, management science etc. In the literature, there are various methods to solve assignment problems in which parameters are represented by triangular or trapezoidal fuzzy numbers. In this paper, the fuzzy assignment problem is formulated to crisp assignment problem using ranking technique of orthocentre of centroid [14]. The numerical example shows that this ranking is a better technique for handling the fuzzy assignment problem.

***Keywords*: Triangular fuzzy numbers, Assignment Problem, Ranking function.**

I.INTRODUCTION

An assignment problem is a special type of linear programming problem where the objective is to assign ***n*** number of persons to ***n*** jobs at a minimum cost (time). Zadeh [17] introduced the concept of fuzzy sets to deal with imprecision, vagueness in real life situations. Since, then tremendous efforts have been spent, significant advances have been made on the development of numerous methodologies and applications to various decision problems. Fuzzy assignment problems have received great attention in recent years.

The term AP (Assignment Problem) first appeared in Votaw and Orden [15]. Hungarian method by Kuhn [10] is widely used for the solution to AP's. Chen [2] proposed a fuzzy assignment model that did not consider the differences of individuals and also proved some theorems. Wang [16] solved a similar model by graph theory.

Lin and Wen [11] investigated a fuzzy AP in which the cost depends on the quality of the job. Dubois and Fortemps [3] proposed a flexible AP which combines with fuzzy theory, multiple criteria decision – making and constraint- directed methodology. Huang and Xu [4] proposed a solution procedure for the APs with restriction of qualification. Mukherjee and Basu [12] proposed a new method for solving fuzzy assignment problems. Kumar et al [9] proposed a method to solve the fuzzy APs, occurring in real life situations.

Kumar and Gupta [7] proposed methods for solving fuzzy AP's with different membership functions . Kumar and Gupta [8] proposed two new methods for solving fuzzy APs and fuzzy travelling salesman problems. Kalaiarasi et al. [5] proposed a fuzzy assignmen model with TFN using Robust ranking technique. Singh and Thakur [13] compared the assignment cost calculated by existing method with the assignment cost which has been found without converting fuzzy assignment problem into crisp AP. Arokiamary and Nithya [1] proposed revised ones assignment method is used to solve the fuzzy assignment problem to find maximum and minimum objective fuction using hexagonal fuzzy number.

II.PRELIMINARIES

In this section, we briefly review some basic concepts of fuzzy numbers and the existing method of ranking fuzzy number.

2.1 Basic Definitions

In this section some basic definitions are reviewed.

Definition 2.1 [6] The characterstic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e $\mu_{\tilde{A}} : X \to [0,1]$. The assigned value indicate the membership grade of the element in the set A.

The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called fuzzy set.

Definition 2.2 [6] A fuzzy set $\tilde{A}$ defined on the universal set of real numbers *R*, is said to be a fuzzy number if its membership function has following characterstics:

(i) $\tilde{A}$ is a convex i.e. $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \text{minimum}(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$ $\forall\, x_1, x_2 \in R$, $\lambda \in [0,1]$.

(ii) $\tilde{A}$ is normal i.e. $\exists\, x_0 \in R$ such that $\mu_{\tilde{A}}(x_0) = 1$.

(iii) $\mu_{\tilde{A}}$ is piecewise continuous.

Definition 2.3[6] A fuzzy number $\tilde{A}$ defined on the universal set of real numbers *R*, denoted as $\tilde{A}$ = (a, b, c), is said to be a triangular fuzzy number if its membership function, $\mu_{\tilde{A}}(x)$, is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)}, & a \le x < b \\ 1, & x = b \\ \dfrac{(x-c)}{(b-c)}, & b < x \le c \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.4 [6] A ranking function is a function *R*: $F(R) \to R$ where F(*R*) is a set of fuzzy numbers defined on the set of real numbers (*R*) which maps each fuzzy number into the real line.

2.2 Existing Ranking Method [14]

In this section the existing ranking method [14] is explained. The existing ranking method based on area, mode, spreads & weights of generalised (non-normal) fuzzy numbers.

The ranking function of the triangular fuzzy no. $\tilde{A}$ = (a,b,c) which maps the set of all fuzzy number to a set of real numbers is defined as

$$R(\tilde{A}) = b(\frac{(b-a)(c-b)+1}{3})$$

mode(m) of triangular fuzzy no. $\tilde{A}$ = (a,b,c) is defined as

$$m = b$$

spread (s) of triangular fuzzy no. $\tilde{A}$ = (a,b,c) is defined as

$$s = (c-a)$$

Left spread (ls) of triangular fuzzy no. $\tilde{A}$ = (a,b,c) is defined as

$$l\,s = (b-a)$$

Right spread (rs) of triangular fuzzy number $\tilde{A}$ = (a,b,c) is defined as

$$r\,s = (c-b)$$

Using the above definitions the ordering of triangular fuzzy numbers can be done.

III. FUZZY ASSIGNMENT PROBLEM

Suppose there are '*n*' jobs to be performed and '*n*' persons are available for doing these jobs. Assume that each person can do one job at a time and each job can be assigned to one person only. Let $\tilde{c}_{ij}$ be the appropriate cost if i^{th} job is assigned to j^{th} person. The problem is to find an assignment x_{ij} (which job should be assigned to which person) so that the approximate total cost for performing all jobs is minimum. The above problem may be formulated as follows

Minimize $\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij}$

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$$\sum_{i=1}^{n} x_{ij} = 1,\ j = 1, 2, 3, \ldots\ldots, n$$
$$\sum_{j=1}^{n} x_{ij} = 1, i = 1, 2, 3, \ldots\ldots., n \quad , \quad x_{ij} = 0 \text{ or } 1$$

IV. PROPOSED METHOD

In this section, a new method is proposed to find the fuzzy optimal solution of fuzzy assignment problems by using existing ranking technique [14]

The steps of proposed algorithm are as follows:

Step 1 - The fuzzy assignment problem is

Minimize $$\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$$

Subject to

$$\sum_{i=1}^{n} x_{ij} = 1,\ j = 1, 2, 3, \ldots\ldots, n$$
$$\sum_{j=1}^{n} x_{ij} = 1, i = 1, 2, 3, \ldots\ldots., n \quad , \ x_{ij} = 0 \text{ or } 1$$

where $\tilde{c}_{ij}$ is fuzzy cost

Step 2 - Convert the fuzzy linear programming problem, obtained in step 1, into crisp linear programming problem by using existing ranking technique [14]

Minimize $$R(\sum_{i=1}^{n}\sum_{j-1}^{n} \tilde{c}_{ij} x_{ij})$$

Subject to

$$\sum_{i=1}^{n} x_{ij} = 1,\ j = 1,2,3,......,n$$
$$\sum_{j=1}^{n} x_{ij} = 1, i = 1,2,3,.......,n \quad ,\ x_{ij} = 0 \text{ or } 1$$

Step 3 Find the optimal solution x_{ij} by solving the crisp linear programming problem, obtained in Step 2.

Step 4 Find the minimum total fuzzy cost by putting the values of x_{ij} in $\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij}$

Step5 - Let using previous steps p different triangular fuzzy numbers $\tilde{c}_1, \tilde{c}_2, ..., \tilde{c}_p$ representing the optimal fuzzy cost be obtained i.e $\tilde{c}_1 \neq \tilde{c}_2 \neq ... \neq \tilde{c}_p$ but $R(\tilde{c}_1) = R(\tilde{c}_2) = ... = R(\tilde{c}_p) = u_1(say)$. Then find the optimal solution of the following crisp linear programming problem.

Minimize mode $(\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij})$

Subject to $R(\sum_{i=1}^{n}\sum_{j-1}^{n}\tilde{c}_{ij}x_{ij}) = u_1$

$$\sum_{i=1}^{n} x_{ij} = 1,\ j = 1,2,3,......,n$$
$$\sum_{j=1}^{n} x_{ij} = 1, i = 1,2,3,.......,n \quad ,\ x_{ij} = 0 \text{ or } 1$$

Case(i)- If by putting the obtained optimal values of x_{ij} in $(\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij})$ a unique triangular fuzzy number, representing the optimal fuzzy cost, is obtained then the obtained triangular fuzzy number will represent the optimal fuzzy cost .

Case(ii)- If more than one triangular fuzzy numbers, representing the optimal fuzzy cost, are obtained then go to Step 4.

Step6- Let using previous steps '*l*' triangular fuzzy numbers $\tilde{c}_1^{'}, \tilde{c}_2^{'},...,\tilde{c}_l^{'}$ obtained i.e. $\tilde{c}_1^{'} \neq \tilde{c}_2^{'} \neq ... \neq \tilde{c}_l^{'}$ but $R(\tilde{c}'_1) = R(\tilde{c}'_2) = ... = R(\tilde{c}'_l) = u_1$ (say) and mode ($\tilde{c}_1^{'}$)= mode ($\tilde{c}_2^{'}$) = ...= mode ($\tilde{c}_l^{'}$) = u_2 (say). Then, find the optimal solution of the following crisp linear programming problem.

Minimize spread $(\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij})$

Subject to

$$R(\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij}) = u_1$$

$$\text{mode}(\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij}) = u_2$$

$$\sum_{i=1}^{n} x_{ij} = 1,\ j = 1, 2, 3, ..., n$$

$$\sum_{j=1}^{n} x_{ij} = 1, i = 1, 2, 3, ..., n$$

$$x_{ij} = 0 \text{ or } 1.$$

Numerical Example 1

Consider an Fuzzy Assignment Problem of assigning '*n*' jobs to '*n*' machines (one job to one machine). Let $\tilde{c}_{ij}$ be the cost matrix whose elements are triangular fuzzy number. The problem is to find optimal assignment so that the total cost of job assignment becomes minimum.

		J_1	J_2
$\tilde{c}_{ij} =$	A	(11,12,14)	(21,21.5,22)
	B	(56,56.5,58)	(13,14,16)

Solution :

Step 1 - In the conformation to model , the fuzzy assignment problem can be formulated in the following mathematical programming form:

$$Min\ [\ (11,12,14)x_{11} + (21,21.5,22)x_{12} + (56,56.5,58)x_{21} + (13,14,16)x_{22}\ ]$$

subject to

$$x_{11} + x_{12} = 1$$
$$x_{21} + x_{22} = 1$$
$$x_{11} + x_{21} = 1$$
$$x_{12} + x_{22} = 1$$

$$x_{ij} = 0 \text{ or } 1\ \forall i, j$$

Step 2 - Convert the fuzzy linear programming problem into crisp linear programming problem by using existing ranking technique [14]

$$Min\ [(11,12,14)x_{11} + (21,21.5,22)x_{12} + (56,56.5,58)x_{21} + (13,14,16)x_{22}]$$

Subject to

$$x_{11} + x_{12} = 1$$
$$x_{21} + x_{22} = 1$$
$$x_{11} + x_{21} = 1$$
$$x_{12} + x_{22} = 1$$

$$x_{ij} = 0 \text{ or } 1\ \forall i, j$$

Step3- The two optimal solutions are obtained (i) $x_{11} = x_{22} = 1$ and $x_{12} = x_{21} = 0$, (ii) $x_{11} = x_{22} = 0$ and $x_{12} = x_{21} = 1$

Step4- Putting the optimal values of x_{ij}, obtained in Step 3, in $[(11,12,14)x_{11} + (21,21.5,22)x_{12} + (56,56.5,58)x_{21} + (13,14,16)x_{22}]$ the optimal fuzzy cost are (24,26,30) and (77,78,80) respectively.

Step5- *R*(24,26,30) = *R*(77,78,80) = 78 but (24,26,30) $\neq$ (77,78,80) Then find the optimal solution of the following crisp linear programming problem.

Minimize

$$\text{mode}\,[(11,12,14)x_{11} + (21,21.5,22)x_{12} + (56,56.5,58)x_{21} + (13,14,16)x_{22}]$$

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$R[(11,12,14)x_{11}+(21,21.5,22)x_{12}+(56,56.5,58)x_{21}+(13,14,16)x_{22}]$ =78

$$x_{11}+x_{12}=1$$
$$x_{21}+x_{22}=1$$
$$x_{11}+x_{21}=1$$
$$x_{12}+x_{22}=1$$
$$x_{ij}=0 \text{ or } 1 \forall i,j$$

Then obtained optimum values are $x_{11}=x_{22}=1$ and $x_{12}=x_{21}=0$ so $A \to J_1, B \to J_2$.

By putting values of x_{ij} in

$[(11,12,14)x_{11}+(21,21.5,22)x_{12}+(56,56.5,58)x_{21}+(13,14,16)x_{22}]$,

a unique triangular fuzzy number (24,26, 30) representing optimal fuzzy cost is obtained.

Numerical Example 2

Three Persons P_1, P_2, P_3 are available to do three different jobs J_1, J_2, J_3. From past records the cost (in dollars) that each person takes to do each job is known and are represented by triangular fuzzy numbers and are shown in Table 2. Find an assignment x_{ij} (which job should be assigned to which person) so that the approximate total cost for performing all the jobs is minimum.

Table 2 – Fuzzy Cost (in dollars)

Person(P) → Job(J) ↓	P_1	P_2	P_3
J_1	(8,8.2,11)	(10,11,12)	(6,6.7,7)
J_2	(4,5,7)	(7,10.8,13)	(16,16.5,17)

J_3	(10,12.5,14)	(14,14.3,16)	(20,22,24)

Solution-

Now by using the proposed method the optimal values

$x_{11} = x_{12} = x_{22} = x_{23} = x_{31} = x_{33} = 0\ \&\ x_{13} = x_{21} = x_{32} = 1$ are obtained.

So $J_1 \rightarrow P_3, J_2 \rightarrow P_1, J_3 \rightarrow P_2$. After putting the values of x_{ij} in objective function, the optimal fuzzy cost (24,26,30) is obtained.

V. ADVANTAGES OF PROPOSED METHOD

In this section advantages of the proposed method over existing methods are shown.

In the above Examples it has shown that two different triangular fuzzy numbers representing optimal fuzzy cost, are obtained and their rank is same. But we cannot say that if rank of two different triangular fuzzy number is same i.e. both triangular fuzzy numbers are equal. Because different fuzzy numbers have different physical interpretation. So by using proposed method a unique triangular fuzzy number representing optimal fuzzy cost will be obtained.

VI.CONCLUSION

A new method is proposed to find the fuzzy optimal solution of fuzzy assignment problems. Since proposed method is based on existing ranking technique [14]. So it is easy to learn and apply in real life situations. The main advantage of proposed method is that a unique triangular fuzzy number, representing fuzzy optimal cost of assignment problem is obtained.

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